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Some Graph Theoretical Properties in the Traditional Construction of Wells a case study of Dukkey wells in Yankari, Bauchi State.

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ABSTRACT
Local wells are constructed to provide good drinking water and some times served as a hiding place for enemies during wars. The application of mathematical concepts involved in the construction is neglected by our comtemporary Mathematicians. This Paper investigates the 99 – Dukkey Wells constructed at the Yankari Game reserved Area of Bauchi State and apply the concept in graph structure to provide some interesting graph theoretical models and structures. The research further examine the graph structures formed which led to obtain some algebraic theoretic properties of graphs which has application in network topology.
KEYWORDS: *Sub graph, complete graph, Integers, Vertices and Edges.*

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I. INTRODUCTION:

Local Wells are constructed with the aim of providing drinking Water to the local inhabitants of a particular area . This is done through construction or digging of holes at a required measurement (in meters inc). This is mostly common in the Northern part of this country(Ibrahim, Dogondaji and Isah, 2002).

The 99 – Dukkey wells are local wells constructed by a Man called Dukkey in the Yankari Game reserved in the 19th century in his attempt to provide drinking water to his people and also to serve as a hiding area or place in case of enemies attack during Fight or War. He then constructed 99 – local wells which were connected together in batches or group so that water can flow from one well to another under the ground level.

The structure of the local wells as constructed can be modeled using the concept of Graph. In the context of Algebra structure, the motion of graph represents a set V(G) of vertices and E(G) of edges connecting such vertices (Bondy and Murty, 1978). Figure I shows a graph models from a group of six dukkey wells structure with properties as shown below.

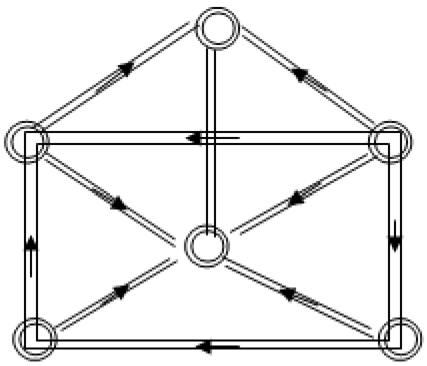
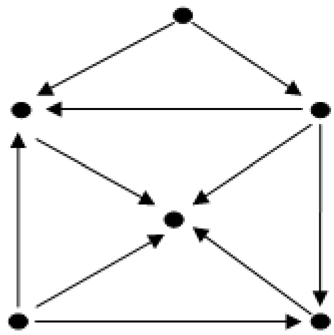


Fig (i) Shows how water flows From one well to another



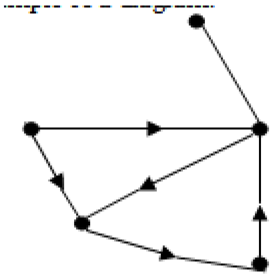
Fig(ii) Shows similar structure using single lines

NOTATIONS

For the better understanding of the paper, we consider the following basic notation.

A Graph: is a Mathematical structure comprising a set of vertices callednodes or points and set of edges E, (sides) which connects the vertices.It is donated by G(V, E) and the order of any graph is designed by the number of vertices in that graph. Thus, for an n – vertex Graph G(G) = n (1)

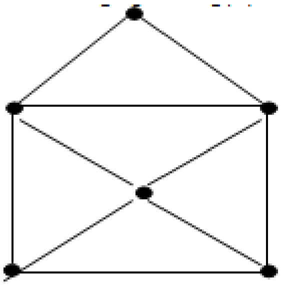
Diagraph: A diagraph is a graph consisting of points called vertices joined by directed lines called arcs (Biggs 1993)
Fig(iii) Shows an example of a diagram.



Fig(iii) diagram

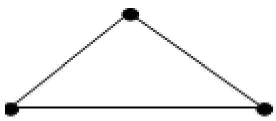
Sub graph: A Graph H is a sub graph of G (written H $\subseteq$ G) if V(H) $\subseteq$ V(G) E(H) $\subseteq$ E(G). V(H) is the restriction of V(G) of E(H). when H $\subseteq$ G but H $\neq$ G we write H $\subset$ G. and call H proper sub graph of G.

If H is a sub graph of G, G is a super graph of H. A spanning sub graph (spanning super-graph of G is a sub graph or (Super sub graph) H with V(H) $\subseteq$ V(G). consider the graph in Fig(iv) below showing different vertices and edges similar to the graph in fig(iii) above.



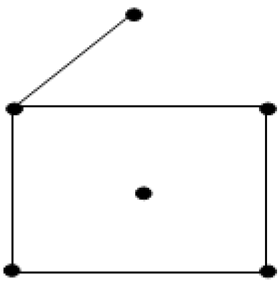
Fig(iv) A graph

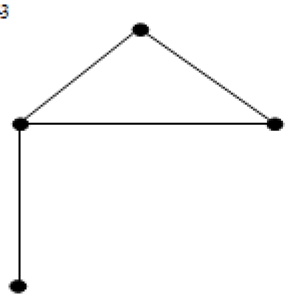
If we consider the graph of fig(iv) above the following are obtained:



Fi(v) Complete graph

K_3





Fig(vi): Spanning sub graph of G Kn = 5

Fig(vii) G – (v, w) Kn = 4

Some special properties of a graph obtained

Complete Graphs
Fig (iv) has one complete sub graphs of order 3. that is K_3, the rest (fig(vi) and (vii)) are how ever not a complete graphs in fact, using other construction models figure (vi) can also be broken into five K_3 complete graphs,

Walk: A walk of length of the G is an alternative sequence of vertices and edges joining two vertices.

Trail: A trail is a walk in which all edges (but nit necessaries all the vertices are distinct)

Path: A path is a walk in which all the vertices and all edges are distinct A labeled Graph is a graph whose vertices have names or labels.
Order of a Graph: This is the number of vertices in G.
Size of Graph: This is the number of edges in G.
Degree: The degree of vertices V is the number of edges incident with V loops count as 2 and denoted by d(E) and deg(E) and a loop is an edge joining vertices to itself (pertesan, 1989)

VARIETIES OF GRAPH
There are different graph varies but few to discus include:
a. **Peterson Graph(1989)**
The Peterson graph is represented in Fig(viii) below

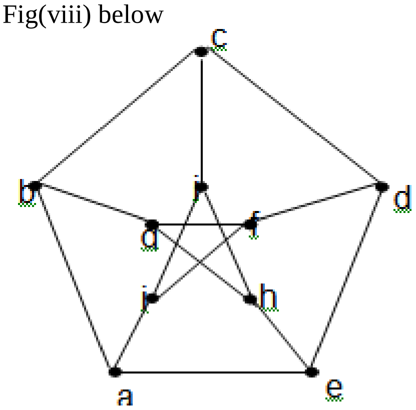


Fig (viii) Peterson Graph.

a. **Moore graph:**
The moore graph is represented in Fig(ix) below

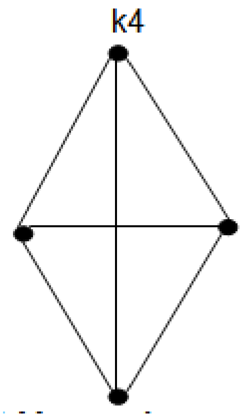


Fig (ix) Moore graph

A + n = 4

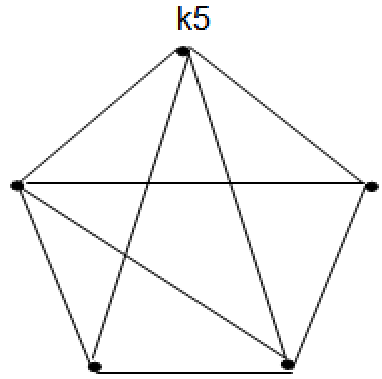


Fig (X) A + n = 5

b. Weightful graph

A typical weightful graph is shown in Fig(x) below. A weight graph is a graph in which each edge has a weight associated with it (J.A Bardy 1976)

THE DUKKEY WELLS

We shall provide here algorithm for the construction of graph in relation to Dukkey wells.

algorithm

1. For a graph $G_{n,\ n=1,\dots 6}$ do
2. $\alpha_i\ \alpha_{i+1} \in E(G)$
3. Repeat until i = 5
4. End if
5. End

We now construct a graph model for the Dukkey wells as follows:

See fig. below.

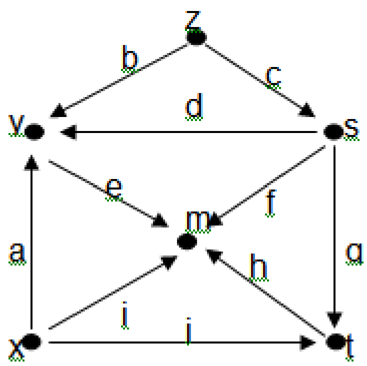
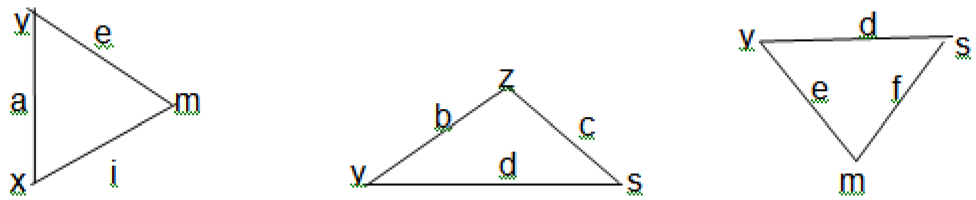


Fig (xi)

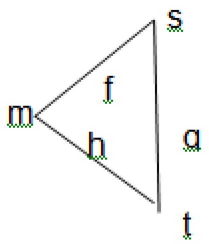
Graph Varieties obtained from Dukkey well

i. Sub graph

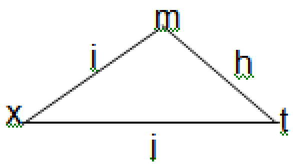
The following represents five complete sub graph of fig(xi)



CG 1.1 CG 1.2 CG 1.3



CG 1.4



CG. 1.5

PROPERTIES

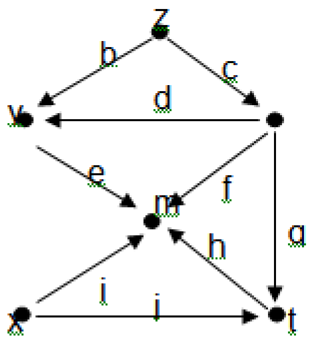
Among the properties of these sub graph is that: Each is regular of degree thus; if direction are not considered it follows that from each one of the three vertices of these sub graphs, there are arcs (edges) linkages up the remaining two. Each sub graph is a connected graph.

ii. Unconnected Sub graph

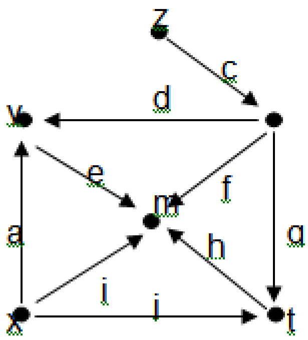
Since the graph in fig(xi) has 10 edges. It follows that: there are 9 sub graphs each unconnected which can be obtained from the same graph.

Note: smaller sub graphs can also be constructed using each of G_i, i = 1, 2, … 9.

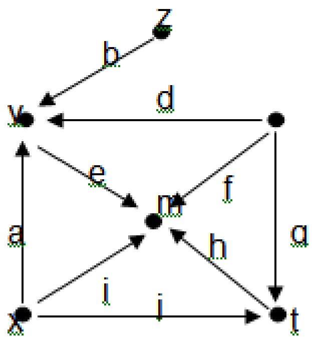
iii. Spanning Sub Graph Of G



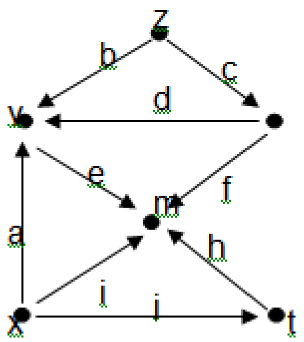
G1



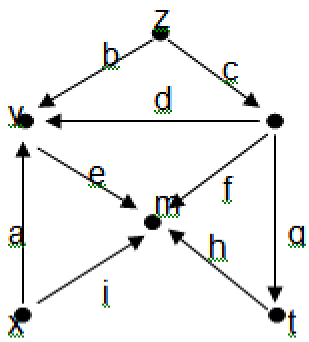
G2



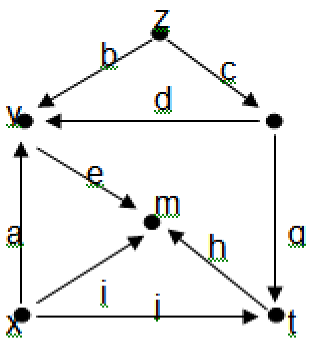
G3



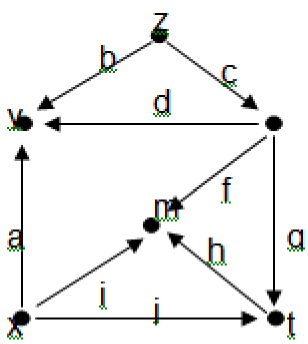
G4



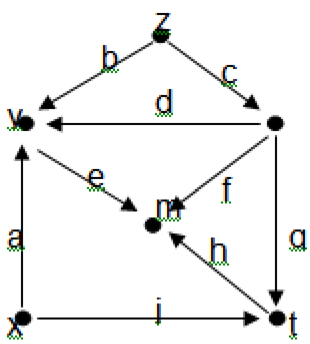
G5



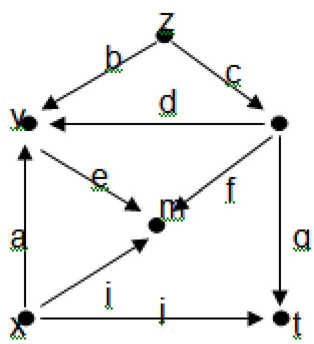
G6



G7



G8



G9

There are nine spanning sub graph of G in Fig(xi)

PROPERTIES

Since directions have been assigned, each G_i, i = 1, 2, … 9. forms walk whose length is determined first by the nature of the graph as well, its relationship with G . The resulting walk of the Spanning sub-graph are as follows:

WALK, TRAIL AND PATH G1 TO G9
Let E(G) = [xy, yz, zs, st, tm, mx, ym, ms, sy, ym, mx, xt, tm]
V(G) = [x, y, z, s, t, m]
Walk (G) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I, x,t, m,I,]
Path (G) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,h, m,I, x,t, x,t,]
Trail(G) = [xa, y,b, z,c, s,d, m,f, t,h]

G1
E(G1) = [yz, zs, st, tm, mx, ym, ms, sy, ym, mx, xt, tm]
V(G1) = [x, y, z, s, t, m]
Walk (G1) = [y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I, x,j, t,h, m,I,]
Trail(G1) = [y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I, x,j,]
Path(G1) = [y,b, z,c, s,d, m,f, t,h x,j]

G2
E(G2) = [xy, zs, st, tm, mx, ym, ms, sy, ym, mx, xt, tm]
V(G2) = [x, y, z, s, t, m]
Walk (G2) = [xa, z,c, s,d, y,e, m,f, s,g, t,h, m,I, x,j, t,h, m,I,]
Trail (G2) = [xa, z,c, s,d, y,e, m,f, s,g, t,h, m,I, x,j,]
Path(G2) = [xa, z,c, s,d, y,e m,f, s,g t,h]

G3
E(G3) = [xy, yz, zs, st, tm, mx, ym, ms, sy, ym, mx, xt, tm]
V(G3) = [x, y, z, s, t, m]
Walk (G3) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I, x,j, t,h m,I,]
Trail (G3) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I, x,j,]
Path(G3) = [xa, y,b, s,d, m,f, t,h]

G4
E(G4) = [xy, yz, tm, mx, ym, ms, sy, ym, mx, xt, tm]
V(G4) = [x, y, z, s, t, m]
Walk (G4) = [xa, y,b, s,d, y,e, m,f, , t,h, m,I, x,j, t,h m,I,]
Trail (G4) = [xa, y,b, s,d, y,e, m,f, t,h, m,I, x,j,]
Path(G4) = [xa, y,b, s,d, m,f, t,h]
G5
E(G5) = [xy, yz, zs, tm, mx, ym, mx, tm]
V(G5) = [x, y, z, s, t, m]
Walk (G5) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I,]
Trail (G5) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I,]
Path(G5) = [xa, y,b, z,c, s,d, y,e m,f, s,g]

G6
E(G6) = [xy, yz, zs, , tm, mx, vm, mx, tm]
V(G6) = [x, y, z, t, m]
Walk (G6) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h, m,I,]
Trail (G6) = [xa, y,b, z,c, s,d, y,e, m,f, s,g, t,h,]
Path(G6) = [xa, y,b, z,c, s,d, m,f, t,h]

G7
E(G7) = [xy, yz, tm, mx, ym, mx, tm]
V(G7) = [x, y, z, s, t, m]
Walk (G7) = [xa, y,b, z,c, s,d, m,f, s,g, t,h, m,I,]

Trail (G7) = [xa, y,b, z,c, s,d, m,f, s,g, t,h,]
Path(G7) = [xa, y,b, z,c, s,d, m,t, t,h]

G8
E(G8) = [xy, yz, zs, tm, mx, ms, st, t,m]
V(G8) = [x, y, z, s, t, m]
Walk (G8) = [xa, y,b, z,c, s,d, m,f, s,g, t,h,]
Trail (G8) = [xa, y,b, z,c, s,d, m,f, s,g, t,h,]
Path(G8) = [xa, y,b, z,c, s,d, m,f, t,h]

G9
E(G9) = [xy, yz, zs, tm, mx, ym, ms, st, tm]
V(G9) = [x, y, z, s, t, m]
Walk (G9) = [xa, y,b, z,c, s,d, y,e, m,t, s,g, m,I,]
Trail (G9) = [xa, y,b, z,c, s,d, y,e, m,f, m,I]
Path(G9) = [xa, y,b, s,d, m,f,]

II. CONCLUSION

Different properties of Graph Structural Model were identified in relation to the Construction of the Dukkey Wells as well as in the configuration set up of the 99 – structure construct. These were studied as blocks, each having five pits. The accompanying graph theoretical properties tally significantly with that of Petersen and Moore graph discovered by Petersen, in 1998.

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