

# Mathematical Modeling of Three Phase Power Transformer Phase-Shift Compensation Differential Protection Using Star/Star Connected Current Transformers

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## ABSTRACT

*In this paper mathematical modeling of three phase power transformer phase-shift compensation for Digital differential protection using star/star connected current transformers was developed to calculate the differential operating currents. In developing the Algorithm, power transformer star and delta windings configurations, current transformer and vector group's configurations parameters were put into consideration. The model performs compensation for current magnitude, phase-angle shift, current transformer mismatch errors and zero-sequence current. Results show that, the resultant operating current  $I_{op}$  flowing through the operating coil of the differential relay summed to zero during normal operation and during external faults, and discriminate internal faults from magnetizing inrush currents. This concludes that there is no fault in the protection zone. This validates the accuracy of the developed model. The model will be used to calculate for differential operating currents in any three-phase power transformer, variants vector group with  $30^{\circ}$  phase angle shifts, and overall differential protection of two or more parallel connected power transformers. The differential protection for any three-phase power transformer will be ideally balanced for all symmetrical and non-symmetrical through-load conditions and external faults irrespective of transformer construction details and actual on load tap changer position.*

**KEY WORDS:** Compensation, Current Transformers, Differential Relay, Mathematical Modeling, Phase angle shifts, Power Transformer, Vector group, Star-Star connection.

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## I. INTRODUCTION

Power transformer is a static piece of Electrical equipment with two or more windings which, by electromagnetic induction, transforms a system of alternating voltage and current into another system of voltage and current usually of different values and at the same frequency for the purpose of transmitting electrical power according to [1, 2]. This static device, totally enclosed and usually oil immersed, and therefore, chances of fault occurrence on them are very rare, but the damaged caused by the faults usually takes much more time and money to repair than are required to repair the damaged caused by the fault on transmission lines [3, 4]. In general, transformers are exposed to internal and external faults. Protection equipment consisting of CTs and relays defines the zone of protection. Faults inside this zone are termed as internal, and outside of this zone as external. Transformer internal faults can cause fire and damage a transformer to an un-repairable degree. The consequences of even a rare fault may be very serious unless the transformer is quickly disconnected from the system. Hence automatic protection of transformers against possible faults is essential and of utmost importance. To protect this costly equipment, microprocessor differential protection relays are widely used now-a-days. They provide high-speed multi-level protection and monitoring of a transformer and trip circuit breakers responsible for isolating this transformer. Protection should be provided against different faults and abnormal operation. According to [5, 6], differential protection is the most reliable scheme, but it is typically used for transformers rated above 5.0 MVA. Protection of three-phase transformers requires that primary and secondary currents of the three-phases are compared individually to achieve differential protection of the three-phase transformer. Under normal load conditions, the currents in the primary and secondary windings are in phase, but the line currents on the star and delta sides of the three-phase transformer are out of phase by  $30^{\circ}$  [7].

According to [8], an internal fault creates a difference between the currents entering and exiting the protected zone. Thus, a protective relay will be subjected to a difference of currents in the secondary windings of CTs. If the operating current exceeds the relay pickup value, the relay will trip the circuit breakers.

## II. DESCRIPTION

### 2.1 Power Transformer Winding Compensation

The four distinct ways by which a three phase two-windings power transformers can be connected includes star/star (Y/Y), delta/star ( $\Delta/Y$ ), star/delta (Y/ $\Delta$ ) and delta/delta ( $\Delta/\Delta$ ). For a star/star (Y/Y), or delta/delta ( $\Delta/\Delta$ ) connections, there is no phase shift between corresponding quantities on the low-voltage and high-voltage windings. However, for delta/star ( $\Delta/Y$ ), or star/delta (Y/ $\Delta$ ) transformer windings connections there is a phase shift in a balanced voltage and currents of typically  $\pm 30^\circ$  depending on transformer connections [9, 10, 11]. In order to correctly apply transformer differential protection it is necessary to properly compensate for the 30 degree phase shift, in electromechanical differential relays, the common approach to the delta / star winding compensation is to connect the CTs on the delta transformer winding in star and the CTs on the star winding in delta as shown in fig: 1 [12], or using interposing CTs to compensate for a) Current magnitude; b) Phase angle shift; and c) provide zero sequence current. This is to ensure that the phase shifts created in the currents of the power transformer are compensated by the CTs, so that the secondary currents are once again in phase. Ideally, net current flowing through the operating coil of the differential relay would be zero during normal operation and during external faults; practically, this is usually impossible in electromechanical relays as small amount of current still flows in the operating coils of the differential relay because of CT ratios mismatch and differences in characteristics of the CTs between CTs on the high-voltage and low-voltage sides of a power transformer as shown in figure 1. The Digital Differential relays see this phenomenon as fault currents and consequently will trip or isolate the circuit [13].

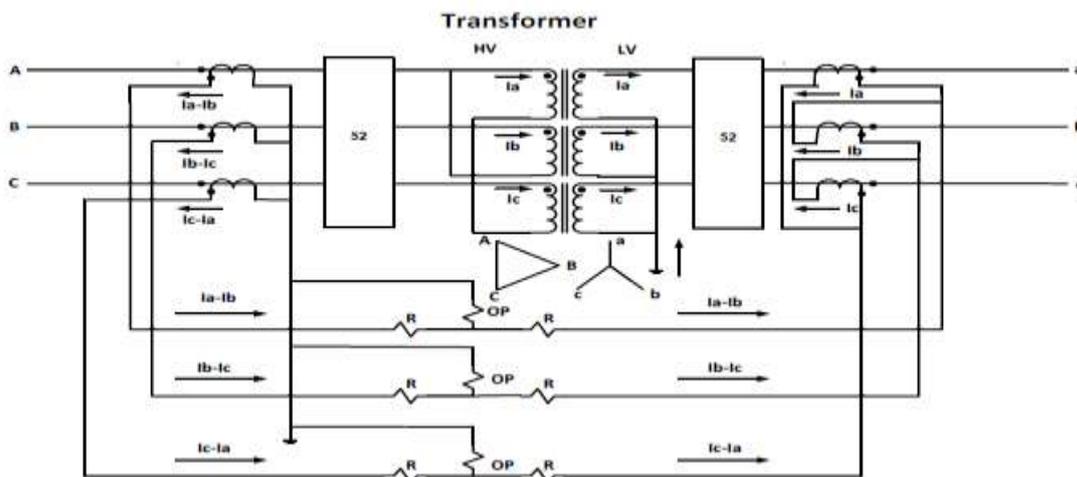


Figure 1: Traditional Electromechanical Relay connection

### 2.2 Power Transformers Differential Protection

Differential protection is one of the most widely used methods for protecting power transformer against internal faults. The technique is based on the measurement and comparison of currents at both sides of transformer's windings. Differential relay protection operates on the basic theory of Kirchhoff's current law which states that the sum of the currents entering the node should equal the sum of the currents leaving the node. The differential relay trips whenever the difference of the currents in both sides exceeds a predetermined threshold. These types of differential relays compare the currents coming in from the high and low side of the transformer and determine if a fault is present within the zone of protection. However, there are several factors that can affect the difference in the currents the relay sees which can result in mis-operations. These factors are CT mismatch errors, phase shift and zero sequence current due to transformer delta-star transformation. In addition, when these factors are present, the differential relays can operate either due to a slight increase in load or during an external fault. CT mismatch errors are inherent in electromechanical relays. These errors are caused by difference in voltage levels which required different current transformer and results in different operating characteristics [14, 15]. If the mismatch error is too large, it can cause the differential relay to mis-operate. The phase shifting of the transformer and its respective vector group is another factor that can cause differential errors. The vector group provides the amount of phase shift that will occur when the current goes from the primary to secondary side of the transformer.

Differential elements compare an operating current with a restraining current. The operating current,  $I_{op}$ , can be obtained from equation (1) as  $I_{op} = I_d = |I_{F1} + I_{F2}|$  (1)

where  $I_d$  is proportional to the fault current for internal faults and approaches zero for any other operating conditions. The restraining current,  $I_{RT}$  is obtained using equation (2) as

$$I_{RT} = k |I_{F1} - I_{F2}|$$

$$I_{RT} = \text{Max}(|I_{F1}| * |I_{F2}|)(2)$$

Where k is a compensation factor, usually taken as 1 or 0.5; The differential relay generates a tripping signal if the differential current,  $I_d$ , is greater than a percentage of the restraining current,  $I_{RT}$ .

$$I_d > \text{SLP} \cdot I_{RT}(3)$$

A typical digital differential relay tripping characteristic is shown in Figure 2[13].

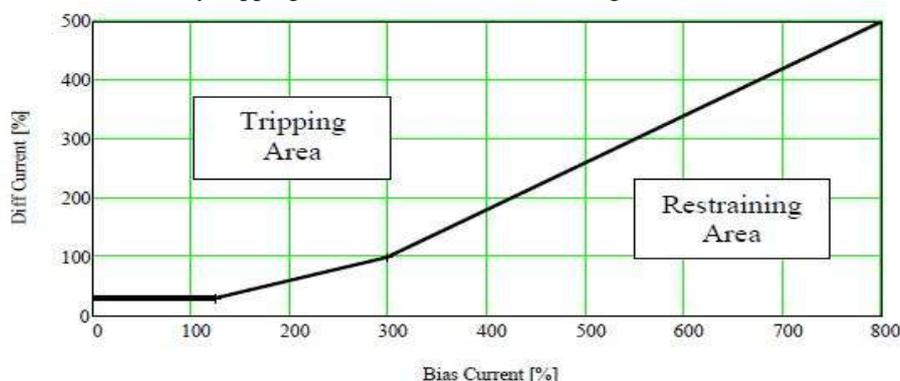


Figure 2: Digital Differential relay tripping characteristic

### 2.3. Power Transformers Vector Groups

Transformer vector group shows the phase difference between the primary and secondary sides of the transformer. The vector group is used to determine the high voltage and low voltage windings arrangement of three-phase transformers. In a two winding three phase power transformer, the primary winding can be connected in various ways either in star, in delta or inter-star (or zigzag); similarly, the secondary winding can be connected in star; in delta or inter-star (or zigzag). Power transformer introduces a phase angle difference between primary and secondary voltages of  $30^0$  degrees phase shift when connected in delta-star or star-delta. Delta and star connections are most commonly used. The zigzag connection finds occasional applications, mostly in specialized constructions like earthing transformer and transformers for multiple pulse circuits. It is common practice to designate a delta winding with letter D and Star winding with letter Y. The lower case letters are applicable for the lower voltage connections (d, y and z). Based on this, it is theoretically possible to connect any pair of windings in a 3 phase transformer in the following pairs of combinations: Dd, Dy, Dz, Yd, Yy, Yz, Zd, Zy and Zz; of this, the first six, are the most commonly encountered ones in practice. Power Transformer winding vector group configurations are referred to by terms such as Dy# or Yy# or Dz# or Yz#; where # is any hour of the clock. The clock is divided into 12 segments, each segment number indicates the number of 30 degree increments the phase angle is shifted. The phase difference varies in steps of 30 degree, the 12'o clock position represents zero or 360 degrees; hence the terms "around the clock" phase shifting. The minute hand is set on 12 o'clock and replaces the line to neutral voltage of the HV winding. This position is always the reference point. For instances, 1'o clock position signifies phase shift of 30 degrees to right; 6'o clock position signifies 180 degrees; and 11'o clock position corresponds to 330 degrees or phase shift of 30 degrees to left. The clock position indicates the angle by which secondary vector leads or lags the primary vector. The phase angle displacement depends on the winding connection details for the specific power transformers. In delta/star or star/delta transformer configurations, positive sequence quantities on the high voltage side (H) lead the corresponding quantities on the low voltage side (X) by 30 degrees. The vector group Dy11 means the Star winding lag Delta winding by phase shift of  $11 \times 30 = 330$  degree or the Delta leads Star by  $-30$  degree. Similarly, Dy1 means a phase shift of  $1 \times 30 = 30$  degree. According to [2, ...] IEC 60076-1 standard, 1997; Lawhead, et al, 2006, the commonly used vector groups Dy1 and Dy11 connections for two-winding three-phase power transformers are shown in Figures 3 and 4 respectively.

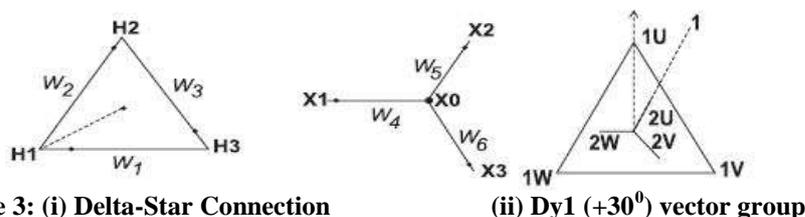


Figure 3: (i) Delta-Star Connection

(ii) Dy1 (+30°) vector group

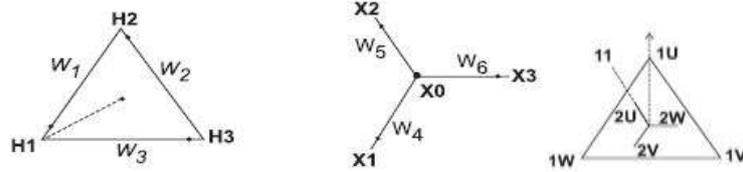


Figure 4:(i) Delta-Star Connection

(ii) Dy11 (-30°) vector group

III. MATERIALS AND METHODS

3.1 Developed Mathematical Model for Power Transformer Phase-Shift Compensation

In developing the Algorithm, power transformer star and delta windings configurations, current transformer and vector group’s configurations parameters were put into consideration.

Let apply the flux balance equation (3.1) of a three-phase, two windings power transformer with three sets each in primary and secondary. Each winding will produce a flux that is proportionate to its load current. The flux levels will be:

$$\begin{aligned} \phi_1 &= N_p K_c I_1 \phi_4 = N_s K_c I_4 \\ \phi_2 &= N_p K_c I_2 \phi_5 = N_s K_c I_5 (1) \\ \phi_3 &= N_p K_c I_3 \phi_6 = N_s K_c I_6 \end{aligned}$$

and  $K\phi_1 = \frac{1}{N_p K_c} (2)$

Where  $\phi$  is the flux proportional to load current in the windings set;  $N_p, N_s$  is the number of turns in primary and secondary winding set; and  $K_c$  is proportionality constant that would be specific to the core. This  $K_c$  factor will cancel out of the current differential equations so we do not need to know its specific value.  $K\phi_1$  is flux proportional to the transformer coil turns. Equation (2) state the current in terms of the core flux in each transformer winding. Introducing matrix representations of the equations, for primary winding set  $W_1$  and secondary winding set  $W_2$ ,

$$\begin{aligned} I_1 W_1 &= \begin{bmatrix} K\phi_1 & 0 & 0 \\ 0 & K\phi_1 & 0 \\ 0 & 0 & K\phi_1 \end{bmatrix} x \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \text{ and, (3a)} \\ I_2 W_2 &= \begin{bmatrix} K\phi_1 & 0 & 0 \\ 0 & K\phi_1 & 0 \\ 0 & 0 & K\phi_1 \end{bmatrix} x \begin{bmatrix} \phi_4 \\ \phi_5 \\ \phi_6 \end{bmatrix} \text{ (3b)} \end{aligned}$$

Condensing the matrixes form, equation becomes  $I_{123} W_1 = K\phi_{pri} x \phi_{123}$  and  $I_{456} W_2 = K\phi_{sec} x \phi_{456} (4)$

Recall the above equations are referring to load flux, for load induced flux, we can assume that in each leg of the transformer the flux that is caused by load current sums to 0 in normal operation.

The equations are:

$$\begin{aligned} \text{Phase 1: } \phi_{w1} + \phi_{w4} &= 0 \\ \text{Phase 2: } \phi_{w2} + \phi_{w5} &= 0 \\ \text{Phase 3: } \phi_{w3} + \phi_{w6} &= 0 \end{aligned} \tag{5}$$

The equation assumes the summation of the flux is zero.

3.1.1 Transformer Windings Configurations

There are six ways to connect a star winding; each modifies the  $W_1, W_2, W_3$  or  $W_4, W_5, W_6$  currents. The two 3x3 matrices represents how the winding current can be transformed by the various star connection. In the condensed matrix equations (6a and b) on the right hand side, these matrices are written as the negative of one of the other as noted in the left column, however, since the primary side of the transformer is delta connected no compensation is required and will use the unit matrix. In both cases, the left hand column matrixes represent each vector group (e.g. Dy1) configuration which is used to compensate the phase shift across the transformer.

$$\begin{aligned} 1A \quad Y_0 1B \quad 1C &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} K\phi & 0 & 0 \\ 0 & K\phi & 0 \\ 0 & 0 & K\phi \end{bmatrix} x \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = 1_{ABC} = 1x K\phi x \phi_{123} \text{ (6a)} \\ 1A \quad Y_6 \text{ (negated } Y_0) 1B \quad 1C &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x \begin{bmatrix} K\phi & 0 & 0 \\ 0 & K\phi & 0 \\ 0 & 0 & K\phi \end{bmatrix} x \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = 1_{ABC} = -1x K\phi x \phi_{123} \text{ (6b)} \end{aligned}$$

There are two basic forms of connecting delta windings, the difference between phases A and B (DAB) connection and the difference between phases A and C (DAC) connections. The current seen by the relay is a summation of two phase currents. Similarly, two 3 x 3 matrices for DAB and DAC are presented in equations (7) and (8) which represents how the winding currents can be transformed by the various delta connections.

$$\mathbf{D}_1(\text{DAB}) \begin{matrix} \mathbf{1A} \\ \mathbf{1B} \\ \mathbf{1C} \end{matrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} K\phi & 0 & 0 \\ 0 & K\phi & 0 \\ 0 & 0 & K\phi \end{bmatrix} \times \begin{bmatrix} \phi 1 \\ \phi 2 \\ \phi 3 \end{bmatrix} = \mathbf{1}_{ABC} = \mathbf{D}_1 \times \mathbf{K}\phi \times \phi_{123} \quad (7)$$

$$\mathbf{D}_{11}(\text{DAC}) \begin{matrix} \mathbf{1A} \\ \mathbf{1B} \\ \mathbf{1C} \end{matrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} K\phi & 0 & 0 \\ 0 & K\phi & 0 \\ 0 & 0 & K\phi \end{bmatrix} \times \begin{bmatrix} \phi 1 \\ \phi 2 \\ \phi 3 \end{bmatrix} = \mathbf{1}_{ABC} = \mathbf{D}_{11} \times \mathbf{K}\phi \times \phi_{123} \quad (8)$$

The difference between the DAB and DAC transformer connection is simply created by swapping two phases (B and C) on the delta connected transformer winding which change the phase sequence.

For calculation of the overall transformation equations, the delta/Star Connected transformer turns ratios are given as:  $\mathbf{D}_y = \sqrt{3} (V_H/V_L)$  (9)

Where  $\mathbf{D}_y$  is Delta/Star connected transformer;  $V_H$  is the High voltage side and  $V_L$  is Low voltage side.

$$\text{The CT ratio factor} = \mathbf{N}^{-1}_{CTHV} \times \mathbf{N}_{CTLV} = \frac{\mathbf{N}_{CTLV}}{\mathbf{N}_{CTHV}} \quad (10)$$

And the overall transformation ratio as:

$$\mathbf{K}\phi = \mathbf{K}^{-1}_{\phi HV} \times \mathbf{K}_{\phi LV} = \frac{\mathbf{N}_{tLV}}{\sqrt{3} \times \mathbf{N}_{tHV}} \quad (11)$$

Hence, the relay magnitude compensation coming into the star connected transformer (Low voltage side) becomes;

$$\mathbf{M}(\theta)_{\text{mag}} = [-1 \times \mathbf{K}\phi_{HV} \times \mathbf{K}\phi^{-1}_{LV} \times \mathbf{N}^{-1}_{CTHV} \times \mathbf{N}_{CTLV}] = -\mathbf{K}_2 \quad (12)$$

### 3.1.2 Current Transformer (CT) Connections

Star CTs connection on the primary and secondary windings is given in equations (13a and b) as:

$$\mathbf{Y}_{CTO} = \begin{matrix} \mathbf{Id}_{a1} \text{ relay} \\ \mathbf{Id}_{b1} \text{ relay} \\ \mathbf{Id}_{c1} \text{ relay} \end{matrix} = \begin{bmatrix} \mathbf{N}_{CT} & 0 & 0 \\ 0 & \mathbf{N}_{CT} & 0 \\ 0 & 0 & \mathbf{N}_{CT} \end{bmatrix} \times \begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_B \\ \mathbf{I}_C \end{bmatrix} = \mathbf{N}_{CT} \times \mathbf{I}_{ABC} \quad (13a)$$

$$\mathbf{Y}_{CT6} = \begin{matrix} \mathbf{Id}_{a2} \text{ relay} \\ \mathbf{Id}_{b2} \text{ relay} \\ \mathbf{Id}_{c2} \text{ relay} \end{matrix} = \begin{bmatrix} -\mathbf{N}_{CT} & 0 & 0 \\ 0 & -\mathbf{N}_{CT} & 0 \\ 0 & 0 & -\mathbf{N}_{CT} \end{bmatrix} \times \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \{-\mathbf{N}_{CT} \times \mathbf{I}_{abc}\} \quad (13b)$$

Where  $\mathbf{N}_{CT}$  is the transformation ratio factor:  $\mathbf{N}_{CT} = \frac{I_{Sec}}{I_{Pri}}$ .

The connections of both CTs are such that the currents  $\mathbf{Id}_{a1}$  and  $\mathbf{Id}_{a2}$  are having a phase difference of  $180^\circ$ .

### 3.2 Developed Mathematical Compensation Model

Combining all compensation techniques presented in section 3.1 into equation (14), the resultant differential operating current ( $I_{op}$ ) algorithm for any two winding; three phase power transformer is represented as

$$\begin{bmatrix} \mathbf{I}_{0P1} \\ \mathbf{I}_{0P2} \\ \mathbf{I}_{0P3} \end{bmatrix} = \begin{bmatrix} \mathbf{Id}_{a1} \text{ relay} & \mathbf{Id}_{a2} \text{ relay} \\ \mathbf{Id}_{b1} \text{ relay} & \mathbf{Id}_{b2} \text{ relay} \\ \mathbf{Id}_{c1} \text{ relay} & \mathbf{Id}_{c2} \text{ relay} \end{bmatrix} = \mathbf{0}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{I}_{a1} \\ \mathbf{I}_{b1} \\ \mathbf{I}_{c1} \end{bmatrix} + \begin{bmatrix} -k2 & 0 & 0 \\ 0 & -k2 & 0 \\ 0 & 0 & -k2 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{I}_{a2} \\ \mathbf{I}_{b2} \\ \mathbf{I}_{c2} \end{bmatrix} = \mathbf{0}$$

$$\text{Thus } \mathbf{I}_{0P123} = [\mathbf{M}(\theta)_{\text{phase}} \times \mathbf{Id}_{a1,b1,c1}] + [\mathbf{M}(\theta)_{\text{Mag}} \times \mathbf{M}(\theta)_{\text{phase}} \times \mathbf{Id}_{a2,b2,c2}] = \mathbf{0} \quad (14)$$

Where  $\mathbf{I}_{0P123}$  = differential relay operating currents;

$\mathbf{Id}_{a1,b1,c1}$  = differential restraint currents at the primary winding;

$\mathbf{Id}_{a2,b2,c2}$  = differential restraint currents at the secondary winding;

$M(\theta)_{\text{phase}}$  = phase compensation at the secondary side of the transformer winding;  
 $M(\theta)_{\text{Mag}}$  = magnitude compensation at the secondary side of the transformer winding.

Equation (14) revealed that when the transformer is operating under normal condition, the resultant operating current on each winding side due to load currents summed to zero.

The compensation model can be achieved by inputting the power transformer parameters correctly from the name plate which includes; (a)Transformer apparent power MVA rating, (b) input and output voltages levels, (c) Transformer turn ratio (d) primary and secondary windings configuration, (e) vector group e.g. Dyn1, (f) System frequency, and (f) Select appropriate current transformers (CTs) ratio relays' primary and secondary sides as well as the normal loading that the user expects which tells the relay what constitutes balanced current magnitudes.

The developed model will be used to calculate for differential operating currents: (a) In any two-windings, three-phase power transformers; (b) For variants vector group with  $\pm 30^\circ$  phase angle shifts; and (c) Differential protection of two or more parallel connected power transformers.

The proposed Mathematical model is dependent on the correct values of the base current and phase angle shift for every power transformer sides available to the differential protection algorithm. These values are obtained as fixed values, determined from the protected transformer rated data and vector group, which are entered as setting parameters by the user (Protection Engineer). With the model, power transformer differential protection can simultaneously:

- a) Provide current magnitude compensation for every side of the protected transformer,
- b) Provide phase angle shift compensation between primary and secondary windings of the power transformer,
- c) Zero sequence current compensation, and CTs ratio mismatch.

#### IV. ANALYSIS AND RESULTS

##### 4.1 Evaluation of the developed Compensation Model using Star/Star connected CTs with Vector group Dy1

Evaluating the developed Digital differential relay operating current using Star–Star connected CTs, the maximum base power rating of a two winding three-phase power transformer is 200MVA, 50Hz, 330KV/ 132 KV (DAB) delta / (Y<sub>0</sub>) Star, 350/875Amperes and Dy1 vector group was analyzed as presented in Table 1. For this application, Digital differential protection solution with vector group (Dy1, DAB) of phase angle shift of  $+30^\circ$  with main CTs star - star connected at both sides of the protected power transformer was evaluated. Assume that the transformer is carrying normal load and that the current in phase A on the star side is the reference phasor.

**Table 1: Base Current Calculation for Dy1 (DAB) Power Transformer**

Windings Configuration	Primary Base Current ( $I_b$ ) Amps.	CT Secondary Current (Amps)
High Voltage, $W_1$ 330KV – Delta ( $\Delta$ )	$I_1 W_1 = \frac{200 \times 10^6}{\sqrt{3} \times 330 \times 10^3} e^{j30}$ $= (303.02 + j174.95),$ $I_2 W_1 = (0.0 + j349.9),$ $I_3 W_1 = (-303.02 - j174.95)$	$I_{a1} = \frac{I_1 W_1}{CTR1} = \frac{350}{400/5}$ $= (3.788 + j2.187),$ $I_{b1} = (0.0 + j2.187),$ $I_{c1} = (-3.788 - j2.187)$
Low Voltage, $W_2$ 132KV- Star ( $\gamma$ ), Taking phase A as reference phasor	$I_1 W_2 = \frac{200 \times 10^6}{\sqrt{3} \times 132 \times 10^3}$ $= (874.77 + j0.0),$ $I_2 W_2 = (-437.385 + j757.573),$ $I_3 W_2 = (-437.385 - j757.573)$	$I_{a2} = \frac{I_1 W_2}{CTR2} = \frac{875}{1200/5}$ $= (3.645 + j0.0),$ $I_{b2} = (-1.8225 + j3.1567),$ $I_{c2} = (-1.8225 - j3.1567)$

Since both primary and secondary windings are to be connected using Star-Star CTs (Y<sub>CT0</sub> and Y<sub>CT6</sub>); and from Table 1 the high winding side current is 350Amps and the low voltage side current is 875 Amps; therefore select CTs of three set each of 400:5A for high voltage side and another three set each of 1200:5A CT ratio for low voltage side being the nearest standard CTs greater than the calculated values.

The CTs on the transformer Delta/Star windings are connected star such that the current  $I_{da1}$  and  $I_{da2}$  are having a phase difference of  $180^\circ$  i.e. Y<sub>CT6</sub> ( $180^\circ$ ) inverted.

From equation (10) CT turn ratio =  $N^{-1}_{CTHV} \times N_{CTLV} = \frac{N_{turn LV}}{N_{turn HV}} = \frac{5/400}{5/1200} = 3$  and

From equation (11), transformer turn ratio is

$K\phi_{HV} \times K^{-1}\phi_{LV} = \frac{N_{yLL}}{\sqrt{3} \times N_{DLL}} = \frac{132}{\sqrt{3} \times 330} = 0.2309$

Multiplying equations (10) by (11) and substituting the values, the magnitude compensation,

$\begin{bmatrix} M(\theta)_{mag} \\ (\theta - 180^\circ) \end{bmatrix} = [-1 \times 0.2309 \times 3] = -0.6928$

In order to calculate the differential currents, the overall magnitude and phase compensation matrixes for the Dy1 (DAB) power transformer is presented in Table 2.

From the relay comparators point of view, it monitors current in two phases of the transformer at a time.

$$\text{Thus } Id_{a2} \text{ Relay} = -K(1_{a2} - 1_{b2}) = -0.69284 (5.4675 + j3.1567) = (-3.788 - j2.187)$$

$$Id_{b2} \text{ Relay} = -K(1_{b2} - 1_{c2}) = -0.69284 (0.0 + j6.3134) = (0.0 - j4.374)$$

$$Id_{c2} \text{ Relay} = -K(1_{c2} - 1_{a2}) = -0.69284 (-5.4675 - j3.1567) = (3.788 + j2.187)$$

From equations (13), the phase compensation on the high voltage side and the overall magnitude and phase compensation on the secondary windings are sums together to obtain differential operating currents ( $I_{op}$ )

$$I_{op1} = Ida_1 \text{ relay} + 1d_{a2} \text{ relay} = [(3.788 + j2.187) + (-3.788 - j2.187)] = 0$$

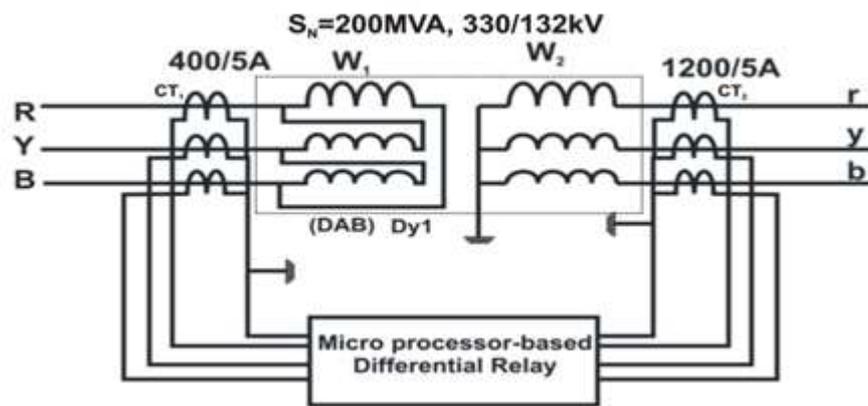
$$I_{op2} = 1d_{b1} \text{ relay} + 1d_{b2} \text{ relay} = [(0.0 + j4.374) + (0.0 - j4.374)] = 0$$

$$I_{op3} = 1d_{c1} \text{ relay} + 1d_{c2} \text{ relay} = [(3.788 - j2.187) + (-3.788 + j2.187)] = 0$$

**Table 2: Phase Angle Compensation for Dy1 (DAB) Power Transformer**

	Magnitude and Phase Compensation Matrix	Differential Relay Restraining Current
High Voltage, $W_1$ 330KV - Delta ( $\Delta$ )	$M(\theta^0)_{\text{phase}} \times Ia_{1,b1,c1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} I_{a1} \\ I_{b1} \\ I_{c1} \end{bmatrix}$	$1 \times Ida_1, b_1, c_1 \text{ relay}$
Low Voltage, $W_2$ 132KV- Star ( $\gamma$ ),	$M(\theta)_{\text{mag}} \times M\theta(+30^0)_{\text{phase}} \times Ia_{1,b1,c1}$ $= \begin{bmatrix} -k & 0 & 0 \\ 0 & -k & 0 \\ 0 & 0 & -k \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} I_{a2} \\ I_{b2} \\ I_{c2} \end{bmatrix}$	$Ida_2, b_2, c_2 \text{ Relay}$ $-k(I_{a2} - I_{b2})$ $= -k(I_{b2} - I_{c2})$ $-k(I_{c2} - I_{a2})$
Relay operating Current ( $I_{op}$ )	$(M(\theta)_{\text{phase}} \times Ia_{1,b1,c1}) + (M(\theta)_{\text{mag}} \times M(\theta)_{\text{phase}} \times Ia_{1,b1,c1}) = 0$	

The results show that, the net current ( $I_{op}$ ) flowing through the operating coil of the differential relay equal to zero during normal operation and during external faults; this conclude that there is no fault in the protection zone.



**Figure 3: Proposed differential relay protection using Star-Star connected CTs**

This validates the accuracy of the developed model. Figure 3 shows schematic diagram of the proposed differential protection using Star-Star connected CTs.

#### 4.2 Validation of the Developed Compensation Model

Results of the developed compensation model were validated as shown in Table 2. Proper selection and application of Star / Star CTs connection on both sides of the transformer windings is critical to the accuracy of the analysis. For star connected main CTs, secondary currents fed to the differential relay are directly proportional to the measured primary currents; are in phase with the measured primary currents; and contain all sequence components including the zero sequence current components. The model will be used to calculate for

differential operating currents: a) In any three-phase power transformer, b) For variants vector group with  $30^0$  phase angle shifts, c) For overall differential protection of two or more parallel connected power transformers, d) the CT mismatch errors and zero sequence current. Several cases were tested and results indicate that the proposed model proved to be very effective in improving the performance of any three-phase power transformer protection schemes based on the current measurement.

## V. CONCLUSION

Evaluation of the developed model using Star-Star Connected CTs for two-winding, three-phase Power Transformer was presented. It shows that:

- a) The mathematical model performs compensation for current magnitude, phase-angle shift, the CT mismatch errors and zero-sequence current. The resultant sums of the differential relay operating current equal to zero. This indicates that there is no fault in the protection zone.
- b) It eliminates common wiring errors that often occur when making up a delta connection.
- c) Star-Delta connected transformer windings using the model do not require interposing or dedicated CTs for other protection or metering functions.
- d) With this model, metering and event reports (measurements) give actual line currents, rather than the summation of two phases as the case with delta/star CTs.
- e) It allows monitoring ground current in the lines and the use of ground relays. It also reduces lead burden for a phase fault.
- f) Digital relays can be used with lower burden star-connected CTs regardless of transformer winding connections and can accommodate virtually any transformer application.

Thus, results of the analysis validate the Digital differential relay compensation model developed. The new compensation model is quite unique, as all necessary transformer data were obtained from the protected power transformer nameplate. The viability of the proposed model to provide differential protection for power transformers has been demonstrated by implementing the proposed techniques in actual field installations at a power transmitting station.

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