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Study of Hysteresis and Eddy Current loss of One-dimensional Magento-thermoelastic problem

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---ABSTRACT---
This paper deals with the propagation of magneto thermoelastic interactions in an isotropic homogeneous perfectly conducting thermoelastic medium in the context of Fourier's law of heat conduction and Ohm's law of electromagnetism. One dimensional application of a thin metallic rod of the finite length of ferromagnetic material which is subjected to a time-dependent magnetic field is solved using Laplace transform and finite Fourier sine transform. Due to time-dependent magnetic fields, conducting currents are produced which give rise to Eddy current and consequently result in heat loss known as Eddy current loss. At the same time due to the time lag between magnetization and demagnetization of magnetic material some amount of energy is lost which is termed Hysteresis loss. We have treated this total loss as the heat source for the problem. The differential equations governing the distribution of temperature fields are formulated and solved. The numerical calculations are carried out for displacement, temperature and stresses. The results obtained are displayed graphically to illustrate the influence of wave frequency, hysteresis loss, Eddy current loss and time-dependent magnetic field.

KEYWORDS; - *Magneto-thermoelasticity, Eddy current, Hysteresis loss, time-varying magnetic field, integral transform*

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I. INTRODUCTION

Investigations of electro-magneto-thermoelastic interactions where we study the relationship and interactivity between strain, temperature and electromagnetic fields in a thermoelastic material are of great practicable significance due to its wide range of applications in various fields like geophysics, damping acoustic waves in magnetic field, designing of heat exchangers, boiler tubes in which elastic deformations occurs due to induced temperature, electrical power engineering, plasma physics and many more.

Magneto-thermoelasticity is a subject where we study the interactions between magnetic, thermal and mechanical fields in a thermoelastic solid in the presence of a magnetic field. Magneto-elasticity is the theory of studying the coupling between electromagnetic and deformation, thermoelasticity combines elasticity and heat conduction to study the coupling theory between temperature field and elastic field. Magneto-thermoelasticity includes the heat conduction theory, classical elasticity theory and electromagnetic theory. These theories are applied to solve the coupling problems of temperature field, electromagnetic field and elastic field of conductive elastic solids. The theoretical foundation of magneto-thermoelasticity was presented by **[1]** and **[2]** and developed by **[3]**. Propagation of plane waves using thermo-elastic solid inside a magnetic field was studied by **[4]** and developed the theoretical framework of the advancement of magneto-thermoelasticity. Propagation of magneto–thermoelastic waves in a non-rotating medium was studied by **[5]**. The above studies were based on the theory of classical coupled thermoelasticity, with interaction among the electromagnetic field, the thermal field, and the elastic field, as well as the dispersion relation, was taken into consideration. Effect of small couplings related to thermoelasticity and magnetoelasticity of an unbounded isotropic medium using Perturbation technique was studied by **[6]**. A one-dimensional thermal shock problem of generalized thermoelastic electrically conducting half-space permeated by a primary uniform magnetic field with thermal relaxation was discussed by **[7]**. A two-dimensional half-space problem was discussed by **[8]** using electro-magneto-thermoelasticity theory which was explored to a non-uniform thermal shock. A mathematical model of generalized magneto-thermoelasticity in a perfectly conducting medium was developed by **[9]**. Problems of magneto-thermoelasticity with thermal relaxation and heat source in the infinite rotating elastic medium in three-dimensional spaces was investigated by **[10]**. Magnetoelastic plane waves in rotating media with uniform angular velocity was studied by **[11]**. A new mathematical model was developed by **[12]** for the equations of the two-temperature magneto-thermoelasticity theory. **[13]** investigated the interaction of a homogeneous and isotropic perfect conducting half-space with rotation, in the context of Lord-Shulman theory. Recently a two-dimensional problem of magneto-thermoelasticity in thermosensitive finite conducting plates with eddy current loss was studied by **[14]**.

The study of the interaction between the magnetic field and the strain field in a thermoelastic solid is receiving considerable attention in recent years due to its many applications in the field of geophysics, plasma physics and related topics. Especially in nuclear fields, the extremely high temperatures and temperature gradients, as well as the magnetic field originating inside nuclear reactors, influences their design and operations. The present article is an attempt to study the effect of eddy current loss (aroused due to Joule heat generated by Eddy current) and Hysteresis loss (aroused due to time lag between magnetization and demagnetization) on one dimensional non-linear HCE as an extension to the research work **[15]**. The derived expressions are computed numerically for steel material and results are presented graphically. Effects of Eddy current loss along with Hysteresis loss and magnetic field quantities are also analyzed. The integral transform technique is used to find the temperature solution. Further thermal and magnetic components of stresses are obtained using this non-dimensional temperature solution.

II. PROBLEM FORMULATION

Consider a one-dimensional isotropic homogeneous perfectly conducting thin rod of finite length c occupying the region $0 \le z \le c$ with temperature-dependent properties. We assume that this rod is subjected to the time-dependent, one-dimensional magnetic field $H_0\Omega(t)$ acting along the y direction. Thus, all variables depend on z and t only. For one dimensional problem, all considered functions will depend on z and t and the displacement vector has one component in $z-$direction. We assume that the components of the magnetic field, induced electric field and current density are

$$\bar{H} = \left(0, H_y(z,t), 0\right), \bar{E} = \left(E_x(z,t), 0, 0\right), \bar{J} = \left(J_x(z,t), 0, 0\right).$$

Ampere-Maxwell's equation which states two possible ways of generation of magnetic field: one is due to electric current and the other is due to changing electric field (called as the displacement current) is given by:

$$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}. \tag{1}$$

The component form (in the absence of displacement current) of the Ampere-Maxwell's equation takes the following form:

$$J_x = -\frac{\partial H_y}{\partial z}. \tag{2}$$

For this one-dimensional problem all the physical variables depend only on the space variable z and time-variable t, therefore the component of displacement vector $\bar{u}$ becomes:

$$u = u_x = 0, \quad v = u_y = 0, \quad w = u_z = w(z,t). \tag{3}$$

We consider the modified Ohm's law which outlines the impact of temperature gradient and charge density ignoring the seemingly small consequence of temperature gradient on the conduction current $\bar{J}$ as **[16]:**

$$\bar{J} = \sigma\left(\bar{E} + \dot{\bar{u}} \times \bar{B}\right), \tag{4}$$

Using equation (2) above the component form of the above equation can be written as:

$$J_x = \sigma\left(E_x - B_y \dot{w}\right), \tag{5}$$

similarly, the component of magnetic flux density is given by:

$$B_y = \mu H_y. \tag{6}$$

Solving the equation (1) and (4) we obtain electric field intensity in component form as:

$$E_x = -\frac{1}{\sigma}\frac{\partial H_y}{\partial z}. \tag{7}$$

Faraday's law of electromagnetism which describes how time-varying magnetic field induces an electric field gives:

$$rot\ \bar{E} = -\frac{\partial \bar{B}}{\partial t}. \tag{8}$$

Using (6) and (7), the component form of the above equation gives the uncoupled equation of magnetic field as:

$$\frac{\partial^2 H_y}{\partial z^2} - \sigma\mu\frac{\partial H_y}{\partial t} = 0, \tag{9}$$

with conditions

$$\begin{aligned} &\text{at } z = c: \quad H_y = H_0\left(1 - e^{t/\sigma\mu}\right) \equiv H_0\Omega(t), \\ &\text{at } z = 0: \quad H_y = 0, \\ &\text{at } t = 0: \quad H_y = 0. \end{aligned} \tag{10}$$

As a result of time-dependent/varying electromagnetic field conducting currents appear in the rod referred to as Eddy current. This Eddy current generates the resistive loss that transforms some form of energy such as kinetic energy into heat which is called Joule heat. This Joule heat gives rise to the Eddy current loss W_E. According to Joule-Lenz law the power of heating generated by an electric current J_x is proportional to the product of its resistance and square of current J_x. Therefore the Eddy current loss is given by

$$W_E(z,t) = \frac{J_x(z,t)^2}{\sigma}. \tag{11}$$

When a magnetization force is applied to a magnetic material, the molecules of the magnetic material are aligned in one particular direction. And when this magnetic force is reversed in the opposite direction, the internal friction of the molecular magnets opposes the reversal of magnetism resulting in Magnetic Hysteresis. To overcome this internal friction, a part of the magnetizing force is used. This work done by the magnetizing force produces heat which causes wastage of energy in the form of heat termed as Hysteresis loss. It is known that Hysteresis loss W_H is proportional to the square of magnetic field amplitude and frequency f therefore it is given by

$$W_H(z,t) = k_H\mu f\, H^2. \tag{12}$$

Neglecting the coupling term η between the temperature and the deformation fields, the governing equation of temperature field in the rod is as follows **[17]**:

$$\nabla^2 T + \frac{W}{\lambda_0} = \frac{C\rho}{\alpha}\frac{\partial T}{\partial t}, \tag{13}$$

$$W = W_E + W_H, \tag{14}$$

where $W(z,t)$ is the heat generated (due to Joule Heat and Hysteresis loss) within the material for $t > 0$ subject to the initial and boundary conditions as follows:

$$\begin{aligned} &\text{at } z = 0, c: \quad T = 0, \\ &\text{at } t = 0: \quad T = 0. \end{aligned} \tag{15}$$

Apart from Eddy current loss and Hysteresis loss the metallic rod placed in a time-dependent magnetic field also suffer the Lorentz force given by the expression:

$$f_z = -\frac{\mu}{2}\frac{\partial}{\partial z}\left(H_y\right)^2. \tag{16}$$

The constitutive equation for stress-displacement-temperature relation and strain-displacement relation is given by Duhamel-Neumann equations:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e - \beta T)\delta_{ij}, \tag{17}$$

$$e = div\,\bar{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}, \tag{18}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{19}$$

The components of the stress field are **[19]**:

$$\sigma_{zz} = (2\mu + \lambda)\frac{\partial w}{\partial z} - \beta T, \qquad \sigma_{xx} = \sigma_{yy} = \lambda \frac{\partial w}{\partial z} - \beta T. \tag{20}$$

The displacement equation of the theory of elasticity, considering the Lorentz force takes the following form:

$$\rho \ddot{u}_i = \sigma_{ik,k} + (J \times B)_i. \tag{21}$$

For considered one-dimensional problem, the above equations of motion become:

$$\rho \ddot{w} = \frac{\partial}{\partial z}\left[(2\mu + \lambda)\frac{\partial w}{\partial z} - \beta T\right] - \frac{\mu}{2}\frac{\partial}{\partial z}\left(H_y\right)^2, \tag{22}$$

with mechanical boundary conditions as follows:

$$\text{at } z = 0, c: \quad \frac{\partial w}{\partial z} = \frac{\beta}{\rho} T,$$
$$\text{at } t = 0: \quad w = \frac{\partial w}{\partial t} = 0. \tag{23}$$

To transform the above equations into dimensionless forms, we define the dimensionless variables as follows:

$$\bar{z} = \frac{z}{c}, \quad \bar{H}_y = \frac{H_y}{H_0}, \quad \bar{f}_z = \frac{c f_z}{\mu H_0^2}, \quad \tau = \frac{t}{\sigma \mu c^2}, \quad \bar{J}_x = \frac{c J_x}{H_0},$$
$$\bar{W}_E = \frac{\sigma c^2 W_E}{H_0^2}, \quad \bar{W}_H = \frac{\sigma c^4 W_H}{\rho \alpha^2 H_0^2}, \quad \bar{T} = \frac{C \rho T}{\mu H_0^2}, \quad \bar{\sigma} = \frac{\sigma}{\mu H_0^2 / 2}. \tag{24}$$

Equations (9) and (10) in the dimensionless forms are taken as:

$$\frac{\partial^2 \bar{H}_y}{\partial \bar{z}^2} - \frac{\partial \bar{H}_y}{\partial \tau} = 0, \tag{25}$$

with the conditions (in dimensionless form) as follows:

$$\text{at } \bar{z} = 1: \quad \bar{H}_y = 1 - e^{-\omega \tau},$$
$$\text{at } \bar{z} = 0: \quad \bar{H}_y = 0,$$
$$\text{at } \tau = 0: \quad \bar{H}_y = 0. \tag{26}$$

Expressions for current density, eddy current loss, Hysteresis loss and temperature field in the dimensionless form are given by

$$\bar{J}_x(\bar{z}, \tau) = \frac{\partial \bar{H}_y(\bar{z}, \tau)}{\partial \bar{z}}, \tag{27}$$

$$\bar{W}_E(\bar{z}, \tau) = \left[\bar{J}_x\right]^2, \tag{28}$$

$$\bar{W}_H(\bar{z}, \tau) = \frac{k_H \mu \sigma c^4 f}{\rho \alpha^2} \bar{H}_y^2, \ (29) \quad \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} + \left(\frac{C\rho}{\sigma \mu \lambda_0 c^2}\right)\bar{W} = \frac{C\rho}{\alpha}\frac{\partial \bar{T}}{\partial t}. \tag{30}$$

Equation (30) is subject to the initial and boundary conditions given by

$$\text{at } \bar{z} = 0, 1: \quad \bar{T} = 0,$$
$$\text{at } \tau = 0: \quad \bar{T} = 0, \tag{31}$$

and the component of Lorentz force in dimensionless form is given by

$$\bar{f}_z(\bar{z}, \tau) = -\frac{1}{2}\frac{\partial}{\partial \bar{z}}\left(\bar{H}_y\right)^2. \tag{32}$$

Stress-displacement relations (20) in the dimensionless form are as follows:

$$\bar{\sigma}_{xx} = \bar{\sigma}_{yy} = \frac{2\lambda}{\mu H_0^2}\frac{\partial \bar{w}}{\partial \bar{z}} - \frac{2\beta}{C\rho}\bar{T},$$

$$\bar{\sigma}_{xx}^M = \bar{\sigma}_{yy}^M = \frac{2\lambda}{\mu H_0^2}\frac{\partial \bar{w}^M}{\partial \bar{z}} = \lambda\left(\bar{H}_y(\bar{z}, \tau)\right)^2,$$

$$\bar{\sigma}_{xx}^{T} = \bar{\sigma}_{yy}^{T} = -\frac{4\beta\mu}{C\rho\left(2\mu+\lambda\right)}\bar{T}(\bar{z},\tau),$$
$$\bar{\sigma}_{zz} = \frac{2\left(2\mu+\lambda\right)}{\mu H_0^2}\frac{\partial\bar{w}}{\partial\bar{z}} - \frac{2\beta}{C\rho}\bar{\mathrm{T}},$$
$$\bar{\sigma}_{zz}^{T} = 0, \quad \bar{\sigma}_{zz}^{M} = \left(2\mu+\lambda\right)\left(\bar{H}_y(\bar{z},\tau)\right)^2. \tag{33}$$

Equations of motion (22), reduces to dimensionless form as:

$$\frac{\partial^2\bar{w}}{\partial\bar{z}^2} = \frac{1}{\left(2\mu+\lambda\right)}\left[\frac{\beta\mu H_0^2}{C\rho}\frac{\partial\bar{T}}{\partial\bar{z}} + \frac{\mu H_0^2}{2}\frac{\partial}{\partial\bar{z}}\left(\bar{H}_y\right)^2\right], \tag{34}$$

with the mechanical boundary conditions as follows:

$$\text{at } \bar{z} = 0,1: \quad \frac{\partial\bar{w}}{\partial\bar{z}} = \frac{\beta\mu H_0^2}{\rho^2 C}\bar{T},$$
$$\text{at } \tau = 0: \quad \bar{w} = \frac{\partial\bar{w}}{\partial\tau} = 0. \tag{35}$$

III. SOLUTIONS

Determination of Magnetic field

To determine Magnetic field expression, we first need to transform the inhomogeneous boundary condition in (26) into a homogeneous one, for this, we introduce a new function:

$$v\left(\bar{z},\tau\right) = \bar{H}_y\left(\bar{z},\tau\right) - h_y\left(\bar{z},\tau\right), \tag{36}$$

And

$$h_y(\bar{z},\tau) = \bar{z}\left(1-e^{-\omega\tau}\right). \tag{37}$$

Now using (36) into (25) we obtain:

$$\frac{\partial^2 v}{\partial\bar{z}^2} - \frac{\partial v}{\partial\tau} = \omega\bar{z}e^{-\omega\tau}. \tag{38}$$

Using (36), the equation (26) becomes:

$$\text{at } \bar{z} = 0,1: \quad v\left(\bar{z},\tau\right) = 0,$$
$$\text{at } \tau = 0: \quad v\left(\bar{z},\tau\right) = 0. \tag{39}$$

Collecting this information, we find that $v(\bar{z},\tau)$ satisfies:

$$\frac{\partial^2 v}{\partial\bar{z}^2} - \frac{\partial v}{\partial\tau} = \omega\bar{z}e^{-\omega\tau}, \tag{40}$$

$$\text{at } \bar{z} = 0,1: \quad v\left(\bar{z},\tau\right) = 0,$$
$$\text{at } \tau = 0: \quad v\left(\bar{z},\tau\right) = 0. \tag{41}$$

To determine the solution of (40), first, we apply finite Fourier sine transform and then Laplace transform **[18]** to (40) to obtain:

$$\tilde{v}^*(\mathrm{n},s) = \frac{(-1)^n}{n\pi}\left(\frac{\omega}{\left(s+\omega\right)\left(s+n^2\pi^2\right)}\right).$$

Now applying inverse Laplace transform and inverse finite Fourier sine transform we obtain:

$$\bar{v}(z,\tau) = 2\omega\sum_{n\geq 1}\frac{(-1)^n}{n\pi}\left(\int_0^{\tau} e^{-\omega\xi}e^{-n^2\pi^2(\tau-\xi)}d\xi\right)\sin(\mathrm{n}\,\pi\,\bar{\mathrm{z}}). \tag{42}$$

Using (42) in (36) we obtain:

$$\bar{H}_y\left(\bar{z},\tau\right) = \bar{z}\left(1-e^{-\omega\tau}\right) + 2\omega\sum_{n\geq 1}\frac{(-1)^n}{n\pi}\left(\int_0^{\tau} e^{-\omega\xi}e^{-n^2\pi^2(\tau-\xi)}d\xi\right)\sin(\mathrm{n}\,\pi\,\bar{\mathrm{z}}). \tag{43}$$

Using equation (43) in (27), the dimensionless current density expression is obtained as:

$$\bar{J}_x(\bar{z},\tau)=\left(1-e^{-\omega\tau}\right)+2\omega\sum_{n\geq 1}(-1)^n\left(\int_0^{\tau}e^{-\omega\xi}e^{-n^2\pi^2(\tau-\xi)}d\xi\right)\cos(n\pi\bar{z}). \tag{44}$$

Using (44) in (28) we obtain Dimensionless form of Eddy Current loss as:

$$\bar{W}_E(\bar{z},\tau)=\left[\bar{J}_x\right]^2=\left[\left(1-e^{-\omega\tau}\right)+2\omega\sum_{n\geq 1}(-1)^n\left(\int_0^{\tau}e^{-\omega\xi}e^{-n^2\pi^2(\tau-\xi)}d\xi\right)\cos(n\pi\bar{z})\right]^2. \tag{45}$$

Similarly, the expression for Hysteresis loss in dimensionless form is obtained as:

$$\bar{W}_H(\bar{z},\tau)=\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\left[\bar{z}\left(1-e^{-\omega\tau}\right)+2\omega\sum_{n\geq 1}\frac{(-1)^n}{n\pi}\left(\int_0^{\tau}e^{-\omega\xi}e^{-n^2\pi^2(\tau-\xi)}d\xi\right)\sin(n\pi\bar{z})\right]^2. \tag{46}$$

The total Heat loss in (dimensionless form) is given by

$$\bar{W}(\bar{z},\tau)=\bar{W}_E+\bar{W}_H=\left[\bar{J}_x\right]^2+\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\bar{H}_y^2,$$

$$\bar{W}(\bar{z},\tau)=\left(1+\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\bar{z}^2\right)\left(1-e^{-\omega\tau}\right)^2$$

$$+4\omega^2\left[\left(\sum_{n\geq 1}(-1)^n\left(\frac{e^{-\omega\tau}-e^{-k_n^2\tau}}{k_n^2-\omega}\right)\cos(k_n\bar{z})\right)^2+\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\left(\sum_{n\geq 1}\frac{(-1)^n}{k_n}\left(\frac{e^{-\omega\tau}-e^{-k_n^2\tau}}{k_n^2-\omega}\right)\sin(k_n\bar{z})\right)^2\right]$$

$$+4\omega\left(1-e^{-\omega\tau}\right)\left[\sum_{n\geq 1}(-1)^n\left(\frac{e^{-\omega\tau}-e^{-k_n^2\tau}}{k_n^2-\omega}\right)\left\{\cos(k_n\bar{z})+\left(\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\right)\frac{\bar{z}\sin(k_n\bar{z})}{k_n}\right\}\right],$$

$$\bar{W}(\bar{z},\tau)=T_1+4\omega^2\left[T_2+T_3\right]+4\omega T_4. \tag{47}$$

Applying the finite Fourier sine transform and the Laplace transform to (47) we obtain:

$$\hat{\bar{W}}^*(n,s)=A_n\left[\frac{1}{s}+\frac{1}{s+2\omega}-\frac{2}{s+\omega}\right]+4\omega^2\left[\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}B_{npq}\times\left(\frac{1}{s+2\omega}-\frac{1}{s+\left(k_q^2+\omega\right)}-\frac{1}{s+\left(k_p^2+\omega\right)}+\frac{1}{s+\left(k_p^2+k_q^2\right)}\right)\right]$$

$$+4\omega\left[\sum_{p=1}^{\infty}C_{np}\times\left\langle\frac{1}{s+\omega}-\frac{1}{s+k_p^2}-\frac{1}{s+2\omega}+\frac{1}{s+\left(k_p^2+\omega\right)}\right\rangle\right], \tag{48}$$

$$A_n=\left[\frac{1-2(-1)^n}{n\pi}-\left[1+(-1)^{n+1}\right]\frac{2}{n^3\pi^3}\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\right],$$

$$B_{npq}=(-1)^{p+q}\left(\frac{1}{k_p^2-\omega}\right)\left(\frac{1}{k_q^2-\omega}\right)\left[\frac{[1-(-1)^{p+q+n}]}{\pi\left(n^2+p^2+q^2\right)^2}\right]$$

$$\times\left[n(\mathrm{n}^2-\mathrm{p}^2-\mathrm{q}^2)-\left(\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\right)\frac{2npq}{(\mathrm{p}+\mathrm{q})\pi}\right],$$

$$C_{np}=\left(\frac{(-1)^p}{k_p^2-\omega}\right)\left\{\frac{(\pi^2\mathrm{n}^3-2n\left(\frac{k_H\mu\sigma c^4 f}{\rho\alpha^2}\right)-\mathrm{p}^2\pi^2\mathrm{n})[1-(-1)^{p+n}]}{\pi^3\left(\mathrm{n}^2-\mathrm{p}^2\right)^2}\right\}. \tag{49}$$

Determination of Non-dimensional temperature

Applying the finite Fourier sine transform and Laplace transform to (30) we obtain:

$$\hat{\bar{T}}^{*}(n,s)=\left(\frac{\alpha}{\sigma\mu\lambda_0 c^2}\right)\times A_n\left[\frac{1}{s(s+B)}+\frac{1}{(s+2\omega)(s+B)}-\frac{2}{(s+\omega)(s+B)}\right]$$

$$+4\omega^2\left[\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}B_{npq}\times\left\langle\frac{1}{(s+2\omega)(s+B)}-\frac{1}{\left[s+\left(k_q^2+\omega\right)\right](s+B)}-\frac{1}{\left[s+\left(k_p^2+\omega\right)\right](s+B)}+\frac{1}{\left[s+\left(k_p^2+k_q^2\right)\right](s+B)}\right\rangle\right]$$

$$+4\omega\left[\sum_{p=1}^{\infty}C_{np}\times\left\langle\frac{1}{(s+\omega)(s+B)}-\frac{1}{\left(s+k_p^2\right)(s+B)}-\frac{1}{(s+2\omega)(s+B)}+\frac{1}{\left(s+\left(k_p^2+\omega\right)\right)(s+B)}\right\rangle\right].$$

Now applying inverse Laplace transform and inverse finite Fourier sine transform to (30) we obtain:

$$\bar{T}(\bar{z},\tau)=\left(\frac{2\alpha}{\sigma\mu\lambda_0 c^2}\right)\times\sum_{n=1}^{\infty}\left[A_nT_1+T_2+T_3\right]\sin(\mathrm{n}\,\pi\bar{z}), \tag{50}$$

where

$$T_1=\frac{1-e^{-B\tau}}{B}+\frac{e^{-2\omega\tau}-e^{-B\tau}}{B-2\omega}-2\frac{e^{-\omega\tau}-e^{-B\tau}}{B-\omega},$$

$$T_2=4\omega^2\left[\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}B_{npq}\times\left\langle\frac{e^{-2\omega\tau}-e^{-B\tau}}{B-2\omega}-\frac{e^{-\left(k_q^2+\omega\right)\tau}-e^{-B\tau}}{B-\left(k_q^2+\omega\right)}-\frac{e^{-\left(k_p^2+\omega\right)\tau}-e^{-B\tau}}{B-\left(k_p^2+\omega\right)}+\frac{e^{-\left(k_p^2+k_q^2\right)\tau}-e^{-B\tau}}{B-\left(k_p^2+k_q^2\right)}\right\rangle\right],$$

$$T_3=4\omega\left[\sum_{p=1}^{\infty}C_{np}\times\left\langle\frac{e^{-\omega\tau}-e^{-B\tau}}{B-\omega}-\frac{e^{-k_p^2\tau}-e^{-B\tau}}{B-k_p^2}-\frac{e^{-2\omega\tau}-e^{-B\tau}}{B-2\omega}+\frac{e^{-\left(k_p^2+\omega\right)\tau}-e^{-B\tau}}{B-\left(k_p^2+\omega\right)}\right\rangle\right].$$

Determination of Magnetic and Thermal components of Displacement and Stresses

The quasistatic solutions of displacement due to temperature change and Lorentz force in terms of its thermal and magnetic components are given by **[15]**,

$$\bar{w}^T(\bar{z},\tau)=\frac{\beta\mu H_0^2}{C\rho\left(2\mu+\lambda\right)}\int\bar{T}(\bar{z},\tau)\,d\bar{z},\qquad \bar{w}^M(\bar{z},\tau)=\frac{\mu H_0^2}{2}\int\left(\bar{H}_y(\bar{z},\tau)\right)^2d\bar{z}. \tag{51}$$

Hence

$$\bar{w}^T(\bar{z},\tau)=\frac{-2\alpha\beta H_0^2}{C\rho\,\mathrm{n}\,\pi\sigma\lambda_0c^2\left(2\mu+\lambda\right)}\times\sum_{n=1}^{\infty}\left[A_nT_1+T_2+T_3\right]\cos(\mathrm{n}\,\pi\bar{z}), \tag{52}$$

and

$$\bar{w}^M(\bar{z},\tau)=\frac{\mu H_0^2}{2}\left[2\bar{z}\left(1-e^{-\omega\tau}\right)^2+2\omega^2\sum_{m\geq1}\sum_{n\geq1}\frac{(-1)^{m+n}}{k_m+k_n}\right.$$

$$\left.\times\left(\frac{e^{-\omega\tau}-e^{-k_m^2\tau}}{k_m^2-\omega}\right)\left(\frac{e^{-\omega\tau}-e^{-k_n^2\tau}}{k_n^2-\omega}\right)\left[\frac{\sin(k_m-k_n)\,\bar{z}}{k_m-k_n}-\frac{\sin(k_m+k_n)\,\bar{z}}{k_m+k_n}\right]\right],$$

$$\bar{w}^M(\bar{z},\tau)=\frac{\mu H_0^2}{2}\left[2\bar{z}\left(1-e^{-\omega\tau}\right)^2+2\omega^2\sum_{m\geq1}\sum_{n\geq1}\frac{(-1)^{m+n}}{k_m+k_n}\left(\frac{e^{-\omega\tau}-e^{-k_m^2\tau}}{k_m^2-\omega}\right)\left(\frac{e^{-\omega\tau}-e^{-k_n^2\tau}}{k_n^2-\omega}\right)\left[\frac{\sin(k_m-k_n)\,\bar{z}}{k_m-k_n}-\frac{\sin(k_m+k_n)\,\bar{z}}{k_m+k_n}\right]\right.$$

$$\left.+4\omega\left(1-e^{-\omega\tau}\right)\sum_{n\geq1}\frac{(-1)^n}{k_n}\left(\frac{e^{-\omega\tau}-e^{-k_n^2\tau}}{k_n^2-\omega}\right)\left[\frac{\sin(k_n\,\bar{z})-\bar{z}k_n\cos(k_n\,\bar{z})}{k_n^2}\right]\right]. \tag{53}$$

We solve equations in (33) further with the help of the above equations and equation (50) to obtain the expressions of thermal and magnetic stresses which are given by

$$\bar{\sigma}_{zz}^T=0, \tag{54}$$

$$\bar{\sigma}_{zz}^{M} = (2\mu+\lambda)\left(\bar{H}_y(\bar{z},\tau)\right)^2, \tag{55}$$

$$\bar{\sigma}_{xx}^{T} = \bar{\sigma}_{yy}^{T} = -\frac{4\beta\mu}{C\rho(2\mu+\lambda)}\bar{T}(\bar{z},\tau), \tag{56}$$

$$\bar{\sigma}_{xx}^{M} = \bar{\sigma}_{yy}^{M} = \lambda\left(\bar{H}_y(\bar{z},\tau)\right)^2. \tag{57}$$

IV. NUMERICAL RESULTS AND DISCUSSION

To illustrate and compare the theoretical results obtained, we now present some numerical results which depict the variations of displacement, temperature, and stress component. The material chosen for the purpose of numerical evaluations is steel, for which we take the following values of the different physical constants **[17]** $\mu_0 = 1.26\times10^{-4}[H/m], \mu = 79.3[GPa], \lambda = 101.91\times10^{9}, \sigma = 7.7\times10^{6}[S/m], C = 502.416[J/kgK]$ $\rho = 7663[kg/m^3], \kappa = 1.4\times10^{-3}[m^2/\sec], v = 0.28, E = 205[GPa], \alpha = 12\times10^{-6}[1/K], c = 1mm, \lambda_0 = 50[W/mK]$.

Figure 1 depicts the variation of the current density $\bar{J}_x$ for different values of $\bar{z}$ varying with the time τ. For $\bar{z}=1$ the value of $\bar{J}_x$ change monotonically which shows a sharp rise followed by a rapid decline when $0<\tau<40$ while the value of $\bar{J}_x$ slowly decreases when $\tau\geq 40$ and then tends to be stable. It can also be seen that at the middle of the plate i. e. for $\bar{z}=0.5$ as well as for $\bar{z}=0$ the current density $\bar{J}_x$ is small and varies slowly in comparison with the variation at the surface. Figure 2 depicts the variation of Eddy current loss with the time τ.

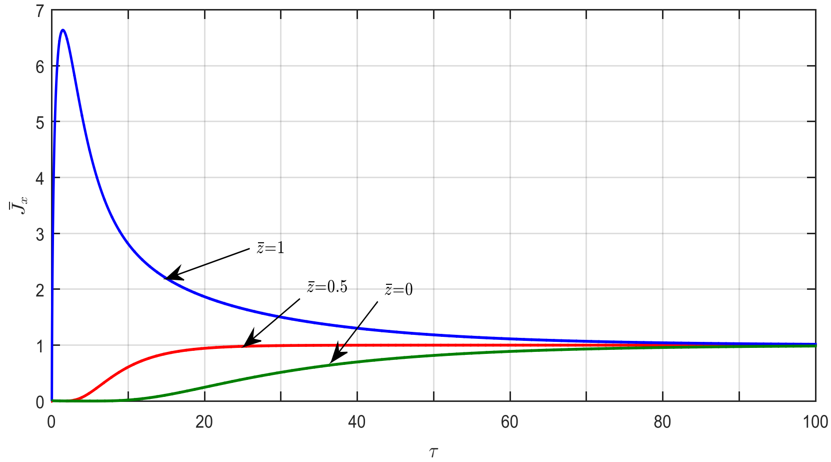

Figure 1. Time Variation of current density $\bar{J}_x$ for $\bar{z}=0,0.5,1$

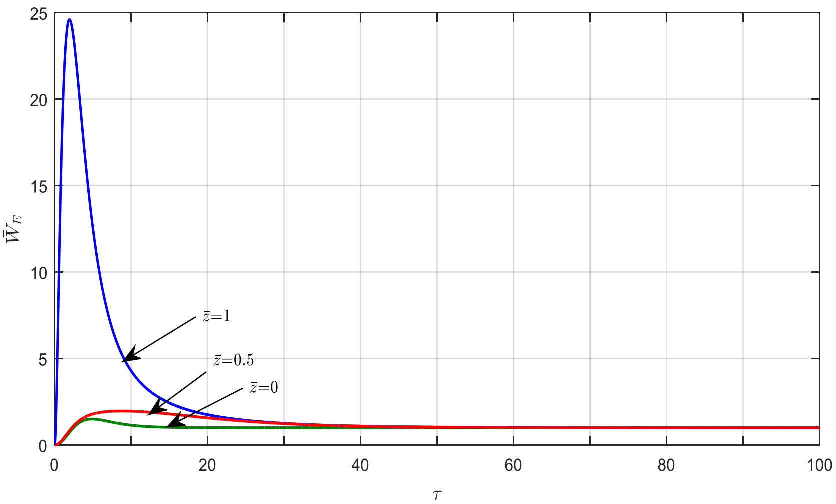

Figure 2. Time Variation of Eddy current loss $\bar{W}_E$ for $\bar{z}=0,0.5,1$

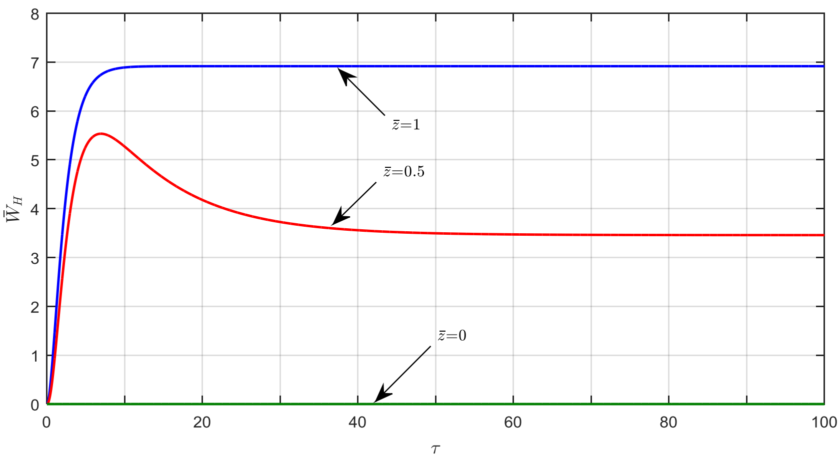

Figure 3. Time Variation of Hysteresis loss $\bar{W}_H$ for $\bar{z}=0,0.5,1$

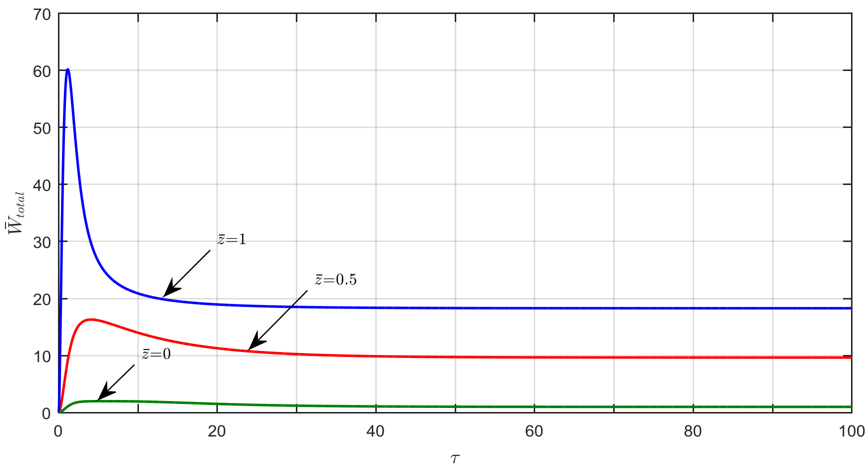

Figure 4. Time Variation of Total Heat loss $\bar{W}_{Total}$ for $\bar{z}=0,0.5,1$

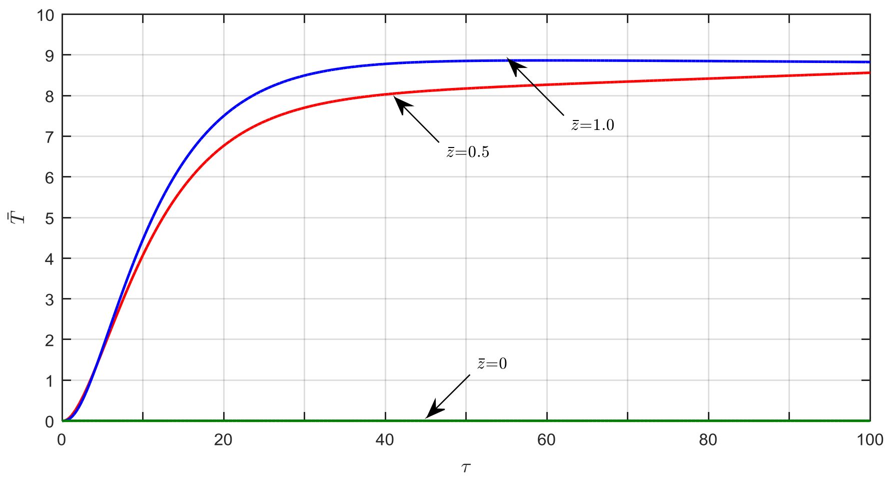

Figure 5. Time evolution of Temperature changes $\bar{T}$

Figure 3 and 4 shows the time variation of Hysteresis loss and total heat loss for $\bar{z}=0,0.5,1.$ Figure 5 shows the time variation of non-dimensional temperature for $\bar{z}=0,0.5,1$ until it attains a steady state. From the graphs it can be seen that for the temperature it takes about 40 seconds to attain steady state. Figure 6 represents the temperature variation in the middle of the rod as a function of time and wave frequency. Figure 7 shows the quasi-static behaviors of thermal stresses $\bar{\sigma}_{xx}^{T}$ with time for $\bar{z}=0,0.5,1.0.$ From the figure it can be seen that the thermal stress $\bar{\sigma}_{xx}^{T}$ is compressive at the surface. Figure 8 and 10 shows the variation of magnetic stress component versus non-dimensional time and distance respectively.

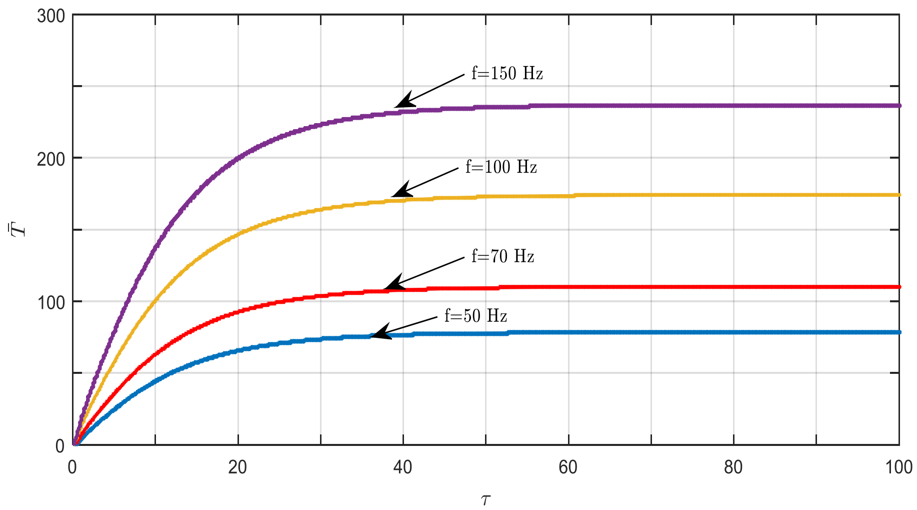

Figure 6. Temperature changes $\bar{T}$ at $\bar{z} = 50\,mm$ as a function of time and wave frequency.

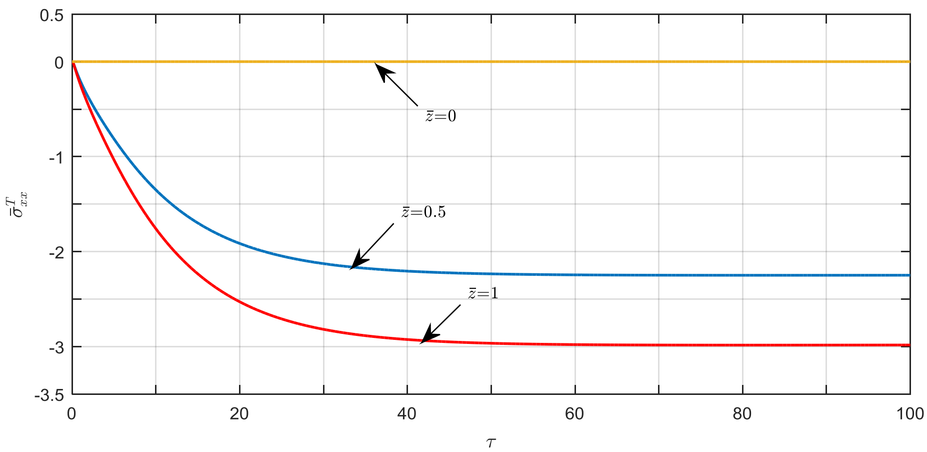

Figure 7. Quasi-static behaviors of thermal stresses $\bar{\sigma}_{xx}^{T}$ versus non-dimensional time.

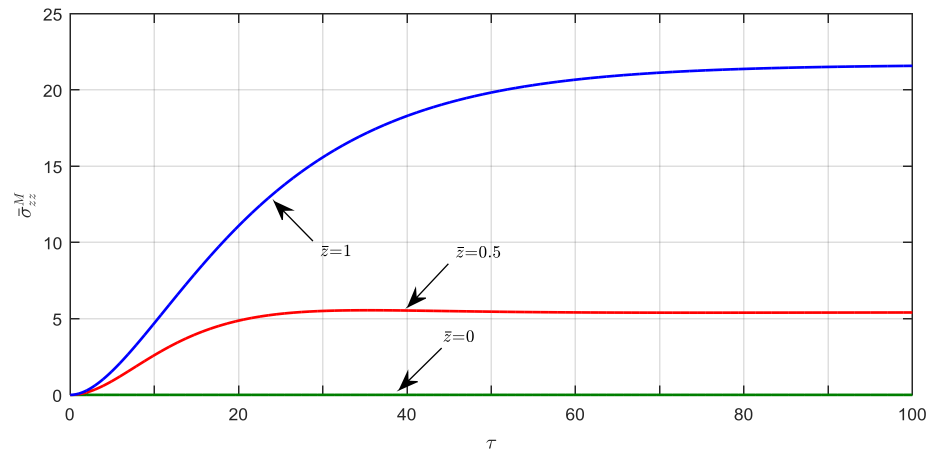

Figure 8. Quasistatic behavior of Magnetic stresses $\bar{\sigma}_{zz}^{M}$ versus non-dimensional time τ

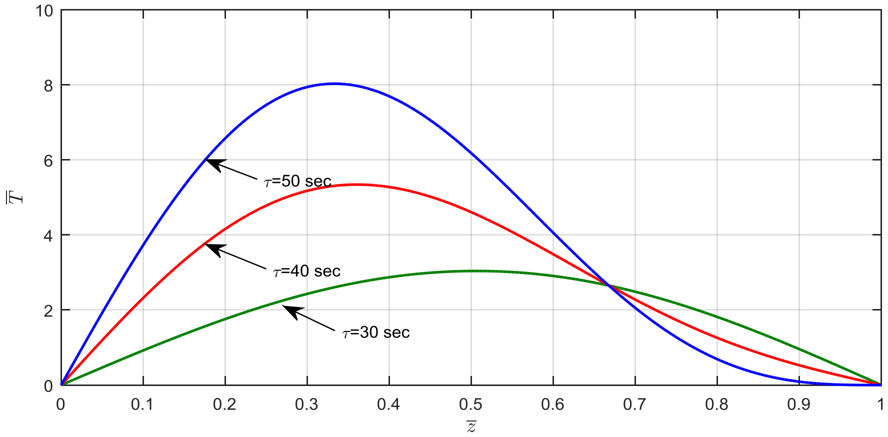

Figure 9. Variation of Non-dimensional Temperature $\bar{T}$ against $\bar{z}$ at for $\tau = 30, 40, 50$sec.

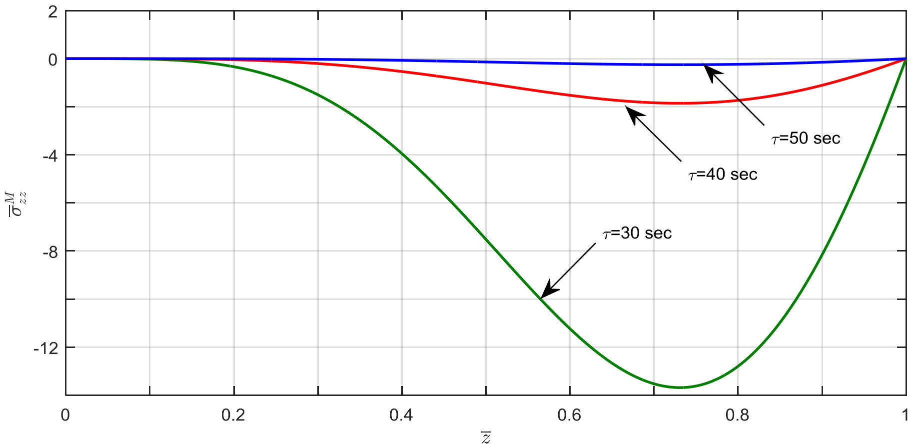

Figure 10. Variation of magnetic stresses component $\bar{\sigma}_{zz}^{M}$ against $\bar{z}$ at $\tau = 30, 40, 50$sec

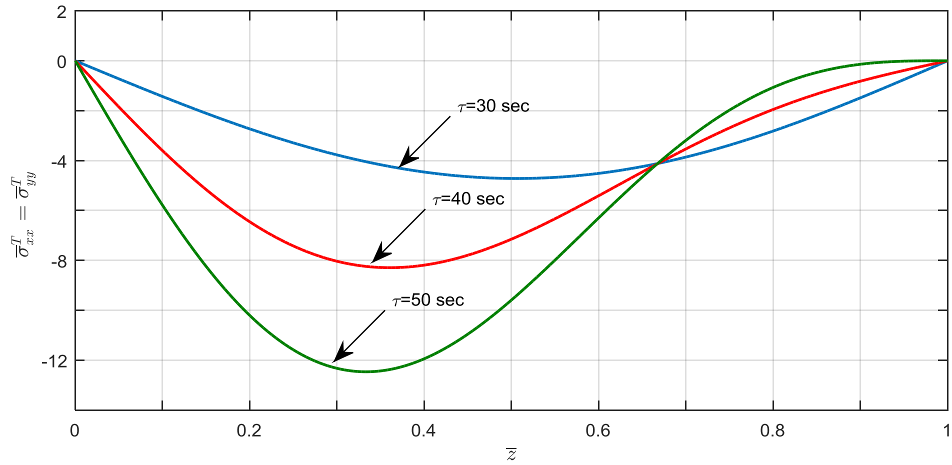

Figure 11. Variation of stresses distribution $\bar{\sigma}_{xx}^{T} = \bar{\sigma}_{yy}^{T}$ against $\bar{z}$ at $\tau = 30, 40, 50$sec

It is evident from figure 9 that the non-dimensional Temperature $\bar{T}$ have coincident point with zero value which is consistent with the boundary conditions applied, afterward it jump to attain maximum value and finally decreases gradually to diminish to zero value as $\bar{z} \to 1$. Figure 13 depicts that stress components $\bar{\sigma}_{xx}^{T} = \bar{\sigma}_{yy}^{T}$ starts with zero value at $\bar{z} = 0$. It decreases at the beginning and starts increasing at $\bar{z} = 0.35$ and finally converges to zero value as $\bar{z}$ increases.

V. CONCLUSION

The main goal of the present work is to study the effect of the Eddy Current loss and the Hysteresis loss on a one-dimensional magneto-thermo-elastic problem subjected to a time-varying magnetic field accompanied with the appearance of eddy current and hysteresis losses. The fundamental solutions are derived using a very suitable method for solving equations governing temperature field in analytical form, that is integral transform technique where we have used Laplace transform and Finite Fourier sine transform and the effect of Eddy current loss and the Hysteresis loss has been studied. The current density is higher at the surface and it decreases with depth. The rate of the decrease depends on the conductivity and permeability of the metal. The conductivity of the material affects the depth of penetration. The intensity of losses decreases exponentially with time. The intensity of Eddy current loss shows a sharp rise followed by a rapid decline after that it decreases slowly with time and tends to be stable. Variation of the intensity of Eddy current loss is small and slow as compared with its variation at the surface. The temperature of the rod increases with an increase in the wave frequency. Under the influence of the wave frequency the rod heats up slowly and after about 40 seconds achieves the maximum temperature and then moves to the steady-state. We note that the solution for thermal stresses is proportional to the temperature of the rod with a negative constant for steel. Therefore, the thermal stresses are compressive at the surface. Some numerical computations for ferromagnetic material like steel have been carried out.

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