

## Comparative Study Of PD, PI And PID Controllers For Control Of A Single Joint System In Robots

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### -----ABSTRACT-----

*This paper looks at the Comparative study of the performance of Proportional Derivative (PD), Proportional Integral (PI), and Proportional Integral Derivative(PID) controllers for Position(Angle of Rotation) control of a Single Joint System comprising of a simple DC Motor as the actuator . In most industrial application of Robots, especially Humanoid Robots, their joints are usually actuated by DC Motors which allows flexible movement or rotation of the joints to perform certain task. The Single Joint System in this work is an Open Loop unstable system , however the application of controllers results in set-point tracking and stabilization , thus the performance of PD, PI and PID controller is studied and compared both in the time and frequency domain for transient and steady state responses.*

**Keywords** - Single Joint System, Robotic system, Humanoid Robot, Set-Point Tracking, Stabilization, Actuator.

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### I. INTRODUCTION

Industrial design of Robots requires that their Joints are normally actuated by simple DC motors which results in the flexible rotation of the Joints. The principle of operation of the system requires that Change in magnetic field of a moving magnet induces an EMF to a coil which then causes rotation of the Armature and therefore producing a resultant torque which causes the arm attached to the motor to rotate[1],[5]. This kind of system is unstable at open loop, making it necessary for the application of a controller to guarantee set-point tracking and stabilization[8]. The three parameters to be compared in this work are the Proportional action, Derivative action and Integral action. The proportional action or gain will drive a system to set-point, the derivative action will help to reduce oscillation at steady state or damp overshoots at transient, and the integral action will deal with steady state errors[9]. [4] had done a similar work by comparing the action of P,PI and PID controllers for speed control of VSI-fed induction motor, and the results showed that each controller contributed its own unique performance to the closed loop system.[6] used PID control for speed control of a DC motor and got some satisfactory result on the steady state performance while [2],[3],[7] have applied PI control for DC motors and confirmed that Integral action really leads to steady state off set cancellation. The application of PD controllers has also been verified by [9] where it was discovered that the system is stabilized from steady state, however steady state errors can still be maintained all through. In reality such a system must always encounter fault and disturbances hence[3] proposed and simulated rotation of this motors encountering faults, [2] has also attempted to observe this faults and parameter states by designing a Kalman Filter for state observation and results obtained has been encouraging and consistent with stability. Other work done on DC motors have focused mostly on speed control hence [5][7][8] have all presented that PID controllers are very reliable and stabilizing when it comes to control of the DC motors as actuators. In this work we are concerned in the comparative performance of PD, PI and PID controllers as the act to control and stabilize a single joint system shown in Fig.1.

### II. METHODOLOGY

The single Joint system can be represented by the figure below which comprises of an arm coupled to the XY coordinate by the action of DC motor as the actuator.

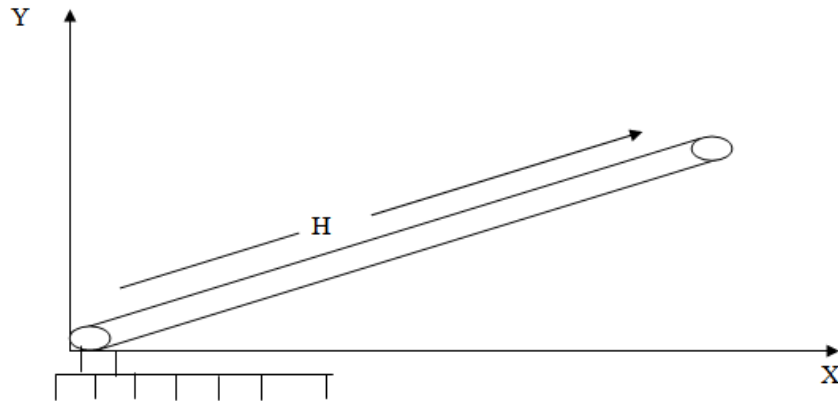


Fig.1: Single Joint System Coupled to the X-Y Axis[1]

The model of the system as derived by [1] can be represented by the equation below;

$$\frac{RJ}{K_2} \frac{d^2\theta}{dt^2} + \frac{RH}{2K_2} Mg \cos\theta + \frac{RT_L}{K_2} + \frac{LJ}{K_2} \frac{d^3\theta}{dt^3} - \frac{LH}{2K_2} Mg \sin\theta \frac{d\theta}{dt} + \frac{L}{K_2} \frac{dT_L}{dt} + K_1 \frac{d\theta}{dt} = U(t) \quad 1$$

The transfer function of the open loop system can be written as;

$$G(s) = \frac{\Delta\theta(s)}{\Delta U(s)} = \frac{K_2}{LJs^3 + RJs^2 + \left( K_1K_2 - \frac{L}{2} HMg \sin\theta_o \right) s - \frac{R}{2} HMg \sin\theta_o} \quad 2$$

Using the table of values as shown in the table below;

Table1: Parameters and their values chosen for the design

Parameter	Value
Resistance of the resistor	0.1Ω
Inductance of the inductor	1.25mH
Motor Torque	0.1kg-m <sup>2</sup>
K <sub>2</sub>	0.5
K <sub>1</sub>	0.4
Weight(Mg)	2N

Hence with this values inputed on the transfer function in equation (2), the open loop transfer function can be written as;

$$G(s) = \frac{4000}{s^3 + 80s^2 + 1592s - 680} \quad 3$$

The general transfer function of the closed-loop system can be written as;

$$TF = \frac{GC}{1 + GC} \quad 4$$

Where G= Transfer function of the plant(system)

C = The controller function.

**The Proportional Derivative Controller;**

The general formula of the proportional derivative controller can be written as;

$$u(t) = k_p e(t) + k_d \frac{de(t)}{dt} \quad 5$$

A P controller system is a type of linear feedback control system[4]. For this kind of open loop system, [4] has been able to summarize the functions of the proportional gain as changing controller gain K can lead to variation in the closed loop dynamics. A large controller gain will result in control system with:

- a) Smaller steady state error meaning good set-point tracking.
- b) Faster dynamics, i.e. broader signal frequency band of the closed loop system and larger sensitivity with respect to measuring noise

- c) Smaller amplitude and phase margin
  - d) High set-point tracking
- and by introducing derivative mode which has the ability to predict what will happen with the error in near future and then to decrease the reaction time of the controller. A controller with Derivative action will lead to
- a) Fast response of the system
  - b) Large disturbances and noise present in the system are handled
  - c) Oscillation cancelling and damping of overshoots
- The combination of both the Proportional and Derivative action is therefore very reliable for the general Single Joint System closed loop system dynamics.

**The Proportional Integral Controller;**

The general formula for a PI controller can be written as;

$$u(t) = k_p e(t) + k_i \int e(t) dt \tag{6}$$

PI controller will eliminate steady state error resulting in set-point tracking. However, it has been observed that integral action has a disadvantage in the sense that it affects the speed and the overall stability of the system. Therefore Proportional action added can assist to increase the speed of response [9]. PI controllers are very often used in industry, especially when speed of the response is not an issue[4].

**The Proportional Integral Derivative Controller;**

The general formula for a PID controller can be written below as;

$$u(t) = k_p e(t) + k_i \int e(t) dt + k_d \frac{de(t)}{dt} \tag{7}$$

The control signal is proportional to the error signal and the proportional gain  $K_p$ . A proportional controller reduces the rise time of a system although it is not entirely eliminated. If an integrator is added, the control signal is proportional to the integral of error and the integral gain  $K_i$ [4]. Integral control will have the function of reducing the steady state error, in principle, to zero value because of the integral action. Derivative control is used to anticipate the future behavior of the error signal by using corrective actions based on the rate of change in the error signal. The control signal is proportional to the derivative of the error and  $K_d$  is the derivative gain. Derivative control will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response[4]. It is also observed that the Derivative control action can never be used alone because this control action is effective only during transient periods [4].

**III. RESULTS AND DISCUSSION**

The results of the performers of the controllers will be summarized in both the time domain and the frequency domain as will be shown by the tables below;

**Time Domain Results;**

**Table 2: Time Domain Results**

Parameter	Steady Error( $E_{ss}$ ) Radians	State in	Time Constant( $\tau$ ) in seconds	Rise Time( $t_r$ ) in seconds	Settling Time( $t_s$ ) in seconds	Maximum Overshoot( $M_p$ ) Radians	in	Percentage Overshoot ( $\%M_p$ ) in percentage
Proportional Derivative ( $K_d$ )	0.07		0.24	0.21	0.50	0		0
Proportional Integral ( $K_i$ )	0		0.76	0.20	3.1	1.19		19
Proportional Integral Derivative (PID)	0		0.17	0.15	0.70	1.07		7

**Frequency Domain Results;**

**Table 3:** Frequency Domain results

Parameter	Gain Margin(GM)	Phase Margin(PM)
<b>Proportional</b>	$\infty$	101.9850°
<b>Derivative (K<sub>d</sub>)</b>		
<b>Proportional</b>	$\infty$	81.5335°
<b>Integral (K<sub>i</sub>)</b>		
<b>Proportional</b>	$\infty$	173.8490°
<b>Integral</b>		
<b>Derivative</b>		
<b>(PID)</b>		

**IV. DISCUSSION**

It can be observed from Table 2 that PD controller has the worst performance when it comes to set-point tracking because it shows a steady state error of about 0.07, while the PI and PID controllers were able to track set-point as required for good control. In terms of speed, PID controller has the fastest response as it shows the fastest time constant of 0.17s and settling time compared to the PD and PI Controllers, however in terms of Damping the PD controller has the best performance because no overshoot was observed at steady state compared to the PI and PID controllers.

In the frequency domain as shown in Table 3, the PID controller shows the best performance because it produces an infinite gain margin and the highest phase margin compared to the other controllers.

**V. CONCLUSION**

In the control of Single Joint Systems normally found in Robotic systems using, Proportional, Integral and Derivative actions, combining all the control gains to form a PID controller gives the best response both in the time and frequency domain, however if oscillations and damping are the primary focus of the control strategy then it will be advisable to apply the PD controller since it gives the best performance in damping oscillations and overshoots.

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