

A Short Review To Compressible Flow

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ABSTRACT

The objective of this work is to review the one-dimensional flow, in a permanent, adiabatic and reversible regime of an ideal gas in nozzles. A conceptual theoretical approach will be made to the state of stagnation and its properties, as well as the relationship between velocity and area in a compressible flow.

Keywords: compressible flow, stagnation, velocity, area.

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I. INTRODUCTION

The compressible flow of a gas is the branch of fluid mechanics that deals with flows and with significant changes in the density of fluids. In general, the flows are generally treated⁽¹⁾ as incompressible when the Mach number is less than 0.3; since in this case the change in density due to speed is negligible. The study of compressible flow is relevant for high-speed aircraft, jet engines, rocket engines, high-speed entry in a planetary atmosphere, pipelines, commercial applications such as abrasive blasting and many other fields. When constant flow velocities approach the speed of sound, compression is inevitable. For example, high-velocity projectiles and combat aircraft move at speeds beyond the speed of sound, while spacecraft and meteorites generally move at a speed equal to that of sound.

During the occurrence of supersonic speeds by a device, there is a strong compression of the air in front of it which gives rise to a discontinuity or shock wave pressure.

II. STATE OF STAGNATION

In questions related to flows, many discussions and equations can be simplified by introducing the concept of isentropic stagnation state and associated properties. The state of isentropic stagnation⁽¹⁾ is the state that the fluid would have suffered if an adiabatic and reversible deceleration to zero velocity had been observed. In order to establish the mathematical conditions of this state first we have to define the concept of Mach number, M. It is the ratio between the real velocity, V, and the speed of sound, which will be represented by the letter a.

$$M = \frac{V}{a} \quad (1)$$

The importance of Mach's number in the analysis of problems involving fluid outflows will be evident in the following paragraphs. This approach, both conceptual and analytical, can be found in the books by ANDERSON⁽¹⁾; FOX and MCDONALD⁽²⁾; HILL & PETERSON⁽³⁾; SHAPIRO⁽⁴⁾.

In isentropic flow in a nozzle occurs variation of velocity, temperature and pressure, but the enthalpy of stagnation remains constant. This approach allows the calculation of fluid properties from initial stagnation conditions and output condition information to any point in the nozzle (cross section).

The figure 1 shows a hypothetical flow occurring through a nozzle.

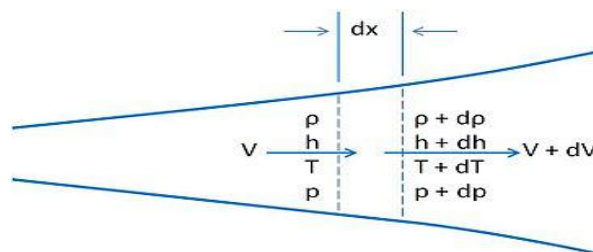


Figure:1 - Illustration of a nozzle ; Source : <https://www.google.com.br/search>

EQUATIONS OF STAGNATION

| | |
|------------------------------------|---|
| Enthalpy of Stagnation | $h_s = c_p T_s = c_p T + \frac{v^2}{2}$ |
| Stagnation temperature | $T_s = T + \frac{v^2}{2c_p}$ |
| Specific heat at constant pressure | $c_p = \frac{kR}{(k-1)}$ |
| Equation of the velocity of sound | $a = \sqrt{kRT}$ |
| Being that | $k = \frac{c_p}{c_v}$ |

Developing mathematically the set of equations mentioned is easily arrived at:

$$\frac{T_s}{T_0} = 1 + \left(\frac{k-1}{2}\right) M^2$$

$$\frac{P_s}{P_0} = \left[1 + \left(\frac{k-1}{2}\right) M^2\right]^{k/k-1}$$

$$\frac{\rho_s}{\rho_0} = \left[1 + \left(\frac{k-1}{2}\right) M^2\right]^{1/k-1}$$

$$M = \frac{V_{\text{FLOW}}}{\sqrt{kRT}}$$

III. ESTADO CRITICO

The critical state of a nozzle flow is defined ⁽²⁾ when it reaches the condition of $M = 1$, which yields the following equations.

$$\frac{T_s}{T_c} = \left(\frac{k+1}{2}\right)$$

$$\frac{P_s}{P_c} = \left[\left(\frac{k+1}{2}\right)\right]^{k/k-1}$$

$$\frac{\rho_s}{\rho_c} = \left[\left(\frac{k+1}{2}\right)\right]^{1/k-1}$$

IV. DIVERGENT CONVERGING NOZZLE

Nozzles ⁽³⁾ are components that exhibit a pressure variation and therefore have the ability to change the velocity of the fluid flowing through the various sections of area. Its uses in the field of Fluid Mechanics and Thermodynamics are diverse, such as wind tunnels, high-energy gas dynamics, and the exhaust system of a turbojet aircraft.

Figure 2 shows a nozzle or convergent and divergent. The cross section presenting the smallest area is called diffuser with throat sections.

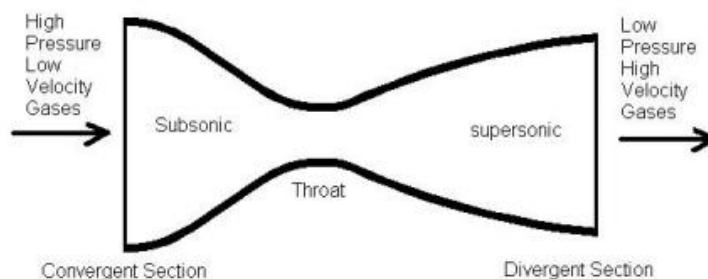


Figure:2 - Divergent divergent nozzle - Source: <https://www.google.com/search>

Our first considerations relate to the conditions that determine whether a nozzle or diffuser should be convergent or divergent and the conditions prevailing in the throat. A characteristic ⁽³⁾ of this flow is caused by the change of speeds when it reaches the Mach limit equal to unity. With this value, shock waves can occur that can prevent the increase of speed, hindering the supersonic flow.

The following relationships can be written to the control volume shown in Figure 2.

By deriving the equation of continuity we have to

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

On the other hand, considering the equation derived from the speed of sound

$$a^2 = \frac{dp}{d\rho}$$

and making some elementary modifications comes

$$\frac{d\rho}{\rho} = \frac{1}{a^2} \frac{dp}{\rho}$$

Finally, taking into account Bernoulli's equation and rewriting in differential form, we have

$$p + \frac{1}{2}\rho V^2 = \text{constant}$$

$$\rho V dV = -dP$$

What allows us to arrive at the next relation between speed and area

$$\frac{dA}{A} = (M^2 - 1) \frac{dV}{V}$$

V. DISCUSSION

The equation $\frac{dA}{A} = (M^2 - 1) \frac{dV}{V}$ allows evaluating⁽⁴⁾ the flow behavior in a convergent-divergent nozzle. For a subsonic flow ($M < 1$) the term $(M^2 - 1)$ will have the same negative signal and the velocity in the flow will be inversely proportional to the cross-sectional area. For a supersonic flow ($M > 1$), the term $(M^2 - 1)$ will have a positive sign and the variation of velocity in the flow will be directly proportional to the cross-sectional area. This analysis⁽⁴⁾ determines that a convergent nozzle is used to achieve the sonic velocity ($M = 1$). To obtain supersonic velocities ($M > 1$) a convergent-divergent nozzle, designed for sonic velocity ($M = 1$) at its throat, is required for the development of supersonic velocity in the divergent section. For $M \rightarrow 0$, which in the limit corresponds to the incompressible flow, the previous equation shows that $A v = \text{const}$, which is familiar to the continuity equation for incompressible flows. For $0 \leq M < 1$ we have subsonic flow, where, an increase in velocity is associated to a decrease of the area and vice versa. For $M > 1$ (supersonic flow), an increase in velocity is associated with an increase in area and vice versa. For $M = 1$, sonic flow, we have $dA/A = 0$, which mathematically corresponds to a minimum or maximum in the area distribution.

VI. FINAL CONSIDERATIONS

The flow in convergent and divergent nozzles is commonly presented from the isentropic point of view, that is, there is no irreversibility provided by the energy dissipation in the flow. This way of studying the phenomenon leads to useful results and is relevant to explain the main characteristics of these types of flow. However, fluid flows are highly dissipative, which makes it interesting to verify the effects of this dissipation in the formulations generally presented and the possibilities of some influences that may be relevant in the understanding of this type of flow. On the other hand, the compressibility effects on the mass flow rate have some unexpected results. We can increase the mass flow through a pipe by increasing the area, increasing the total pressure or decreasing the total temperature. But the effect of increasing speed is a bit hard to figure out. Mass flow rate limitation is called flow blocking. The phenomenon is called nozzle clogged flow.

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