The International Journal of Engineering and Science (IJES)
|| Volume || 7 || Issue || 12 Ver.I || Pages || PP 63-68 || 2018 ||
ISSN (e): 2319 – 1813 ISSN (p): 23-19 – 1805

# Position study of a four-bar mechanism using MATLAB

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-------------------------------------------------------***ABSTRACT***-------------------------------------------------------

*In this work a didactic program was developed to perform the analysis of the position of a generic mechanism of four bars. The program was carried out through the mathematical analysis of the closed loop equation of the mechanism and was implemented in the GUIDE platform of MATLAB through functions programmed in the MATLAB environment. The result allows the user to find precisely the angular positions and the final configuration of the four-bar mechanism.*

***KEYWORDS:*** *four link age, ICT, analysis position, mathematical analysis*

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Date of Submission: 20-12-2018 Date of acceptance: 04-01-2019

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## I. INTRODUCTION

Recent advances in the development of information and communication technologies (ICT) have been very important in the teaching and learning processes; currently students of different levels are more inclined towards digital technologies based on computer systems. In the teaching area, ICTs have allowed the implementation of different theories and models of learning in both theoretical and practical courses. The use of ICT in education dynamically promotes autonomous learning, student-centered learning, problem solving, cooperative, collaborative and social learning, ultimately allowing students to achieve integrated learning.

The use of ICT in the area of mechanical engineering is presented as a very powerful tool in the teaching and training process of students, based on the information stated above, the use of ICT to facilitate the assimilation of knowledge and techniques used in learning the analysis of mechanisms is presented as an area of opportunity within the training process of professionals and technicians, that having to perform repetitive and complex calculations, the student can make and learn from mistakes when viewing and interpreting data from manuals, books or figures.

In the study of the mechanisms it is important to perform a position analysis of each of its members. To define the position of a rigid body, it is necessary to specify not only its origin coordinates (x, y), but also its length and orientation in space. There are different methods to perform the position analysis of a mechanism, the most basic method is graphing, where through a hand drawing or with some computer aided design (CAD) software, the position of the links of a mechanism is completely characterized, however, the graphical method is a method that involves a lot of processing time and is not feasible for the measurement of positions in multiple locations.

Although the graphical method to analyze the position of a mechanism is simple and direct, the most accurate results are obtained through the analytical method. In the analytical method, the development of the equations is required to obtain the angular positions of output as a function of the lengths of the links and the value of the angular position of entry [1, 2, 3].

A four-bar mechanism is the simplest mobile mechanism used in the design of machines. The four-bar mechanism consists of four rigid bodies called bars or links with a fixed link [4]. The four links are joined by pivots to form a closed loop (see Figure 1).

In this work, a didactic computational tool is created to help the students who study subjects such as "analysis and synthesis of mechanisms" to strengthen the knowledge necessary for the position analysis of a four-bar mechanism.

## II. METHODS

***Mathematical analysis***

In the present work, a program was created in the GUIDE graphic environment of MATLAB, that allows the student to know the positions of the generic four-bar mechanism shown in Figure 1, as a starting point, we have the analysis of the equation of closed loop (ec.1) of the mechanism through the complex variable diagram represented in Figure 2 [5].

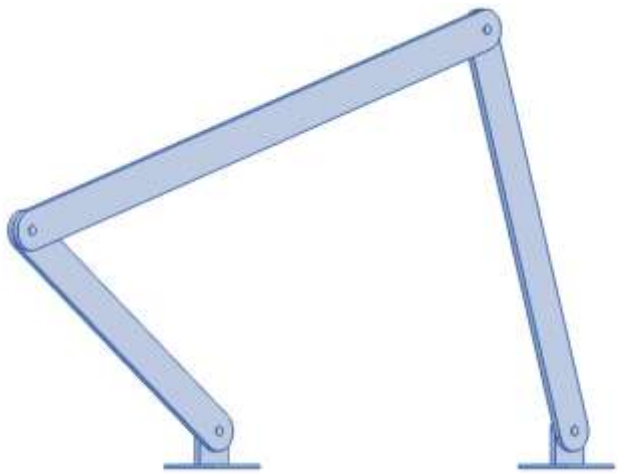

**Figure1. Configuration of a four-bar mechanism to be studied.**

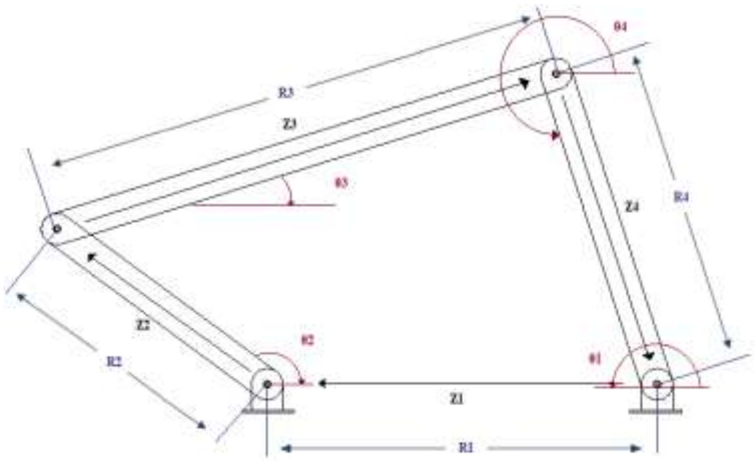

**Figure2. Representation of the variablesin the mechanism to be studied.**

$$z_1 + z_2 + z_3 + z_4 = 0 \qquad (1)$$

where the representation in exponential form for the complex numbers of each link is:

$$z_n = R_n e^{i\theta_n}$$

The variables in the closed-loop equation are the lengths of the four links: R1, R2, R3 and R4, as well as the angular positions of the four links represented by θ1, θ2, θ3 and θ4. The reference system is taken such that θ1 = π. To solve the eq. (1) it must be presented by a maximum of two unknowns and thus be able to solve algebraically Eq. 1 (real part and imaginary part). The position analysis for this four-bar configuration presents 4 different cases, where z1 and z2 are known variables.

**Case 1. Two unknown lengths ($R_3$ and $R_4$)**

The solution of ec.1 in this case is determined by Eq.2:

$$\begin{bmatrix} R_3 \\ R_4 \end{bmatrix} = -\frac{1}{\sin(\theta_4-\theta_3)} \begin{bmatrix} sin\theta_4 & -cos\theta_4 \\ -sin\theta_3 & cos\theta_3 \end{bmatrix} \begin{bmatrix} R_1 cos\theta_1 + R_2 cos\theta_2 \\ R_1 sin\theta_1 + R_2 sin\theta_2 \end{bmatrix} (2)$$

**Case 2. An unknown length and angle**
**Case 2a. Unknown length and angle of the same link.**

Assuming that R3 and θ3 are unknown. Mathematically, the solution to this case is trivial and is represented by Eq. 3

$$z_3 = -(z_1 + z_2 + z_4) (3)$$

**Case 2b. Unknown length and angle of different links.**
Assuming that R4 and θ3 are the variables to be found, then the solution of the closed-loop equation is expressed by Eq. 4.

$$R_4 = -R_1 \cos(\theta_1 - \theta_4) - R_2 \cos(\theta_2 - \theta_4) \pm \frac{\sqrt{1-q^2}}{q} [R_1 sin(\theta_1 - \theta_4) - R_2 sin(\theta_2 - \theta_4)] (4)$$

where:

$$q = sen(\theta_3 - \theta_4) \quad (5)$$

$$sin(\theta_3 - \theta_4) = \frac{-R_1 \sin(\theta_1-\theta_4) - R_2 \sin(\theta_2-\theta_4)}{R_3} (6)$$

And the solution will be dependent on the eq. (4) - (6)

**Case3. Two unknown angular positions.**
Assuming that the positions θ3 and θ4 are the variables to be found, the solution of Eq. (1) will be two groups of valid equations, the first group are the eq. (7) - (8):

$$\theta_3 = \theta_T + \cos^{-1}\left(\frac{R_4^2 - R_T^2 - R_3^2}{2 R_T R_3}\right) \qquad (7)$$

$$\theta_4 = \theta_T + 2\,\pi - \cos^{-1}\left(\frac{R_3^2 - R_T^2 - R_4^2}{2 R_T R_4}\right) \qquad (8)$$

And the second pair of solutions are the eq. (9) - (10): are:

$$\theta_3 = \theta_T + 2\,\pi - \cos^{-1}\left(\frac{R_4^2 - R_T^2 - R_3^2}{2 R_T R_3}\right) \qquad (9)$$

$$\theta_4 = \theta_T + \cos^{-1}\left(\frac{R_3^2 - R_T^2 - R_4^2}{2 R_T R_4}\right) \qquad (10)$$

where:

$$Z_T = R_T e^{i\theta_T} = R_1 e^{i\theta_1} + R_2 e^{i\theta_2} \qquad (11)$$

***Flow diagram and algorithms***

For the programming of the application, the flow diagram of the necessary operations was made and discussed, which is shown in Figure 3. The start of the program is through the selection of the system for measuring the angular positions of the mechanism to be studied (degrees sexagesimal or radians). The next step in the structure of the program is to select some of the 3 cases of study (4 with case 2b), then it proceeds to ask for the input variables necessary to perform the calculation of the two possible unknowns depending on the case of study. In this step it is necessary to convert the string type variables to numerical variables for its mathematical treatment. With the established case, we resort to programming equations (2), (4), (6), (9) and (10) to find the particular unknowns, as shown in Figure 4.

To show the final graph of the studied four-bar mechanism, line tracing functions were used through two points, establishing for each line an origin relative to the previous link, and drawing the links of different colors: red, yellow, blue and green for better identification, as shown in Figure 5.

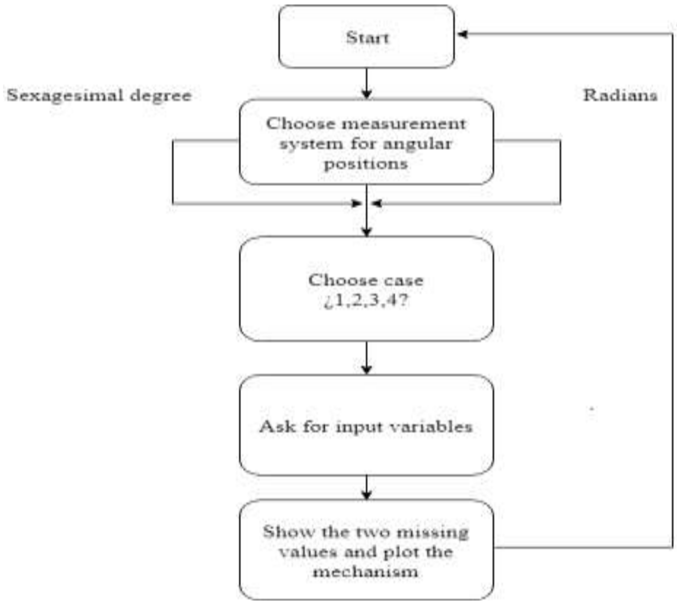


**Figure3. Flow diagram of the program developed for position analysis of a four-bar mechanism**

**Figure4. Code for the acquisition of variables and calculation of unknowns**

**Figure5. Code made for the tracing of the four-bar mechanism**

## III. RESULTS AND DISCUSION

In the development of the graphical interface [6,7], it was sought to be very user friendly, with only two main windows being obtained at the end. The first window is the start window and allows selecting the type of case required and the system of angular units to be selected, see Figure 6.

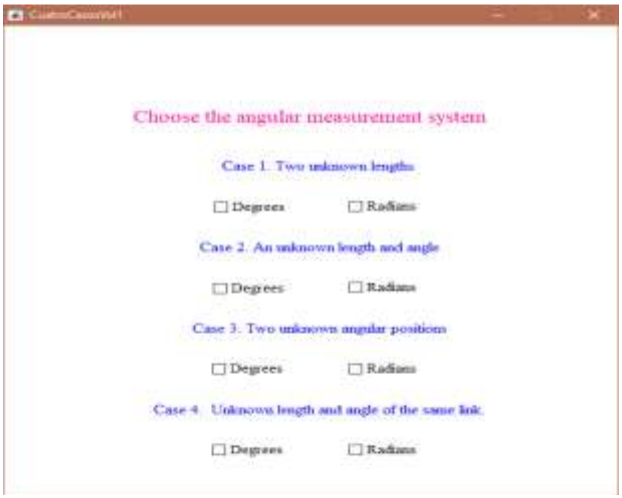


**Figure6. Start window to perform the position analysis of a four-bar mechanism**

Then the process of acquiring the necessary information is carried out through a graphical user interface and through a window it is displayed: i) the mathematical solution of the four-bar mechanism and ii) the graphic representation of the solved mechanism. In this same window the program allows the return to the start window to make the selection of another possible case. Figure 6 shows the dimensions for a four bar linkage so and Table 1 has the information that characterizes the mechanism. To find the values of the lengths of link 3 and link 4 analytically, equation 2 is used and the results shown in equation 13 are obtained.

$$\begin{bmatrix} R_3 \\ R_4 \end{bmatrix} = -\frac{1}{sin(63.435° - 180°)} \begin{bmatrix} sin(63.435°) & -\cos(63.435°) \\ -sin(180°) & \cos(180°) \end{bmatrix} \begin{bmatrix} 300\cos(270°) + 640.3124\cos(308.663°) \\ 300 sin(270°) + 640.3124 sin(308.663°) \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} 800.0158 \\ 894.4019 \end{bmatrix} \quad (13)$$

| Link | Length (unit arbitraries) | Angle (Degrees) |
|---|---|---|
| 1 | 300 | 270 |
| 2 | 640.3124 | 308.663 |
| 3 | Unknown | 180 |
| 4 | Unknown | 63.435 |

**Table1. Lengths and angular positions of a four-bar mechanism**

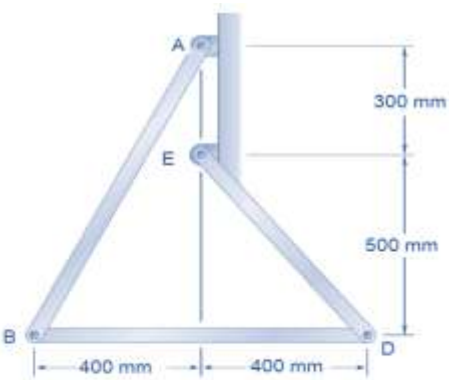


**Figure7. Four-bar mechanism to study**

The results when performing the simulation are shown in Figure 8.

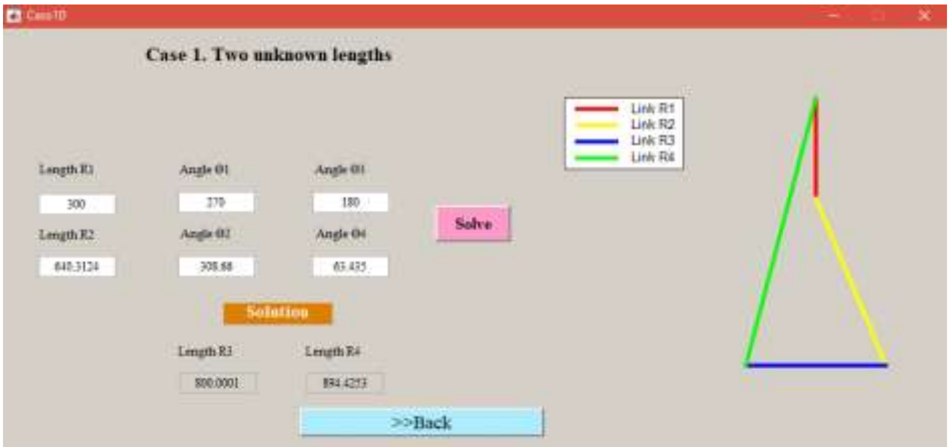


**Figure8. Window of calculation and results in the analysis of position of a four-bar mechanism**

## IV. CONCLUSION

Through the MATLAB platform, a didactic program was developed to find the angular position and, where appropriate, the length of the links that make up a four-bar mechanism. A simulation was obtained that characterizes a mechanism of four bars using MATLAB to be able to find the angular positions and the lengths of each one of the links in dependence on the known variables. The application shows a very powerful tool in the use of teaching the analysis of mechanisms for students of mechanical engineering or mechatronic engineering. The program been helpful to strengthen the competence to analyse the mechanisms in two dimensions through graphical and analytical methods.

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Erasto Vergara Hernández. "Position study of a four-bar mechanism using MATLAB" The International Journal of Engineering and Science (IJES), 7.12 (2018): 63-68