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An Analysis Of The Concept Of Exponential Functions In History And Textbooks In Vietnam

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ABSTRACT
In general, a mathematical object's appearance and the institution that teaches it because of specific educational requirements differ significantly. This study aims to clarify the characteristics of exponential functions in history; then, it is compared in the teaching scope in Vietnam's math textbooks. The method of analyzing and synthesizing scientific documents, books, and articles is used to make essential comments on features such as the scope of use, problems, and arising conditions of functions in its history and Vietnamese textbooks. Analysis results have shown a disparity in these characteristics of exponential functions in two selected ranges. Finally, there are some pedagogical approaches to exponential functions to teach them historical respect and show a mathematical connection with other subjects.
Keywords: *concept, exponential function, characteristics, problem, mathematics education.*

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I. INTRODUCTION

Each mathematical concept arises from its history and has its characteristics, such as the scope of use, emerging problems and conditions, and contexts. Historically, the exponential function is widely used in Physics, Chemistry, and biology as a high school tool. At the tertiary level, a function $y = e^x$ - the particular case of exponential functions - plays a vital role in the theory of differential equations. Exponential functions have many different applications, such as predicting population growth, computation in astronomy, economics, finance and banking, radioactive decay, and heat reduction of objects in the natural environment, especially modeling the above phenomena. Therefore, the selection of the exponential function to study is appropriate, meeting the three problems raised by the program as above. Regarding the definition of an exponential function, there are many different processes. In which the concept of power still plays a crucial role in the description of exponential functions. The exponential definition's first process is strongly influenced by the exponential expansion of powers with positive, zero, negative, rational, and real integers. For example, an exponent is defined as an inverse function of the logarithmic function, and so the logarithmic function is an inverse function of the exponent. The expansion of the concept of powers can be done through exponential functions. There are some studies associated with exponential functions:

In his idea of the exponential law before 1900 (1978), Lorenzo J. Curtis presented the seeds for exponential formation that have been around since Babylon, 2000 BC. It is a matter of interest rates on deposits. The document shows how to perform this calculation by a table of interest rates that are predetermined at equal predetermined intervals. At any given time, interest is calculated based on interpolation. Lorenzo J. Curtis also points to another scientific evidence for exponentials that was set around 1650 BC. It is evidence of the emergence of computational needs for exponentially increasing phenomena. An exponential law (the law of increasing, decreasing in quantity over time) is gradually formed human understanding. However, the purpose of its appearance is to serve the computational needs of human life (in the form of puzzles), which has not become a study of mathematics. The solution to this problem was presented in the early 20th century by mathematician and scientific historian Neugebauer using a small number of addition and multiplication operations. Besides, Lorenzo J. Curtis also showed that exponential functions are discovered indirectly through related studies, such as in construction relics in geometry.

Further work, Napier's superior construction of the logarithms by Denis Roegel (2012), will show the implicit appearance of the concept of the base. This work shows that Napier defined logarithms according to the kinetic and proportional-theoretic approaches, without the idea of radix in his definition. Still, base, *e* appears implicitly in the description. More than 100 years later, the new *e* number was defined, and Leonhardo Euler 1748 (1707 - 1783) presented in *Introductio in Analysin infinitorum*, the result $e = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + ...$ is the base of today's Napier logarithms.

Related to the topic of exponential functions, doctoral thesis A case study of a secondary mathematics teacher's understanding of exponential function: an emerging theoretical framework by April Dawn Strom (2008). This thesis's research describes a case study of the essential math teacher's conception as he works through a set of exponentially related activities. The research tools are designed to learn his skill for repeating multiplication in the context of exponential manipulation. This work is a study of a specific case of teachers' conceptions about the exponential series of related activities set by the researcher.

A doctoral thesis about "A teaching experiment in covariational reasoning and exponential growth" was carried out by Carlos Castillo-Garsow (2010). The thesis describes how students can teach "thinking" about exponential behavior by the positive effects of two change quantities. The view has built exponential actions from different impacts. The thesis has described a teacher's teaching experiment, proposing a series of tasks based on the mission type of discovery teaching interview. The purpose of the thesis experiment is to determine the student's actions on the covariance effects and its effects on mathematics, which are related to exponential growth. This work is a reference to teaching methods in which knowledge is exponential.

There are works on the history of exponential functions, and educators worldwide have done this mathematical object teaching research. So in the Vietnamese context, what exponential content will educators choose to teach students? What are these contents similar and different from their history?

The research purpose:

- Clarifying important features of exponential functions in history and Vietnamese math textbooks.
- Proposing pedagogical approaches showing the orientation of teaching exponential functions with the connection between mathematics and other subjects.

II. METHODOLOGY

The method used is to analyze and synthesize scientific works, books, and articles to point out the characteristics of exponential functions in history and Vietnamese math textbooks. In it, researchers investigate the process of forming and developing the concept of exponential functions (especially the process of expanding the idea of power), the relationship of the concept with other images of mathematics and science (including the construction of real numbers and the concept of continuous functions. Besides, the applications and influences of exponentials for other fields are also clarified.

III. RESULTS AND DISCUSSION

A. The exponential functions in history

Problems with the seed of exponential formation appeared very early in Babylonian times, such as the problem of compound interest, which was found in archaeological artifacts dating back to 2000 BC. Then the problem is solved by a given compound interest table at equal predetermined intervals. At any given time, interest is calculated based on linear interpolation. The problem has created the implicit appearance of the exponential by a corresponding table between the number of years and the proceeds whose expression is the power expression. The concept of powers with rational exponents was thought of and expressed through linear interpolation.

The idea of a function, in general, was also noticed during this time. The mathematical work Almagest (Book of Astronomy) of Ptolemy (100 - 170) shows many examples of functional relations tables between sets of quantities. In it, many examples show that "The Babylonians previously created many tables corresponding to the square, as well as in astronomy there were predictions of the time when the phenomena of different celestial bodies appeared. ". However, the primitive tables represent only discrete values. With the interpolation method, Ptolemy added the calculation of function values (according to the table) corresponding to any given value of the "independent variable."

Early studies of exponential functions can be found primarily in the concept of powers. The 14th and 15th centuries had symbols such as Nicole Oresme used fraction powers to represent roots, Chuquet used hostile powers to describe inverse, and powers did not represent powers also followed. Initially, the symbols with words, characters such as latus, quadratus, latus cubi to represent *x*, x^2, x^3; Q. for x^2, C. for x^3, biqq or qq. for x^4, Ss for x^5, Cq. for x^6, SsB for x^7, Triq. or qqq for x^8, Cc. for x^9. The most apparent notation today is an upper-right index published by René Descartes in 1637's La Gkomktrie. In the other direction, exponentials are discovered indirectly through related studies. In one case, it is noted that the exponential function (or at least some properties of the exponential function) is detected through the construction locus in geometry.

Since the 17th century, the concept of power has been studied more gradually, especially the expansion of powers. Accordingly, it is found that there is research to expand the concept of powers from positive integers to negative integers and fractions such as $a^{\frac{1}{2}}$, $a^{\frac{3}{2}}$, $a^{\frac{5}{2}}$, represent $\sqrt{a}$, $\sqrt{a^3}$, $\sqrt{a^5}$; a^{-1}, a^{-2}, a^{-3}. Then, the concept of power is opened to real exponent through Napier's logarithmic table by the formula $a^b = (e^{\ln a})^b = e^{b\ln a}$, then $a^b = t \Leftrightarrow e^{b\ln a} = t \Leftrightarrow b\ln a = \ln t$, using a logarithmic table to find t such that $lnt = blna$. Next, the power to the imaginary exponent emerged by discovering the formula $e^{+x\sqrt{-1}} + e^{-x\sqrt{-1}} = 2\cos x$ and $e^{+v\sqrt{-1}} = \cos v + \sqrt{-1}\sin v$ of L. Euler. The imaginary exponent has occurred while the power to the real exponent has yet to be found a big note. By the 18th century, the idea of an upper and lower bound of power with irrational exponents equal to powers with rational exponents without the need for logarithms was proposed by Euler.

In the 19th century, A. L. Cauchy published the expression $((a))^x = e^{xla}.e^{2kx\pi i}$, where *l* is the logarithmic value according to the table of $|a|$, e=2.718…, π=3.141…, k=0, ±1, ±2,…This does not help compute based on known powers (up to this point) because powers with base e are undefined; also, the logarithm of the table does not come from base *e*. The e-number was then officially defined by L. Euler in "Introductio in Analysin infinitorum" (1748). Thus the expression $((a))^x = e^{xla}.e^{2kx\pi i}$ carries a more sense of the relationship between powers and Napier logarithm rather than a power to real exponent definition. The function is generally defined at this point, but the exponent is still not defined or drawn from another function.

The development of the concept of powers (exponential expressions) up to this point can be summarized with the following schema: powers with exponents: positive integers $\Rightarrow$ negative numbers $\Rightarrow$ fractions $\Rightarrow$ imaginary numbers. By the 19th century, when the concept of limit and Cantor's principle of the sequence of nested and constriction segments came into being, the upper and lower intercept of power with irrational exponents equals powers With rational exponent, there is the official solution. Then, the power to the irrational exponent will be defined. Exponential functions are also formally introduced by expressions. This approach also shows that the scientific obstacles to the limit and the confidentiality of Q in R are the most significant difficulty in defining powers with irrational exponents (an important component that makes up the functional expression. exponent) by upper and lower bound by powers of rational exponents. Additionally, the above definition shows that a continuum function's interference into an exponential function is through the concept of limit.

Thus, the process of exponential generation and development has undergone a very long period, independent of the logarithmic function. The result of the logarithmic and exponential functions being two inverse functions of each other is a consequence of their relationship. Scientific analysis also shows that the birth and development of exponential functions are not entirely associated with the concept of power expansion. In particular, powers with positive rational exponents appeared in ancient times. Real representatives' powers come with pure imaginary representatives. The scientific obstacle to the exponential concept is building actual numbers, limited functional concepts, continual functionalities, and approaches. The exact definition of these concepts makes the official description of the exponential very late. The absence of a real problem is also why powers with irrational exponents come into existence late and have no place in practice.

When becoming a mathematics theory, exponentials substantially impact the development of science and technology, including other mathematical ideas. For the convenience of presentation and follow-up, we will present the effects of exponentials in each area as follows.

Differential equations are used to model phenomena in nature, changing to predict their changes in the future. A phenomenon in nature that is mathematically modeled, satisfying the first differential equation $y' = ky$ shows that the rate of change is proportional to its initial scale. In particular, the familiar phenomena are referred to as population growth (bacteria), individual population growth (body), decrease in radioactivity, chemical reactions rate in the populations proportional to the levels of the essence of reaction (compound interest rate is the amount ratio) in the heat in the surrounding areas. In the future, the phenomena will change over time. These phenomena are modeled to exploit in practice. In the chapter's limit, we select and present several models in the next section of the chapter, especially those applied in daily human life and related models to the school curriculum. Moreover, the exponential function plays a particularly important role in many fields because of its presence in many expressions of the differential equations of levels representing the change in things.

There are equations in the presence of exponential expression a^x. Equations are used to solve many mathematics problems and solve science and technology problems such as archaeology, finance, natural increase, and decreased phenomena. The most straightforward equation has the form of $y = A_0 e^x$, then helping to find unknown quantities in fields such as archaeology, finance, population, and many other phenomena in science and technology. For example, we need the exponential equation $M = M_0 e^{0,06t}$ in a requirement: "How long does it take an invested capital to double its value if the interest rate is 6%/year continuously compounded?"

As is well known, mathematics in engineering science falls under the category of applied mathematics. The real computational value or computational technique is more interested in this field than creating the mathematical concept. The above definition shows that trend. The exponential function $f(x) = a^x$ is defined in a format that describes the practice of calculating function values through the concept of powers.

The need for the second consideration of modeling is the reasonableness of problem results compared with reality. Because the modeling problem uses exponential *e*, function values should consider the precision required to achieve reliable results in practice corresponding to the phenomenon under consideration. The above construction modeling uses the expression $P(t) = P(0)e^{kt} = 2560e^{kt}$ to calculate the population, so the product's accuracy depends on the population's size. The approximate value of the exponentiation expression can be determined e^{kt}. Models of this form include the decay of radioactive substances, the concentration of a substance in a chemical reaction, and the reduction of the object's heat compared to the surrounding environment. The decrease in heat (temperature rise in the case of warming) of an item is proportional to the difference between the object's temperature and the ambient temperature if it is not too great (Newton's Law on hypothermia).

Similar to the above models, the physical meaning of the exponent is shown in this modeling. The difference in the model is that it also depends on atmospheric temperature (Newton's hypothermia law) and the change in object temperature according to the current temperature, natural phenomena, which can be changed over time, produce a practical sense of mathematical function. The continuity of the function here is shown in the time continuity, and checking the association's suitability by graphing shows the meaning of the real phenomenon's function continuity.

Thus the above models show the physical significance of - the rate of change (increase/decrease) over time and the actual (sense of a model) power value as the real exponent of the exponent, where both the modeling and the real value of the actual model under discussion are small in exponent value. These are the meanings associated with the exponential shown through the models just examined. Therefore, exponential modeling allows learners to detect the meanings associated with it in real situations while also enhancing their mathematics skills in practice. Indeed, in the above survey models, it is found that the rate of change depends on the initial scale (number, magnitude) of the phenomenon, on the surrounding environment, such as the change in temperature. The object is also dependent on the ambient temperature.

The above models also show clearly the meaning of the concept of variables and the concept of dependence in reality, which is also a specific difficulty in teaching and learning a function. All of the above factors will be essential observations for designing an exponential teaching project, which considers the match between exponential knowledge in high school and its practical application in the fields.

B. The exponential functions in textbooks in Vietnam

In Vietnam, the current high-school math textbooks are classified by groups of boards. To have the books' exponential analysis content more detailed and detailed, we choose the books belonging to the natural sciences department for this analysis. Other volumes are subject to additional research as they differ. On the topic of exponential functions, these textbooks' approach is similar according to the process: defining the concept first, then the examples of applying concepts, and the last practical problems. Compared with the development of exponential functions in history, there has been a pedagogical transformation of the exponential development process that the textbook has performed. This transformation ignores the temporal and spatial sequence of exponential development, so it is beneficial to simplify exponential knowledge to be taught in high school. So comfortable leads to inadequate reflection of the actual exponential development process.

The exponential function definition uses the concept of powers to real exponents. The expansion of powers (or transformations) from powers with rational exponents to real numbers by the concept of limits. This move is not proven because "for high school students, there is not enough basis to prove it" and requires students to accept, "just let students have a rough idea of this concept building concept."

Thus, the exponential definition's pedagogical transformation has transformed an exponential development process in a long time, including repetition at each stage of development with associated obstacles into one process development, continues one after another, and leads to the perfect definition without barriers. However, the birth and growth of exponentials are not wholly associated with expanding the concept of power. The power to positive rational exponents appeared in ancient times, and powers with pure virtual exponents came before powers to real exponents. Scientific obstacles are associated with the construction of real numbers, the concept of limiting functions, continuous functions, and approximate calculation. Therefore, the above pedagogical transformation has helped to simplify the knowledge of exponential functions in high schools, and at the same time, help "overcome" obstacles with exponential functions.
Some problems associated with exponentials should be considered as follows:
Problem 1: According to the 1-year compounding method, a person deposits VND 15 million into the bank at an interest rate of 7.56% per annum. Assuming the interest rate does not change, how much will that person earn (both capital and interest) after five years? (Round to the second decimal place).

Problem 2: Know that a radioactive uranium substance and its mass are reduced by 0.4% daily (i.e., daily radioactive rate -0.4%). The abundance of that substance after n days is calculated using the compound interest formula $C = C_0 e^{ni}$, where C_0 is the mass of the radioactive material on day one, and *i* is the daily radioactive rate. After 50 days of radiation, how much of that substance will be left in 30 grams?

In addition to showing practical significance for the concept of powers with negative rational exponents, the above problems have resulted in a minimal reduction in radioactive material compared to the original mass. If no further attention is paid, the number can be rounded. As a result, the mass of the radioactive material does not change in the group.
Problem 3: When pouring a cup of hot tea in a small room, the cup of tea cools down according to the rule: The difference between the temperature of the teacup and the room temperature decreases exponentially with the exponential time; in other words, the difference is a function defined by the formula $f(t) = ka^t$, where *t* (minutes) is time, *k* and *a* are some suitable constants (that is called the "New Zealand law of cooling").
Know that the room temperature is 20^0C, the tea's temperature at first pouring (i.e., when *t* = 0) is 85^0C, and then 5 minutes is 72^0C.
a) Determine the constants *k* and *a*.
b) Calculate the temperature of the teacup 10 minutes after pouring.
c) It is assumed that one can drink the teacup when its temperature does not exceed 55^0C. How many minutes after pouring the tea above can be drunk? (in unit accuracy).

The above problem is a problem solving the actual situation of growth and decline, which can be used exponentially mathematical modeling to solve mathematical knowledge skills to solve and evaluate real-world problems for students. The number of exercises of this type is the largest compared to other formats. These exercises' main requirement is to immediately apply a given or learned formula to calculate the value of an unknown quantity when the cost of additional amounts of growth or decline phenomena such as population growth, compound interest, disintegration. Thus, there has been a pedagogical simplification or transformation here in solving a practical problem of the increase and decrease phenomena in nature. This approach may not create favorable conditions for students to mobilize exponential knowledge to solve practical problems; this is an effect of pedagogical transformation.

C. Some comments after analyzing exponential functions contents in history and Vietnamese math textbooks

In Vietnamese textbooks, exponential functions are a particular case in the mathematical modeling of natural phenomena that grow (decrease) over time; the meaning associated with exponential functions is also shown through the models. Furthermore, the models also offer the practical significance of the power values; Variable concepts, dependent concepts in practice. Meanwhile, the above mathematical organizations have not yet fully shown the meanings of exponential functions. Mathematical modeling of problems, real situations on the exponential topic can improve this.

Besides, the definition of an exponential function by exponentiation with a real exponent adopts the limit of the rational sequence but found that the type of task required to consider the power's value to the real exponent that its accuracy will correspond to an actual decision.

Each teaching institution in high schools has different purposes and requirements depending on the program designer's conception, the educational administrators' regulations, national rules such as the program's length, and internal controls use knowledge to teach. Concerning expertise, the above models demonstrated physical meaning when referring to the exponential functions. Thus, the rate of change (increase/reduction) over the period, the real value (model significance) of power for the actual exponent, when the value of the exponent in modeling and the actual value in the existing model is low. However, these models are absent from the exponential case in textbooks.

IV. CONCLUSION

The clarification of mathematical knowledge in history and the textbook has shown the difference. Specifically, the exponent characteristics of the range appear the problem, the conditions that arise knowledge. Therefore, some proposals associated with exponential teaching to respect historical aspects and pedagogical factors show the interdisciplinary relationship between mathematics and other subjects as follows:

1) The exponential teaching content structure enhances practical problems applying mathematical modeling to help supplement the applicable meanings and other connective meanings of exponential science functions. Simultaneously, it allows students to mobilize mathematical knowledge to solve practical problems and interdisciplinary problems, especially economics and finance issues. This solution is based on knowledge analysis results exponentially in history, and according to the general education program's current innovation requirements in Vietnam.

2) They enhance information technology's application on exponential topics with project-based teaching, practical activities, and experience solving practical problems with exponential knowledge. Exponential functions must be taught in the study findings; real-life situations should be fully used to help students learn. The students experience a more real-world problem in terms of exponential functions that computer tools can use to explain and validate the most "right power value with real exponents, which is exceptionally irrational.

3) Here is the solution to the exponential building process.

- Step 1: Approach the concept of exponential functions from the actual problem of interest rates.

This suggestion is a scientific knowledge-based approach to exponential generation and development. Accordingly, the exponential function was derived from the need to calculate people's interest rates in Babylon. On the other hand, the approach also shows the practical significance of the exponential function.

- Step 2: Institutionalize the power of irrational exponents by "approximation type" with powers with rational exponents. The exponent definition is a function given by the formula $f(x) = a^x$, where a is a positive real number; other than 1, the determination domain is all reals.

- Step 3: Teaching exponential modeling of growth phenomena (decline).

This stage is a convenient implementation step for exponential knowledge. Perform this step to develop modeling capabilities, apply math to solving practical problems. Carry out practical activities and experience to apply exponential knowledge to practical problems, especially economic and financial issues.

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