

## Unsteady MHD Free Convective Heat And Mass Transfer Flow Past An Inclined Surface With Heat Generation

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### -----ABSTRACT-----

Because of great importance of thermal instability in nature, chemical processes, separation processes, and industrial applications as well as in geophysical and astrophysical engineering, the effect of thermal diffusion on unsteady MHD free convective heat and mass transfer flow past an inclined surface with heat generation has been investigated in this paper. Numerical as well as differential perturbations of solutions for the primary velocity field, secondary velocity field, temperature distribution as well as concentration distributions are obtained for associated parameters using the explicit finite difference method. The obtained results are discussed with the help of graphs to observe effects of various parameters on the above mentioned quantities. Finally, important findings of the investigations are concluded.

**Keywords:** MHD, free convection, mass transfer flow, inclined surface, heat generation. *Mathematics Subject Classification (MSC):* 76Dxx, 76Sxx

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### NOMENCLATURE

Gr : Grashof Number	$u$ : Velocity component in x-direction
Gm : Modified Grashof Number	$v$ : Velocity component in y-direction
Sc : Schimidt Number	L : Length of the boundary layer
$B_0$ : Magnetic Field	$\nabla$ : Differential operator
M : Magnetic Parameter	<b>Greek Symbols:</b>
R : Rotation	$\delta$ : Boundary layer thickness
Pr : Prandtl Number	$\mu$ : Co-efficient of viscosity
$\alpha$ : Heat Source Parameter	$\nu$ : Co-efficient of kinematic viscosity
Re : Reynolds Number	$\rho$ : Density of the fluid in the boundary Layer
S : Suction Parameter	
t : Time	<b>Subscripts:</b>
$\tau$ : Shear Stress	w : Condition at wall
J : Current Density	$\infty$ : Condition at infinity
Nu : <u>Nusselt</u> Number	

### I. INTRODUCTION

The effect of thermal diffusion and magnetic field on MHD boundary layer flow has become significant in the field of mechanical and chemical engineering . MHD heat transfer has great importance in the liquid metal flows, ionized gas flow in an nuclear reactor and electrolytes. Many researchers works on radiation of heat and through these have modern applications in army, nuclear power plant, parts of aircraft and heat radiation with or without magnetic field. The free convection and mass transfer flow of an electrically conducting fluid past an inclined surface under the action of induced magnetic field has effective application in various sectors such as in astrophysics, geophysics and many engineering problems. In light of these applications, Umemura and Law (1990) generalized a formulation for the natural convection boundary layer flow over a flat plate with arbitrary inclination. They found that the flow characteristics depend not only on the extent of inclination but also on the distance from the leading edge. Hossain et al. (1996) investigated the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Bestman and Adjepong (1998) studied the unsteady hydro-dynamics free convection flow with radiative heat transfer. Anghel et al. (2001) presented a numerical solution of free convection flow past an inclined surface. Chen (2004) studied the

momentum, heat and mass transfer characteristics of MHD natural convection flow over a permeable, inclined surface with variable wall temperature and concentration, taking into consideration the effects of ohmic heating and viscous dissipation. Mohammad M. Rahman (2010) studied hydromagnetic heat and mass transfer flow over an inclined heated surface with variable viscosity and elastic conductivity. Das et al. (2010) investigated the effects of mass transfer flow past an impulsively started infinite vertical plate with constant heat flux in the presence of chemical reaction.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Vajravelu and Hadjinicolaou (1993) studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. In that study they considered that the volumetric rate of heat generation  $q''' [w.m^3]$  should be  $q''' = Q_0(T - T_\infty)$  for  $T \geq T_\infty$  and equal to zero for  $T < T_\infty$ , where  $Q_0$  is the heat generation/absorption constant. The above relation is valid as an approximation of the state of some exothermic process and having  $T_\infty$  as the onset temperature. When the inlet temperature is not less than  $T$ , they used  $q''' = Q_0(T - T_\infty)$ . The effect of conjugate conduction heat transfer along a thin vertical plate with non-uniform heat generation was studied by Mendez and Trevino (2000). Hossain et al. (2004) studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation /absorption. Vasu et al. (2011) investigated radiation and mass transfer effects on transient free convection flow of a dissipative fluid past an semi-infinite vertical plate with uniform heat and mass flux. Seth et al. (2012) studied unsteady hydromagnetic couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of an inclined magnetic field taking Hall current into account. Asymptotic behavior of the solution was analyzed for small and large values of time  $t$  to highlight the transient approach to the final steady state flow and the effects of Hall current, magnetic field, rotation and angle of inclination of magnetic field on the flow-field. They showed that hall current and rotation tend to accelerate fluid velocity in both the primary and secondary flow directions while Magnetic field has a retarding influence on the fluid velocity in both the primary and secondary flow directions.

Magneto-hydrodynamics (MHD) flow problems have become considerable attention to the researchers because of its significant applications in industrial manufacturing processes such as plasma studies, petroleum industries, MHD power generator cooling of clear reactors, boundary layer control in aerodynamics and crystal growth. Many authors have studied the effects of magnetic field on mixed, natural and force convection heat and mass transfer problems. Indeed, MHD laminar boundary layer behavior over a semi-infinite vertical plate is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. This problem has also an important bearing on metallurgy where MHD techniques have recently been used. Considering the aspects of rotational flows, model studies were carried out on MHD free convection and mass transfer flows in a rotating system by many investigators. Hossain (1992) studied the effects of viscous and Joule heating on the flow of viscous incompressible fluid past a semi-infinite plate in presence of a uniform transverse magnetic field. The combined effects of forced and natural convection heat transfer in the presence of a transverse magnetic field from vertical surfaces are also studied by many researchers. In recent years, considerable attention has been given to study the heat transfer characteristics in boundary layer flow of a Newtonian fluid past a flat plate because of its extensive application in production engineering. Alam et al. (2013) applied similarity transformations to investigate heat and mass transfer characteristics of MHD free convection of steady flow of an incompressible electrically conducting fluid over an inclined plate under the influence of an applied uniform magnetic field taking into account the effects of Hall current.

The aim of the present work is to study free convective heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting viscous incompressible fluid in the presence of a magnetic field and heat generation. The investigation has been made for solving the system of nonlinear partial differential equations. For this purpose an explicit finite difference technique has been used for which non-similar solutions of the coupled non-linear partial differential equations are sought.

## II. MATHEMATICAL FORMULATION

We consider an unsteady two-dimensional hydro-magnetic flow of a viscous incompressible, electrically conducting fluid past a semi- infinite inclined plate with an acute angle  $\alpha$  to the vertical. The governing equations under the usual Boussinesq's and boundary layer approximations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = g \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\alpha + g\beta^*(C - C_\infty)\cos\alpha - \frac{\sigma B_0^2}{\rho}u \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

With the corresponding initial and boundary conditions:

$$\text{At } t = 0 \quad u = 0, v = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ everywhere} \quad (5)$$

$$u = 0, v = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ at } x = 0$$

$$t > 0 \quad u = 0, v = 0, T \rightarrow T_w, C \rightarrow C_w \text{ at } y = 0 \quad (6)$$

$$u = 0, v = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ at } y \rightarrow \infty$$

where x, y are the coordinates in Cartesian coordinate system; u, v are components of fluid velocity respectively; g is the local acceleration due to gravity;  $\beta$  is the thermal expansion coefficient;  $\beta^*$  is the concentration expansion coefficient;  $\nu$  is the kinematic viscosity;  $\rho$  is the density of the fluid;  $B_0$  is the constant induced magnetic field;  $\sigma$  is the electrical conductivity;  $\kappa$  is the thermal conductivity;  $C_p$  is the specific heat at the constant pressure; D is the coefficient of mass diffusivity.

Since the solutions of the governing equations (1) - (4) under the initial condition (5) and boundary condition (6) will be based on finite difference method it is require to make the said equations dimensionless. For this purpose we now introduce the following dimensionless quantities;

$$X = \frac{xU_0}{\vartheta}, Y = \frac{yU_0}{\vartheta}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \tau = \frac{tU_0^2}{\vartheta},$$

$$\bar{B}_0 = \sqrt{\frac{\mu_e B_0}{\rho U_0}} \bar{T} = \frac{T - T_\infty}{T_w - T_\infty} \quad \bar{C} = \frac{C - C_\infty}{C_w - C_\infty}$$

Now, from the above dimensionless variables we have

$$X = \frac{xU_0}{\vartheta} \Rightarrow xU_0 = \vartheta X \Rightarrow x = \frac{\vartheta X}{U_0} \Rightarrow \partial x = \frac{\vartheta}{U_0} \partial X$$

$$Y = \frac{yU_0}{\vartheta} \Rightarrow yU_0 = \vartheta Y \Rightarrow y = \frac{\vartheta Y}{U_0} \Rightarrow \partial y = \frac{\vartheta}{U_0} \partial Y$$

$$U = \frac{u}{U_0} \Rightarrow u = UU_0 \Rightarrow \partial u = U_0 \partial U$$

$$V = \frac{v}{U_0} \Rightarrow v = VU_0 \Rightarrow \partial v = U_0 \partial V$$

$$\tau = \frac{tU_0^2}{\vartheta} \Rightarrow tU_0^2 = \vartheta \tau \Rightarrow t = \frac{\vartheta}{U_0^2} \tau \Rightarrow \partial t = \frac{\vartheta}{U_0^2} \partial \tau$$

$$\bar{T} = \frac{T - T_\infty}{T_w - T_\infty} \Rightarrow T = T_\infty + \bar{T}(T_w - T_\infty) \Rightarrow \partial T = (T_w - T_\infty) \partial \bar{T}$$

$$\bar{C} = \frac{C - C_\infty}{C_w - C_\infty} \Rightarrow C = C_\infty + \bar{C}(C_w - C_\infty) \Rightarrow \partial C = (C_w - C_\infty) \partial \bar{C}$$

Using these relations we obtained the following non-linear coupled partial differential equations in terms of dimensionless variables

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + Gr\bar{T}\cos\alpha + Gm\bar{C}\cos\alpha - MU \quad (8)$$

$$\frac{\partial \bar{T}}{\partial \tau} + U \frac{\partial \bar{T}}{\partial X} + V \frac{\partial \bar{T}}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \bar{T}}{\partial Y^2} + Q\bar{T} \quad (9)$$

$$\frac{\partial \bar{C}}{\partial \tau} + U \frac{\partial \bar{C}}{\partial X} + V \frac{\partial \bar{C}}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 \bar{C}}{\partial Y^2} \quad (10)$$

where,  $Gr = \frac{\vartheta g \beta (T_w - T_\infty)}{U_0^2}$  is the Grashof number;  $Gm = \frac{\vartheta g \beta^* (C_w - C_\infty)}{U_0^2}$  is the Modified Grashof number;

$M = \frac{\sigma \vartheta B_0^2}{\rho U_0^2}$  the Magnetic parameter;  $Pr = \frac{\vartheta \rho c_p}{k}$  the Prandtl number;  $Q = \frac{Q_0 \vartheta}{\rho c_p U_0^2}$  the Hat generation and

$Sc = \frac{\vartheta}{D_m}$  is the Schmidt number.

Also the associated initial and boundary conditions become

$$\tau = 0 \quad U = 0, \bar{T} = 0, \bar{C} = 0 \tag{11}$$

Everywhere,

$$\tau \geq 0 \quad \begin{cases} U = 0, \bar{T} = 0, \bar{C} = 0 \text{ at } X = 0 \\ U = 0, \bar{T} = 1, \bar{C} = 1 \text{ at } Y = 0 \\ U = 0, \bar{T} = 0, \bar{C} = 0 \text{ as } Y \rightarrow \infty \end{cases} \tag{12}$$

### III. NUMERICAL SOLUTION

To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes where X-axis is taken along the plate and Y-axis is normal to the plate. Here we consider that the plate of height  $X_{max} (= 100)$  i.e. X varies from 0 to 100 and regard  $Y_{max} (= 25)$  i.e. Y varies 0 to 25. There are  $m=125$  and  $n=125$  grid spacing in the X and Y directions respectively as shown in Fig. a.

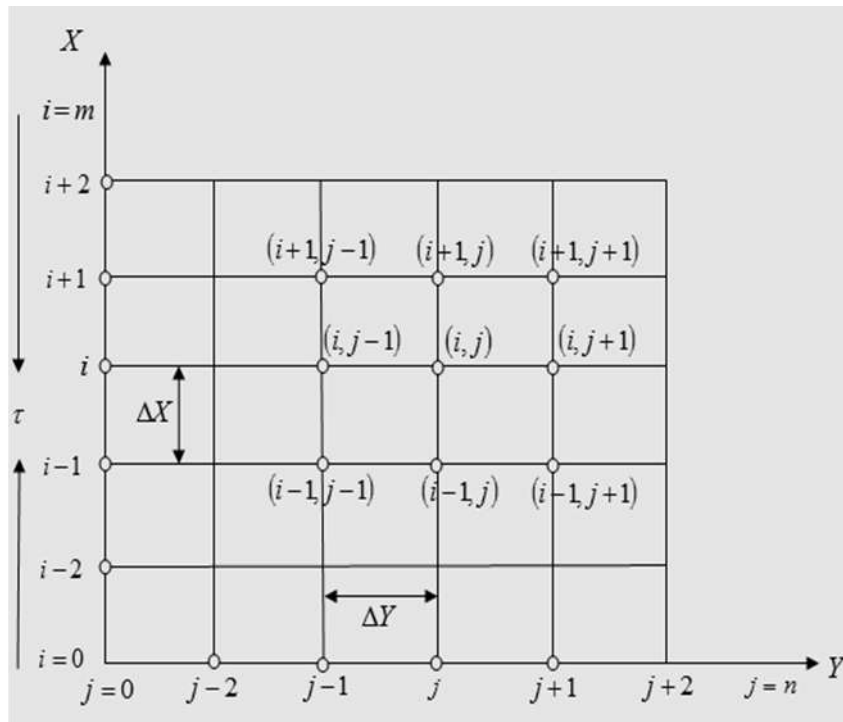


Fig. a: The finite difference space grid

Let  $U', V', \bar{T}', \bar{C}'$  are denoted the values of  $U, V, \bar{T}, \bar{C}$  at the end of a step of time respectively.

Using the explicit finite difference approximation we have,

$$\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \tag{13}$$

$$\begin{aligned} \frac{U'_{i,j} - U_{i,j}}{\Delta \tau} + U_{i,j} \frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j} \frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \\ = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + Gr \bar{T}'_{i,j} \text{Cosa} + Gm \bar{C}'_{i,j} \text{Cosa} - M U_{i,j} \end{aligned} \tag{14}$$

$$\frac{\bar{T}'_{i,j} - \bar{T}_{i,j}}{\Delta \tau} + U_{i,j} \frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta X} + V_{i,j} \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta Y}$$

$$= \frac{1}{Pr} \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta Y)^2} + Q \bar{T}_{i,j} \quad (15)$$

$$\frac{C'_{i,j} - C_{i,j}}{\Delta \tau} + U_{i,j} \frac{C_{i,j} - C_{i-1,j}}{\Delta X} + V_{i,j} \frac{C_{i,j+1} - C_{i,j}}{\Delta Y} = \frac{1}{Sc} \frac{C_{i,j+1} - 2C_{i,j} + C_{i,j-1}}{(\Delta Y)^2} \quad (16)$$

Boundary conditions:

$$\begin{aligned} U_{i,j}^0 &= 0, V_{i,j}^0 = 0, \bar{T}_{i,j}^0 = 0, \bar{C}_{i,j}^0 = 0 \\ U_{0,j}^n &= 0, V_{0,j}^n = 0, \bar{T}_{0,j}^n = 0, \bar{C}_{0,j}^n = 0 \\ U_{i,0}^n &= 0, V_{i,0}^n = 0, \bar{T}_{i,0}^n = 1, \bar{C}_{i,0}^n = 1 \\ U_{i,L}^n &= 0, V_{i,L}^n = 0, \bar{T}_{i,L}^n = 0, \bar{C}_{i,L}^n = 0 \quad \text{where } L \rightarrow \infty \end{aligned} \quad (17)$$

Here the subscripts  $i$  and  $j$  designate the grids points with  $x$  and  $y$  coordinates respectively and superscript  $n$  represents a value of time,  $\tau = n\Delta\tau$  where  $n = 0, 1, 2, 3, \dots$ . From the initial condition (11), the values of  $U, \bar{T}, \bar{C}$  are known at  $\tau = 0$ . During any one time-step, the coefficients  $U_{i,j}$  and  $V_{i,j}$  appearing in equation (14)-(16) are treated as constants. Then at the end of any time-step  $\Delta\tau$ , the new temperature  $\bar{T}'$ , the new concentration  $\bar{C}'$ , the new velocity  $U', V'$  at all interior nodal points may be obtained by successive application of equations (16), (15), (14) and (13) respectively. This process is repeated in time and provided the time-step is sufficiently small  $U, V, \bar{T}, \bar{C}$  should eventually converge to values which approximate the steady state solution of equations (1) - (4). These converged solutions are shown graphically in Figs. 1 - 15.

#### IV. RESULTS AND DISCUSSION

For the purpose of discussing the result of the problem, the approximate solutions are obtained for different parameters. We have calculated the unsteady state numerical values of the velocity  $U$ , temperature  $\bar{T}$  and concentration  $\bar{C}$  for different values of Magnetic parameter  $M$ , Heat generation parameter  $Q$ , Prandtl number  $Pr$ , Schmidt number  $Sc$ , Angle of inclination  $\alpha$ .

The values of Schmidt number  $Sc$  are taken for hydrogen ( $Sc = 0.22$ ), water vapor ( $Sc = 0.60$ ) and carbon dioxide ( $Sc = 0.94$ ). Throughout the calculations, physical variables  $Gr = 12$  and  $Gm = 6$  are taken which correspond to a cooling problem that is generally encountered in nuclear engineering in connection with cooling of a reactor. Finally the values of  $M, Q$  and  $\alpha$  are chosen arbitrarily. The flow behaviors are discussed graphically along with the obtained unsteady state solutions. The profiles of velocity, temperature and concentration versus  $Y$  are presented in Figs 1 - 15.

The velocity profiles have been shown in Figures 1, 4, 7, 10 and 13. The effect of Magnetic field parameter  $M$  on the velocity profile is represented in Fig. 1. From this figure it is observed that the increase of magnetic field leads to a decrease in the velocity field indicating that the magnetic field retards the flow field. The effect of Heat generation parameter  $Q$  on the velocity profile is shown on Fig. 4. It is seen from this figure that when heat is generated the buoyancy force increases, which induces the flow rate to increase, giving rise to the increase in the velocity profile. The effects of Prandtl number  $Pr$  on the velocity is shown in Fig. 7. The velocity  $U$  decreases when Prandtl number  $Pr$  increases. Figure 10 shows the effect of Schmidt number  $Sc$  on velocity of the air ( $Pr = 0.71$ ) boundary layer. It is observed that increasing the Schmidt number decreases the velocity. In Fig. 13, the effect of inclination of the surface on velocity is shown where we observe that the fluid (air) velocity is decreased for increasing angle  $\alpha$ . The fluid has higher velocity when the surface is vertical ( $\alpha = 0$ ) than when inclined because of the fact that the buoyancy effect decreases due to gravity components ( $g \cos \alpha$ ), as the plate is inclined.

The temperature profiles have been shown graphically in Figs. 2, 5, 8, 11 and 14. The effect of magnetic field parameter is represented by Fig. 2. We see that an increase in the magnetic field leads to rise in the temperature distribution. When the value of Heat generation parameter increases, the temperature distribution also increases rapidly which is shown in Fig. 5. It is seen from Fig. 8 that when Prandtl number  $Pr$  increases, the temperature distribution leads to a fall. The temperature distribution increases with the increase of Schmidt number. It is also noticed from Fig. 11 that the variation in the thermal boundary layer is very small due to moderate change in Schmidt number. The effect of inclination of temperature is shown in Fig. 14 where we see that when inclination  $\alpha$  increases, the temperature of air boundary layer also increases.

The concentration profiles have been represented graphically in Figs. 3, 6, 9, 12 and 15. We see from Fig. 3, the concentration profile increases with the increases of Magnetic field parameter  $M$ . With the increase of Heat generation parameter  $Q$  and Schmidt number  $Sc$ , the concentration profiles decrease as shown in Figs. 6 and 12, respectively. We observe a rise in the concentration profiles with the increase of Prandtl number  $Pr$  and inclination  $\alpha$  as shown in Figs. 9 and 15, respectively.

The effects of the above –mentioned parameters on the local skin–friction coefficients ( $Cf$ ), local Nusselt number ( $Nu$ ) and the local Sherwood number ( $Sh$ ) for air ( $Pr = 0.71$ ) and water ( $Pr = 7.0$ ) are shown in Table 1.

**Table 1:** Numerical values of  $Cf, Nu$  and  $Sh$  for  $Gr = 12, Gm = 6, Sc = 0.22, M = 0.25$  and  $\alpha = 30^\circ$ .

Pr	M	Q	Cf	Nu	Sh
0.71	0.25	0.50	2.8114	0.0464	0.2580
0.71	1.00	0.50	2.3428	-0.1360	0.2334
0.71	1.50	0.50	2.1508	-0.2648	0.2237
0.71	0.25	0.50	2.8114	0.0464	0.2580
0.71	0.25	1.00	3.5287	-0.7431	0.2957
0.71	0.25	1.50	4.6943	-2.2242	0.3413
7.00	0.25	0.50	2.4523	-0.9334	0.2235
7.00	1.00	0.50	2.3672	-1.8918	0.2023
7.00	1.50	0.50	2.2569	-2.3219	0.1845
7.00	0.25	0.50	2.4523	-0.9334	0.2235
7.00	0.25	1.00	4.1744	-7.1718	0.2703
7.00	0.25	1.50	6.8052	-20.7015	0.3181

There are many works found on the heat and mass transfer related topic in different cases. Alam (2006) investigated on MHD free convective Heat and mass transfer flow past an inclined surface with heat generation. Which is almost similar to this investigation but he used shooting iteration method for numerical solution. His investigation was in steady state. Here we have used explicit finite difference method for numerical solution. The explicit finite difference method is fast and inexpensive computationally. The finite difference method using a rectangular grid provides accurate velocity and displacement fields close to the inclusion boundary. In the present study, we have considered unsteady-state condition; because in practical applications the concept of steady state is not significantly used.

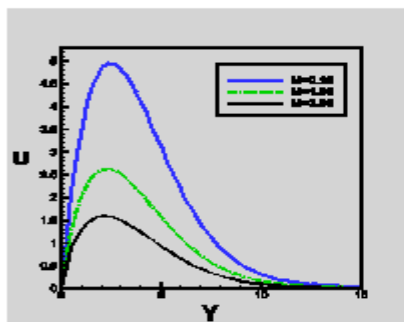


Fig. 1: Velocity profile for different values of  $M$ .

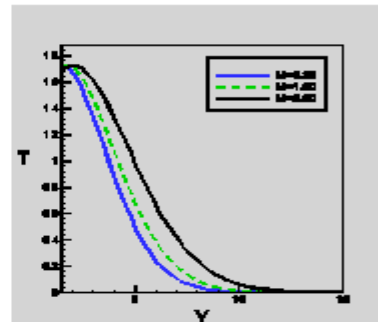


Fig. 2: Temperature profile for different values of  $M$ .

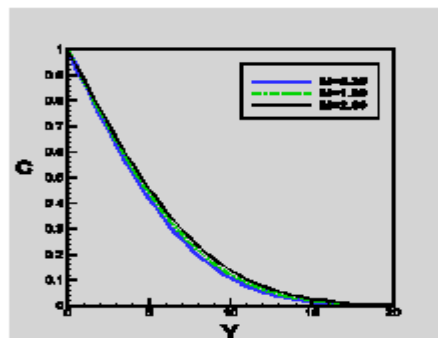


Fig. 3. Concentration profile for different values of  $M$ .



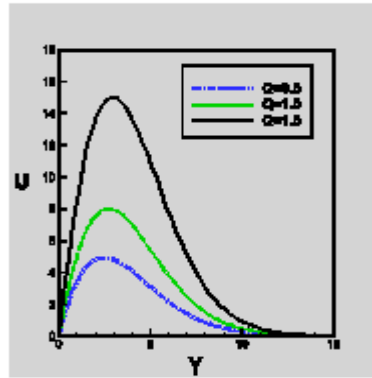


Fig. 4. Velocity profile for different values of  $Q$ .

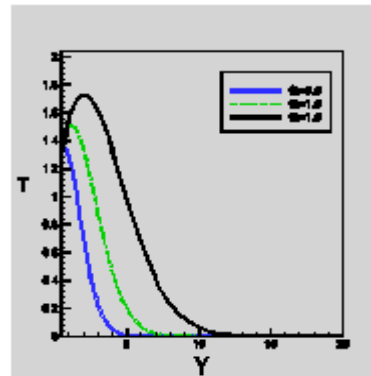


Fig. 5. Temperature profile for different values of  $Q$ .

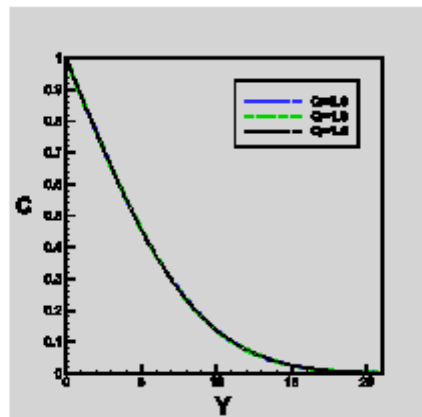


Fig. 6. Concentration profile for different values of  $Q$ .

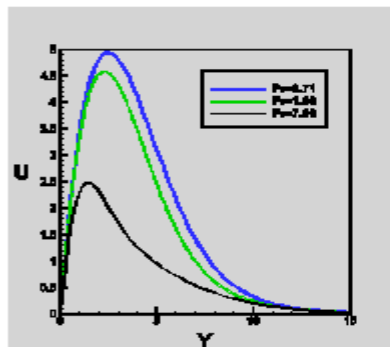


Fig. 7. Velocity profile for different values of  $Pr$ .

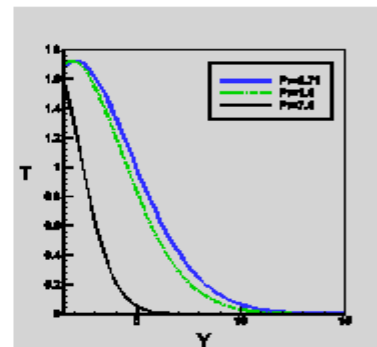


Fig. 8. Temperature profile for different values of  $Pr$ .

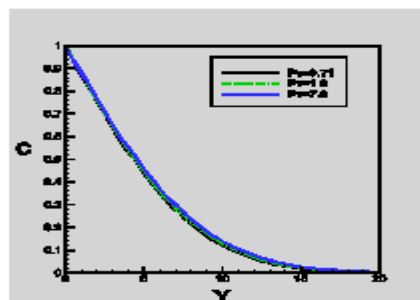


Fig. 9. Concentration profile for different values of  $Pr$ .

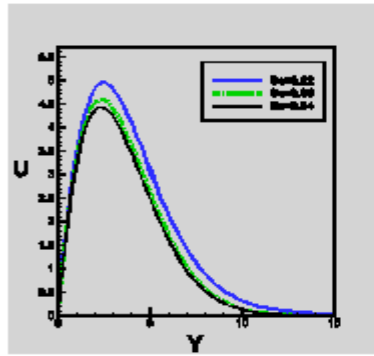


Fig. 10. Velocity profile for different values of  $Sc$ .

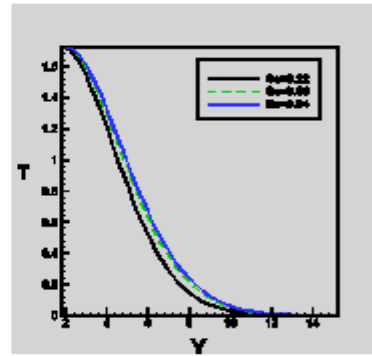


Fig. 11. Temperature profile for different values of  $Sc$ .

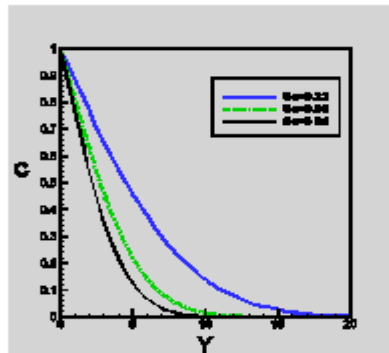


Fig. 12. Concentration profile for different values of  $Sc$ .

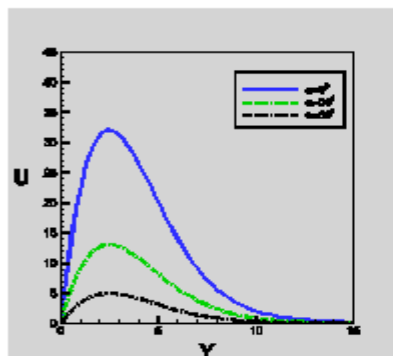


Fig. 13. Velocity profile for different values of  $\alpha$ .

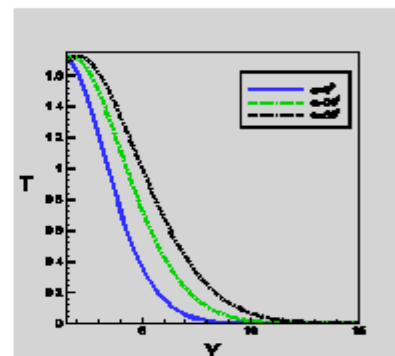


Fig. 14. Temperature profile for different values of  $\alpha$ .

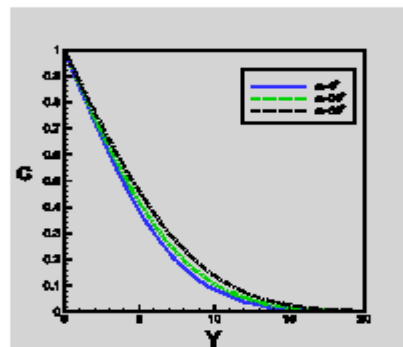


Fig. 15. Concentration profile for different values of  $\alpha$ .



## V. CONCLUSIONS

In this study, an unsteady MHD free convective heat and mass transfer flow past an inclined surface with heat generation. The governing system of dimensionless coupled non-linear partial differential equations are numerically solved by using explicit finite difference method. The results are discussed for magnetic parameter  $M$ , heat generation parameter  $Q$ , Prandtl number  $Pr$ , Schmidt number  $Sc$  and angle of inclination  $\alpha$  on velocity  $U$ , temperature  $\bar{T}$  and concentration  $\bar{C}$ . In this study, the physical variables Grashof number  $G_r = 12$ , and modified Grashof number  $G_m = 6$  are taken which correspond to a cooling problem that's generally encounter in nuclear engineering in connect with cooling of a reactor. The results show that

1. The velocity increases with the increase of  $Q$  while it decreases with the increase of  $M$ ,  $Pr$ ,  $\alpha$  and  $Sc$ .
2. The temperature increases with the increase of  $M$ ,  $Q$ ,  $Sc$  and  $\alpha$  while it decreases with the increase of  $Pr$ .
3. The Concentration increases with the increase of  $M$ ,  $Pr$  and  $\alpha$  while it decreases with the increase of  $Q$  and  $Sc$ .

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