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Minimization the Mass of Planetary Gear Train using Different Types of Materials

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--------------------ABSTRACT--------------------
Planetary gear trains can be found in such high-technological systems as automotive transmissions, aircraft motors and wind turbines. They are distinguished with compact dimensions, less noise and higher torque-to-weight ratios, when compared to parallel gearboxes. Minimization the mass of planetary gear train can improve the efficiency of the gearbox through transmitting more torque values. An optimization model for minimization of planetary gear train mass is studied here under constraints of bending and contact stresses. The design variables used are the module, gear teeth width, number of teeth for both sun and planet gears, inner diameter of both sun and planet gears and the outer diameter of the ring gear. Different types of materials have been used in this study. The optimization problem has been formulated and solved by Genetic Algorithm using the MATLAB optimization Toolbox routines. Center distance between sun gear and planet gear for the optimized results has been calculated and figured Results indicated that the optimum mass of planetary gear train has maximum values in case of Cast Iron while has minimum values in case of plastic material.

KEYWORDS;- *Optimization–Planetary gear train–Center Distance– Tooth bending stress – Contact stress*

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Nomenclature

P_{in}	*Input Power of Motor (KW)*	K_o	*Power Source and Driven Machine Factor*
N_{in}	*Input Speed (rev/min)*	Y_N	*Stress Cycle Life*
h_s	*Gear Tooth Height (m)*	Z_N	*Pitting Resistance Stress Cycle*
α	*Pressure Angle*	C_p	*Elastic Coefficient Factor*
a	*Center Distance (m)*	K_f	*Fatigue Concentration Factor*
ρ	*Density of Gear Material*	C_H	*Hardness Ratio Factor*
K_v	*Dynamic Factor*	I	*Pitting Geometry Factor*
Y_j	*Modified Lewis Factor*	K_R	*Reliability Factor*
K_a	*Application Factor*	m	*Module*
K_s	*Size Factor*		
K_m	*Load Distribution Factor*		
K_i	*Idler Factor*		
K_B	*Rim Thickness Factor*		

I. INTRODUCTION

Among all power transmitting machine elements, gears are the most important mechanical element [1]. Gears are employed in wide range of applications such as aeronautics, automobile, machine tools, wind turbines, children toys and house electrical instruments [2].The simplest of all gears is the spur gears; they offer considerable precision and high power transmission efficiency [3]. Planetary gear train is a form of gearbox structure which consists of four elements. The four elements are mainly; a sun gear, one or more planet gears, a ring gear and an arm (planet carrier) [4] as shown in Figure 1. Sun gear is always located at the center and transmits torque to planet gears orbiting around the sun gear. The planet gears are mounted on an arm or carrier (surrounded by the ring gear) that fixes the planets in an orbit relative to each other [5]. Planetary gears are found in many variations and arrangements in order to meet a broad range of speed ratios in the design requirements. One element of the planetary gear train configuration is fixed and the other elements are rotated

according to the input and output members. Different configurations can be easily obtained by re-arranging input member, outer member and stationary member.
Planetary gearboxes have a various application in different mechanical systems, such as industrial drives, rotorcraft, automobiles, wind turbines, etc., where they can offer compact dimensions, less noise, high gear ratio and higher torque-to-weight ratios, especially compared to standard parallel axis gear trains [6, 7, 8]

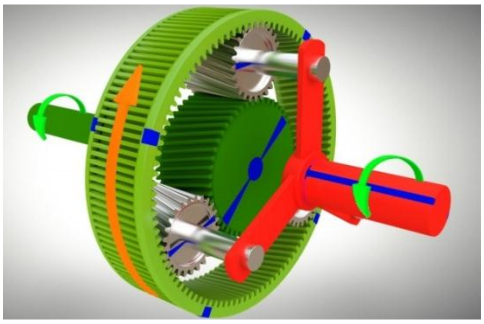

Figure 1: Planetary Gear train

S.B. Nandeppagoudar, et al [9] designed a three stage planetary gear box suitable for machine tool application. Reduction of material has been one the critical aspects of any design along with reduction in deformation and stress factors, which increases the life of the product as presented by Fatmir Azemi, et al [10], authors presented design, analysis and shape optimization in order to reduce material of spur gears and focuses on static analysis. Finite element analysis has been used to implement optimization and maintaining stress and deformation levels. Brahim Mahiddini, et al [11] presented a two level design optimization methodology for a simple reducer. The first level is based on using an analytical formulation for the design problem based on classical gear and beam theory, while the second level was used to construct a fine CAD model of the reducer. In fact, the design of the planetary gear train is highly complicated and involves many aspects. This complexity leads to many design variables, mathematical formulations, constraints, and many influencing factors as studied by Kaoutar Daoudi, et al [12]. They described a multi-objective optimization for the epicyclical gear train system using the GA. The purpose of this study is to minimize the weight and the centre distance of one pair of spur gears. This objective was accomplished by means of the GA under some constraint such as bending strength, contact stress and each dimension conditions of gears, which must be satisfied. The results are calculated by using MATLAB tools of Genetic algorithm for three types of materials, which are alloy steel, cast iron, and epoxy glass composites. The multi-objective genetic algorithm theory is used to optimize the structure of the automobile gearbox as shown by JIN Xiangjie et al [13].S. Padmanabhan et al [14] optimized a two-stage gear reducer with major conflict functions like minimization of gear material volume, minimization of centre distance, maximization of power and maximization of efficiency as objectives with design stresses as the constraints. We have considered two different types of materials for this study Based on the MATLAB multi-objective genetic algorithm toolbox Paridhi Rai and Asim Gopal Barman [2], have extended a gear design model to include more AGMA geometrical factors along with some modifications compared to earlier studies. The design optimization of the spur gear set is carried out using two powerful optimization tools, simulated annealing and real-coded genetic algorithm. Tae Hyong Chong et al [15] used Genetic Algorithm for optimum design of minimum volume for 2 parallel stages and planetary gear train

II. MATERIALS AND METHODOLOGY

According to the geometry for the gear profile, the basic dimensions of a gear are verified as shown in Figure 2

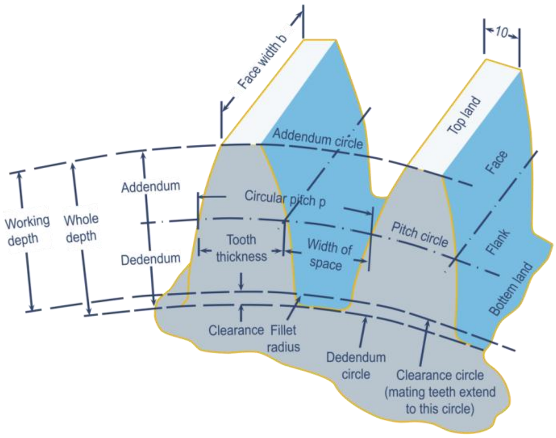

Figure 2: Gear geometry

The gear size is one of the most important variables in design optimization of gear set used in many applications [2].

Pitch circle diameter $d_P = m * z$

Addendum $h_a = m$

Dedendum $h_f = 1.157m$

Clearance $C = 0.157m$

Planetary (Epicyclic) Gear train (which consists of sun, planet and ring gear)
z_s = No of teeth in sun wheel gear
z_p = No of teeth in planet gear
z_r = No of teeth in annulus ring gear

$$a = \frac{m * (z_s + z_p)}{2}$$

The three gear forms must share the same module *m to* mesh properly

$$d_{p\_r} = 2 * d_{p\_p} + d_{p\_s} \qquad (1)$$

$$d_{r\_s} = d_{p\_s} - 2 * h_f$$

$$d_{r\_p} = d_{p\_p} - 2 * h_f$$

$$d_{r\_r} = d_{p\_r} + 2 * h_f$$

$$d_{o\_s} = d_{p\_s} + 2 * h_a$$

$$d_{o\_p} = d_{p\_p} + 2 * h_a$$

$$d_{i\_r} = d_{p\_r} - 2 * h_a$$

Where d_p pitch diameter, d_r root diameter, d_o outer diameter and d_i inner diameter
The subscripts s, p and r are used for sun, planet and ring gear respectively

Spur Gear Teeth Stresses

Gears in gear boxes sometimes get short life due to wear and breakage by repetitive load during operation time [8], In planetary gear train, the gears should have some specific properties in order to maintain their function during working [16] Ultimate tensile strength (UT) to prevent failure against static loads., Fatigue strength (FS) to withstand dynamic loads, Low coefficient of friction (CF) and sufficient hardness (H) [16]
The two primary failure modes for gears [5] (as shown in Figure 3) are:

1) **Tooth Breakage** - from excessive bending stress, and
2) **Surface Pitting/Wear** - from excessive contact stress

Bending stress plays a significant role in gear design wherein its magnitude is controlled by the nominal bending stress and the stress concentration due to the geometrical shape. The bending stress is indirectly related to shape changes made to the cutting tool [17]

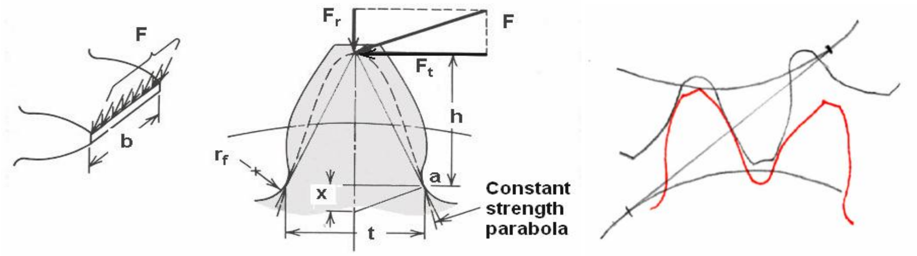

Figure 3: Bending and surface contact stresses on gear tooth

$$\sigma_b = \frac{F_t}{b*m*Y_j} * K_s * K_a * K_v * K_m * K_B * K_f \quad (2)$$

Where F_t is the tangential force acting on the tip of the gear tooth, Y_j is approximately 0.34

$$K_v = \frac{50}{50+\sqrt{V}} \text{ V in ft/min} \quad (3)$$

Linear speed V is

$$V = \frac{\pi * d_{p\_s} * N_{in}}{60} \quad (4)$$

$$\sigma_c = C_p \sqrt{\frac{F_t}{b*d_{p\_s}*I} * K_o * K_s * K_m * K_v} \quad (5)$$

$$C_p = \sqrt{\frac{1}{\pi * \left(\frac{1-(\mu_s)^2}{E_s} + \frac{1-(\mu_p)^2}{E_p} \right)}} \quad (6)$$

Where μ_s and μ_p are coefficient of friction for both sun and planet gears (the same material)
E_s and E_p are modulus of elasticity for both sun and planet gears (the same material)

$$T = \frac{60 * P_{in}}{2 * \pi * N_{in}} \quad (7)$$

$$F_t = \frac{T}{d_{p\_s}/2} \quad (8)$$

$$\sigma_y = 0.75 * \sigma_u \quad (9)$$

$$\sigma_{b\_all} = 0.6 * \sigma_y * \frac{Y_N}{K_R * SF} \tag{10}$$

$$\sigma_{c\_all} = \sigma_y * \frac{Z_N * C_H}{K_R * SF} \tag{11}$$

Parameters considered for material selection

Some considerations should be taken while choosing the materials of gears for design and manufacturing the gearbox. These considerations are mechanical properties, manufacturing considerations, availability and cost [2].
The mechanical properties of the different materials used in this study are presented in Table 1 and the values of data input for the optimization problem that can be assumed [3 and12] as presented in Table2

	Modulus of Elasticity (E)	**Ultimate Tensile Strength (σ_u)**	**Density ρ**	**Poisson's Ratio μ**
	GPa	Mpa	Kg/m^3	
Alloy Steel	209	600	7800	0.29
Stainless Steel	200	620	8000	0.27
Cast Iron	140	150	7200	0.28
Aluminum	72	257	2600	0.33
Brass	101	300	8450	0.35
Ceramic	410	380	3150	0.14
Plastic Polyethylene PET	2.5	50	1300	0.37
Composite	60	130	1750	0.25

Table 1: Mechanical Properties of different materials [18, 19]

P_{in}	1.1, 2.2, 3, 4, 5.5, 7.5	K_o	1.25
N_{in}	1500	Y_N	1
α	20	Z_N	1.1
Y_j	0.34	K_R	1
K_a	1	K_f	1
K_s	1	C_H	1
K_m	1.2	K_B	1
K_i	1	I	0.14
S.F	1.5		

Table 2: Data Input

To calculate the mass of the planetary gear train, the masses of sun gear, planet gears and ring gear are to be calculated as shown below:

Sun Gear

$$A_s = \frac{\pi}{4}\left(d_{r\_s}{}^2 - d_{i\_s}{}^2\right) + \left(0.5 * \frac{\pi}{4}\left(d_{o\_s}{}^2 - d_{r\_s}{}^2\right)\right) \tag{22}$$

$$M_S = V_S * \rho = A_S * b * \rho \quad (23)$$

Planet Gear

$$A_p = \frac{\pi}{4}\left(d_{r\_p}{}^2 - d_{i\_p}{}^2\right) + \left(0.5 * \frac{\pi}{4}\left(d_{o\_p}{}^2 - d_{r\_p}{}^2\right)\right) \quad (24)$$

$$M_P = V_P * \rho = A_P * b * \rho \quad (25)$$

Ring Gear

$$A_r = \frac{\pi}{4}\left(d_{o\_r}{}^2 - d_{r\_r}{}^2\right) + \left(0.5 * \frac{\pi}{4}\left(d_{r\_r}{}^2 - d_{i\_r}{}^2\right)\right) \quad (26)$$

$$M_r = V_r * \rho = A_r * b * \rho \quad (27)$$

$$M = M_s + (3 * M_p) + M_r$$

Genetic Algorithm in MATLAB Toolbox

Optimization is the process of finding the minimum or maximum value that a particular function attains and finding the value for the independent variables of objective function [20]. GA is a heuristic method, which is based on natural selection, the process that drives biological evolution. GA starts by generating a population of individuals in the space search. The choice of the size of population and the manner for representing the individual solutions are very important in order to promote the success of the method [11]. In fact, it is a series of random iterations and evolutionary computations which simulate the process of selection, crossover, and mutation occurred in natural selection and population genetics [12]

Formulation Optimization Model

The optimization model using MATLAB Toolbox routines consists of three files, objective function file, nonlinear constraints function file and genetic algorithm solver file.
The minimal mass design optimization problem can be formulated as follows:

Find the design variables vector

$$x = \left(z_s, z_p, m, b, d_{i\_s}, d_{i\_p}, d_{o\_r}\right) \quad (30)$$

Which minimizes the objective function

$$f = Min(M) \quad (31)$$

Subject to the behavior constraints:

$$\sigma_b \le \sigma_{b\_all} \quad (32)$$

$$\sigma_c \le \sigma_{c\_all}$$

$$6 \le \frac{b}{m} \le 12$$

$$z_p > z_s$$

And the side constraints (limits in mm): $\vec{X}_L \leq \vec{X} \leq \vec{X}_U$

$$17 \leq z_s \leq 30 \tag{33}$$

$$17 \leq z_p \leq 40$$

$$1 \leq m \leq 5$$

$$10 \leq b \leq 40$$

$$20 \leq d_{i\_s} \leq 40$$

$$20 \leq d_{i\_p} \leq 40$$

$$300 \leq d_{o\_r} \leq 700$$

z_s and z_p are integer

III. RESULTS AND DISCUSSION

The results obtained from the optimization model have been collected in the next table, Table 3

Input Power	Material Type	Optimal Solutions		Center Distance
		$x_{opt} = (Z_S, Z_P, m, b, ds_S, ds_P, d_r)$	M_{opt}	a
1.1 KW	Alloy Steel	(30, 40, 2, 12, 38, 40, 300)	4.2554	70
	Stainless Steel	(30, 40, 2, 12, 38, 40, 300)	4.3645	70
	Cast Iron	(24, 29, 4, 31, 40, 40, 422)	19.7983	106
	Aluminum	(22, 28, 3, 18, 40, 40, 302)	2.0282	75
	Brass	(21, 28, 3, 18, 39, 40, 300)	6.578	73.5
	Ceramic	(25, 30, 3, 23, 40, 40, 328)	3.7353	82.5
	Plastic	(21, 28, 3, 18, 39, 40, 300)	1.012	73.5
	Composite	(20, 24, 4, 25, 40, 40, 352)	2.6454	88
2.2 KW	Alloy Steel	(21, 28, 3, 18, 39, 40, 300)	6.0722	73.5
	Stainless Steel	(19, 29, 3, 18, 35, 40, 300)	6.3537	72
	Cast Iron	(25, 30, 5, 34, 40, 40, 546)	37.1066	137.5
	Aluminum	(26, 32, 3, 18, 40, 40, 347)	2.7335	87
	Brass	(23, 28, 3, 24, 28, 40, 305)	9.0473	76.5
	Ceramic	(25, 30, 4, 24, 40, 40, 437)	7.2076	110
	Plastic	(23, 28, 3, 22, 40, 40, 305)	1.2576	76.5
	Composite	(25, 30, 4, 31, 40, 40, 437)	5.1721	110
3 KW	Alloy Steel	(22, 28, 3, 18, 39, 40, 302)	6.0932	75
	Stainless Steel	(22, 28, 3, 18, 40, 40, 302)	6.2405	75
	Cast Iron	(29, 35, 5, 34, 40, 40, 634)	50.364	160
	Aluminum	(27, 33, 3, 22, 40, 40, 358)	3.5668	90
	Brass	(28, 34, 3, 22, 40, 40, 369)	12.348	93
	Ceramic	(20, 24, 5, 32, 40, 40, 440)	9.8085	110
	Plastic	(25, 30, 3, 25, 40, 40, 328)	1.6756	82.5
	Composite	(30, 36, 4, 29, 37, 40, 522)	6.9944	132
4 KW	Alloy Steel	(24, 29, 3, 20, 30, 40, 317)	7.5651	79.5
	Stainless Steel	(24, 29, 3, 18, 40, 40, 317)	6.904	79.5
	Cast Iron	x	x	x
	Aluminum	(22, 27, 4, 24, 40, 40, 392)	4.7494	98
	Brass	(19, 23, 4, 35, 40, 40, 337)	16.3045	84
	Ceramic	(30, 36, 4, 30, 40, 40, 522)	13.007	132
	Plastic	(21, 26, 4, 26, 40, 40, 377)	2.3714	94
	Composite	(24, 29, 5, 38, 40, 40, 527)	9.377	132.5
5.5 KW	Alloy Steel	(24, 29, 3, 27, 40, 40, 317)	10.097	79.5
	Stainless Steel	(25, 30, 3, 22, 40, 40, 328)	9.074	82.5
	Cast Iron	x	x	x
	Aluminum	(24, 29, 4, 28, 40, 40, 422)	6.4575	106
	Brass	(24, 29, 4, 29, 40, 40, 422)	21.7364	106
	Ceramic	(26, 32, 5, 34, 40, 40, 577)	18.204	145
	Plastic	(18, 22, 5, 30, 40, 40, 402)	3.146	100
	Composite	(29, 35, 5, 35, 40, 40, 634)	12.6014	160
7.5 KW	Alloy Steel	(22, 27, 4, 24, 40, 40, 392)	14.248	98

	Stainless Steel	(20, 24, 4, 26, 40, 40, 352)	12.577	88
	Cast Iron	x	x	x
	Aluminum	(22, 24, 5, 34, 40, 40, 440)	8.6019	115
	Brass	(20, 24, 5, 36, 40, 40, 440)	29.6007	110
	Ceramic	(29, 35, 5, 36, 40, 40, 634)	23.3306	160
	Plastic	(20, 24, 5, 32, 40, 40, 440)	4.048	110
	Composite	x	x	x

Table 3: Optimization Model Results

The output mass in Kg and gears dimensions in mm

From the results obtained in the previous table, a bar graph has been drawn relating optimum mass with respect to input power for different gear materials as shown in Figure 4 below. The x values in the above table indicated that constraints are violated so the algorithm couldn't attain the objective solution for these cases.

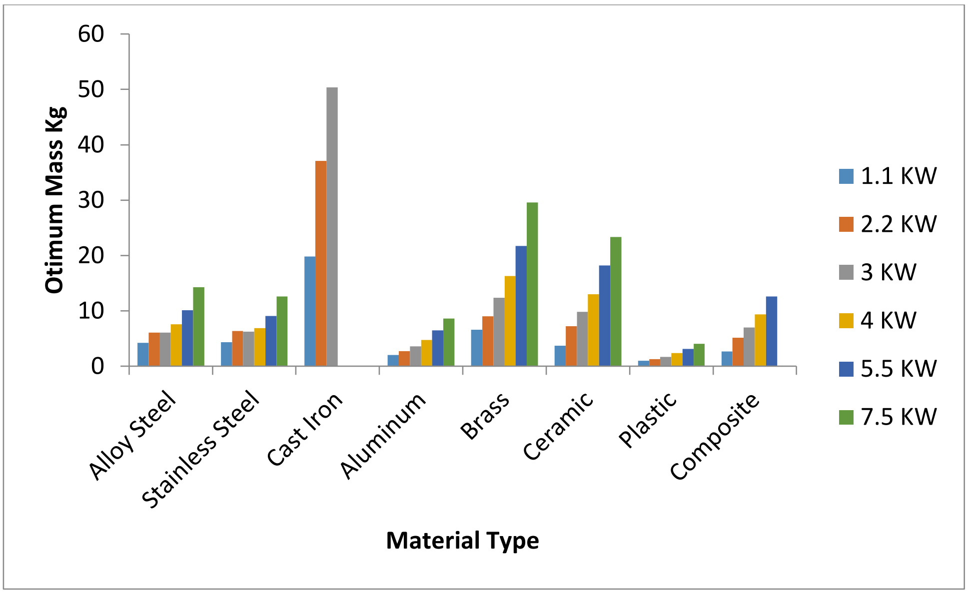

Figure 4: Optimum mass of planetary gear train related with input power for different materials

Results obtained have shown that for each material type studied in this research, the optimum mass is increased with increasing the input power from 1.1 KW to 7.5 KW at the same input speed.
Comparing the materials studied, it is found that cast iron has the maximum output mass between different materials because it has low ultimate tensile strength as shown in table 1 while in case of plastic material it has the minimum values of optimum mass because it has very low density.
In case of alloy steel and stainless steel, they have moderate and nearest values of optimum mass
Also in case of brass and ceramic, they have nearest values and high optimum mass
Composite material and aluminum have low and nearest values of optimum mass
The different values of optimum mass for the studied materials are due to the different mechanical properties of each material

The center distance between the sun gear and planet gear has been calculated for all cases studied and a relationship has been drawn between material type and center distance for different input power as shown in Figure 5 below

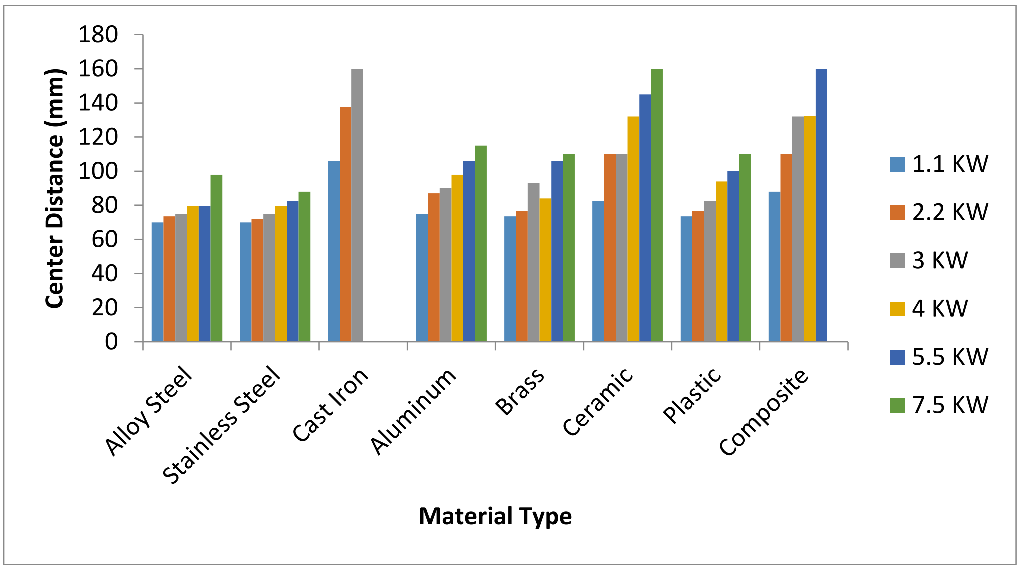

Figure 5: Center distance between sun and planet gear related with input power for different materials

As shown in the above figure, the center distance is increased with increasing the input power for all different materials. The highest values of center distances have observed in the case of cast iron, ceramic and composite materials.

IV. CONCLUSION

An optimization problem has been formulated using MATLAB optimization toolbox routings implementing genetic algorithm to obtain the minimal mass of planetary gear train. Linear, nonlinear and side constraints have been formulated related to design variables.

Results obtained have shown that the optimization model succeeded in arriving the optimum values of design variables.

Results indicated that the optimum mass of planetary gear train has maximum values in case of Cast Iron while has minimum values in case of plastic material.

The center distance is increased with increasing the input power for all different materials

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