

# Inventory Models for Non-Instantaneous Deterioration Items with Stock Dependent Demand under Trade Credit Policy with Inflation

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## Abstract-

*In this paper, we investigate an inventory model with non-instantaneous deteriorating items, stock-dependent demand, partial backlog shortages, and trade credit. Some factors, like as trade credit and inflation, are introduced in this model. The demand rate is a linear function of time and the model has non-instantaneous deterioration that is time dependent. Demand and perhaps deterioration alter inventory levels over a cycle. This study demonstrates that the overall average is maximal when the total trade credit period is shorter than the whole cycle length. To explain the development model, a numerical example is used. The ideal solution is also subjected to a sensitivity analysis in terms of important parameters.*

**Keywords:-** holding cost, inflation, Trade credit, stock dependent demand, non-instantaneous deterioration.

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## I. INTRODUCTION

This chapter is concerned with determining the optimal economic order quantity (EOQ) for the basic inventory model with non-instantaneous deteriorating items and stock dependent demand under trade credit policy with inflation. A simple approach for determining the optimal EOQ, along with the various mathematical supporting arguments, is proposed. The mathematical model which considered has the particularity that items in stock experience some type of a delay deterioration effect. Additionally, demand for the items is influenced by the amount held in stock.

Khanra et al. (2011) proposed an EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. In this model, the deterioration rate was assumed to be constant and the time varying demand rate was taken to be a quadratic function of time. In this direction, Sharma et al. (2012) investigated the optimal supplier/retailer's replenishment decisions under two levels of trade credit policy with in the economic order quantity framework. This model dealt with supplier/retailer's inventory replenishment problem under two levels of trade credit in one replenishment cycle. In the year (2013), Singh et al. discussed on EOQ model for deteriorating items with linear demand, variable deterioration and partial backlogging. In the same year (2013), Singh et al. presented an extended model of Khanra et al. (2011), in this model to allow for a variable rate of deterioration, when delay in payment was permissible.

Yang C.T. (2014) developed an inventory model under a stock-dependent demand rate and stock-dependent holding cost rate with relaxed terminal conditions. In year (2014), an inventory model of price and stock dependent demand rate with deterioration under inflation and delay in payment, was formulated by Pal et al. Goel et al. (2015) examined a supply chain model with stock dependent demand, quadratic rate of deterioration with allowable shortage. Khurana et al. (2015) developed a two echelon supply chain production inventory model for deteriorating products having stock-dependent demand under inflationary environment. This model was developed for finite time horizon. The shortages were allowed and partially backlogged.

In this study, we reviewed the paper developed by K.-S. Wu, L.-Y. Ouyang, and C.-T. Yang (2006) on the topic an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. With a view of extending it to models with permissible delay in payment given by L.Y. Ouyang, Yang, (2006) on an inventory model for non-instantaneous deteriorating items with permissible delay in payment. After young's model many concepts are discovered. C.-T. Chang, et al. (2015) developed a model on Optimal pricing and ordering policies for non-instantaneously deteriorating items under order-size-dependent delay in payments.

This paper is extension of paper "On an optimal replenishment policy for inventory models for non-instantaneous deteriorating items with stock dependent demand and partial backlogging". (2018). In this paper we introduce some variables as trade credit and inflation. i.e. in this chapter we develop an inventory model for

Inventory models for non-instantaneous deterioration items with stock dependent demand under trade credit policy with inflation. By analyzing inventory models, a very useful inventory replenishment policy is given. After solving this model finally, we go through some numerical examples which provides solution to the theoretical concepts. Sensitivity analysis of optimal solution with respect to major parameters is also carried out.

## II. ASSUMPTIONS AND NOTATIONS

This model developed under the following assumptions and notations.

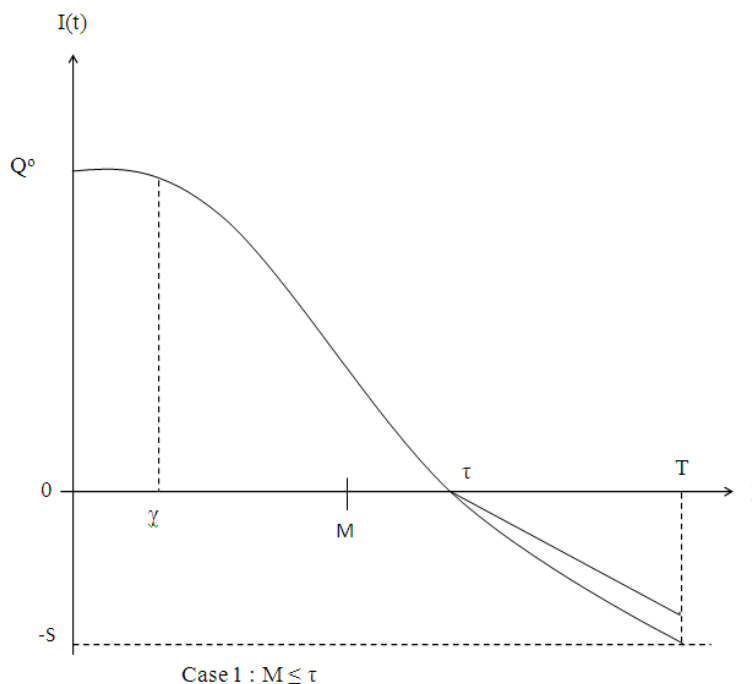
- (1) A single product is held in stock over an infinite planning horizon.
- (2) The planning horizon is made-up of identical cycle of length  $T$ , where  $T > 0$ .
- (3) The inventory level is at its maximum level at the beginning of the cycle.
- (4) During a cycle the inventory level is affected by demand and possibly deterioration.
- (5) Items experience deterioration once they spent at least some known time  $> 0$  in stock.
- (6) Deteriorated items are not repaired while in stock.
- (7) Shortages are partially backlogged.
- (8) Inflation is considered in this model with rate of inflation ( $r > 0$ ).
- (9) Trade credit policy is considered with trade period  $M > 0$ .
- (10) The replenishment rate is infinite.
- (11) The demand rate,  $D(t)$ , at some time  $t > 0$ , is given by

$$D(t) = \begin{cases} a + b(t), & I(t) > 0 \\ a, & I(t) \leq 0 \end{cases}$$

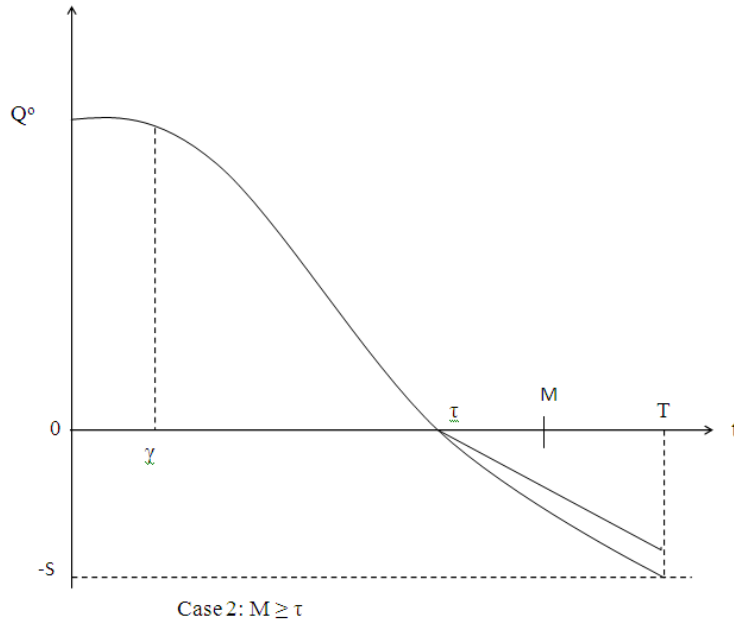
- (12) The deterioration rate is denoted by  $\theta \geq 0$ ;
- the length of time of nonnegative inventory is denoted by  $\tau \geq 0$  ( $\tau \geq T$ );
- the backlogging parameter  $\delta > 0$ ;
- the holding cost  $h > 0$ ;
- the backlogging cost  $s > 0$ ;
- the opportunity cost  $o > 0$ ;
- the unit cost  $c_2 > 0$ ;
- the set-up cost  $A > 0$ .

**Mathematical Modelling:** We considered a model starting with initial state having inventory level  $Q^0$ . In this model we considered non instantaneous deterioration so, deterioration will start after the time ' $\gamma$ '.

In this model we apply trade credit policy with trade period ' $M$ '. so there is two case for this problem, whose graphical representation is given below for both cases. For first case we consider  $M \geq \tau$  and in second case we consider  $M \leq \tau$ . Shortage is allowed is in this model so, we considered that shortage occurs after time ' $\tau$ ' in both cases.



**Figure: 1**



**III. MATHEMATICAL FORMULATION**

The differential equations are given for the three duration of time in which we use initial conditions to solve the differential equations. For finding inventory level for first duration we use initial condition.

$$I_1(\gamma) = I_2(\gamma)$$

For finding inventory level for second duration we use initial condition.

$$I_2(\tau) = 0$$

For finding inventory level for third duration we use initial condition.

$$I_3(\tau) = 0$$

On developing and solving the given equations we get as follows.

$$\frac{dI_1(t)}{dt} = -(a + bI_1(t)) \dots \dots \dots \text{equ(1)} \dots \dots \dots 0 \leq t \leq \min\{\gamma, \tau\}$$

$$\frac{dI_2(t)}{dt} = -a - (b + \theta)I_2(t) \dots \dots \dots \text{equ(2)} \dots \dots \dots \min\{\gamma, \tau\} \leq t \leq \tau$$

$$\frac{dI_3(t)}{dt} = -\frac{a}{1 + \delta(T - t)} \dots \dots \dots \text{equ(3)} \dots \dots \dots \tau \leq t \leq T$$

from (1), (2), (3).

using  $I_1(\gamma) = I_2(\gamma)$

$$I_1(t) = \frac{a}{b} (e^{b(\gamma-t)} - 1) + \frac{a}{b + \theta} (e^{b(\tau-t) + \theta(\tau-\gamma)} - e^{b(\gamma-t)}) \dots \dots \dots \text{eq(4)}$$

$$I_2(t) = \frac{a}{b + \theta} (e^{(b+\theta)(\tau-\gamma)} - 1) \dots \dots \dots \text{equ(5)}$$

with  $I_3(\tau) = 0$

$$I_3(t) = \frac{a}{\delta} \left( \log \frac{1 + \delta(T - t)}{1 + \delta(T - \tau)} \right) \dots \dots \dots \text{equ(6)}$$

**IV. COST COMPONENTS**

Cost components for inventory problem are given as follows.

**Set up cost = A**

**Holding cost:**

$$\begin{aligned}
 &= h \int_0^{\tau} e^{-rt} I(t) dt \\
 &= h \int_0^{\gamma} I_1(t) e^{-rt} dt + h \int_{\gamma}^{\tau} I_2(t) e^{-rt} dt \\
 &= h \left[ \frac{a}{b} \left( \frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r} \right) + \left( \frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma}) (1 - e^{-(b+r)\gamma}) \right) \right. \\
 &\quad \left. + \frac{a}{(b+\theta)} \left( \frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r} \right) \right]
 \end{aligned}$$

**Deteriorating cost:**

$$\begin{aligned}
 &= D \int_0^{\tau} \theta e^{-rt} I(t) dt \\
 &= D \theta \int_0^{\gamma} I_1(t) e^{-rt} dt + D \theta \int_{\gamma}^{\tau} I_2(t) e^{-rt} dt \\
 &= D \theta \left[ \frac{a}{b} \left( \frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r} \right) + \left( \frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma}) (1 - e^{-(b+r)\gamma}) \right) \right. \\
 &\quad \left. + \frac{a}{(b+\theta)} \left( \frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r} \right) \right]
 \end{aligned}$$

**Shortage cost:**

$$\begin{aligned}
 &= -S \int_{\tau}^T \theta e^{-rt} I_3(t) dt \\
 &= -\frac{S \theta a}{\delta^2} \log(1 + \delta(T - \tau))
 \end{aligned}$$

**Opportunity cost:**

$$\begin{aligned}
 &= o \int_{\tau}^T \left( 1 - \frac{1}{(1 + \delta(T - t))} \right) D dt \\
 &= \frac{o a}{\delta} (\delta(T - \tau) - \log(1 + \delta(T - \tau)))
 \end{aligned}$$

**Cost for trade period:**

**Case:1** when  $(M \leq \tau)$

**Interest pay (I.P.):** The amount of money which is paid by retailer to the loan agency comes under consideration to interest pay. This is the amount which is taken by retailer, to pay supplier, from any external loan agency due to trade period is less than the time at which complete inventory sold.

$$\begin{aligned}
 I.P. &= c i_p \int_M^\tau I(t) dt \\
 &= \frac{c a i_p}{b + \theta} \left( \frac{(e^{(b+\theta)(\tau-M)} - 1)}{b + \theta} + (M - \tau) \right)
 \end{aligned}$$

**Interest earn (I.E):** The amount of money which is earned by retailer on the amount of inventory which is sold, will come under this amount.

$$\begin{aligned}
 I.E. &= P i_e \int_0^M D t dt \\
 &= P i_e \left[ \frac{(a-1)\gamma^2}{2} - \frac{a}{b} \left( \gamma + \frac{(1-e^{b\gamma})}{b} \right) + \frac{a}{b+\theta} \left( \frac{\gamma+1}{b} - e^{(b+\theta)(\tau-\gamma)} (b\gamma-1) + \frac{(e^{(b+\theta)(\tau-\gamma)} - e^{b\gamma})}{b^2} \right) \right. \\
 &\quad \left. + a \left( 1 + \frac{ab}{b+\theta} \right) \frac{(M^2 - \gamma^2)}{2} - \frac{ab}{(b+\theta)^3} \{ (M(b+\theta) - 1) e^{(b+\theta)(\tau-M)} - (\gamma(b+\theta) - 1) e^{(b+\theta)(\tau-\gamma)} \} \right]
 \end{aligned}$$

**Case: 2** when  $(M \geq \tau)$

**Interest pay (I.P.) = 0**

**Interest earn:**

$$\begin{aligned}
 I.E. &= P i_e \left( \int_0^\tau D t dt + (M - \tau) \int_0^\tau D dt \right) \\
 &= P i_e \left[ \frac{(a-1)\gamma^2}{2} - \frac{a}{b} \left( \gamma + \frac{(1-e^{b\gamma})}{b} \right) + \frac{a}{b+\theta} \left( \frac{\gamma+1}{b} - e^{(b+\theta)(\tau-\gamma)} (b\gamma-1) + \frac{(e^{(b+\theta)(\tau-\gamma)} - e^{b\gamma})}{b^2} \right) \right. \\
 &\quad \left. + a \left( 1 + \frac{ab}{b+\theta} \right) \frac{(\tau^2 - \gamma^2)}{2} - \frac{ab}{(b+\theta)^3} \{ (\tau(b+\theta) - 1) - (\gamma(b+\theta) - 1) e^{(b+\theta)(\tau-\gamma)} \} \right] + (M - \tau) a \tau
 \end{aligned}$$

## V. TOTAL COST FOR THE GIVEN MODEL

**For case: 1**

**T.C<sub>1</sub>.** = Set up cost + Holding cost + Deteriorating cost + opportunity cost + Shortage cost + Interest pay – Interest earn.

$$\begin{aligned}
 &= A + h \left[ \frac{a}{b} \left( \frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r} \right) + \left( \frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma}) (1 - e^{-(b+r)\gamma}) \right) \right. \\
 &\quad \left. + \frac{a}{(b+\theta)} \left( \frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r} \right) \right] \\
 &\quad + D \theta \left[ \frac{a}{b} \left( \frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r} \right) + \left( \frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma}) (1 - e^{-(b+r)\gamma}) \right) \right. \\
 &\quad \left. + \frac{a}{(b+\theta)} \left( \frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{S \theta a}{\delta^2} \log(1 + \delta(T - \tau)) \\
 & + \frac{oa}{\delta} (\delta(T - \tau) - \log(1 + \delta(T - \tau))) \\
 & + \frac{ca i_p}{b + \theta} \left( \frac{(e^{(b+\theta)(\tau-M)} - 1)}{b + \theta} + (M - \tau) \right) \\
 & - P i_e \left[ \frac{(a-1)\gamma^2}{2} - \frac{a}{b} \left( \gamma + \frac{(1 - e^{b\gamma})}{b} \right) + \frac{a}{b + \theta} \left( \frac{\gamma + 1}{b} - e^{(b+\theta)(\tau-\gamma)} (b\gamma - 1) + \frac{(e^{(b\tau+\theta(\tau-\gamma)} - e^{b\gamma})}{b^2} \right) \right. \\
 & \left. + a \left( 1 + \frac{ab}{b + \theta} \right) \frac{(M^2 - \gamma^2)}{2} - \frac{ab}{(b + \theta)^3} \{ (M(b + \theta) - 1) e^{(b+\theta)(\tau-M)} - (\gamma(b + \theta) - 1) e^{(b+\theta)(\tau-\gamma)} \} \right]
 \end{aligned}$$

**Case: 2**

**T.C<sub>2</sub>** = Set up cost + Holding cost + Deteriorating cost + opportunity cost + Shortage cost – Interest earn.

$$\begin{aligned}
 & = A + h \left[ \frac{a}{b} \left( \frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r} \right) + \left( \frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau-\theta\gamma} - e^{-b\gamma}) (1 - e^{-(b+r)\gamma}) \right) \right. \\
 & \left. + \frac{a}{(b+\theta)} \left( \frac{(e^{(b+\theta)\tau-(b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r} \right) \right] \\
 & + D \theta \left[ \frac{a}{b} \left( \frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r} \right) + \left( \frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau-\theta\gamma} - e^{-b\gamma}) (1 - e^{-(b+r)\gamma}) \right) \right. \\
 & \left. + \frac{a}{(b+\theta)} \left( \frac{(e^{(b+\theta)\tau-(b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r} \right) \right] \\
 & - \frac{S \theta a}{\delta^2} \log(1 + \delta(T - \tau)) \\
 & + \frac{oa}{\delta} (\delta(T - \tau) - \log(1 + \delta(T - \tau))) \\
 & - P i_e \left[ \frac{(a-1)\gamma^2}{2} - \frac{a}{b} \left( \gamma + \frac{(1 - e^{b\gamma})}{b} \right) + \frac{a}{b + \theta} \left( \frac{\gamma + 1}{b} - e^{(b+\theta)(\tau-\gamma)} (b\gamma - 1) + \frac{(e^{(b\tau+\theta(\tau-\gamma)} - e^{b\gamma})}{b^2} \right) \right. \\
 & \left. + a \left( 1 + \frac{ab}{b + \theta} \right) \frac{(\tau^2 - \gamma^2)}{2} - \frac{ab}{(b + \theta)^3} \{ (\tau(b + \theta) - 1) - (\gamma(b + \theta) - 1) e^{(b+\theta)(\tau-\gamma)} \} + (M - \tau) a \tau \right]
 \end{aligned}$$

**VI. TOTAL AVERAGE COST FOR INVENTORY MODEL**

**For case: 1 (A.P<sub>1</sub>)**

$$\begin{aligned}
 &= \frac{1}{T} [A + h \{ \frac{a}{b} (\frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r}) + (\frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma})(1 - e^{-(b+r)\gamma}) \\
 &+ \frac{a}{(b+\theta)} (\frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r}) ] \\
 &+ D\theta [ \frac{a}{b} (\frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r}) + (\frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma})(1 - e^{-(b+r)\gamma}) \\
 &+ \frac{a}{(b+\theta)} (\frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r}) ] \\
 &- \frac{S\theta a}{\delta^2} \log(1 + \delta(T - \tau)) \\
 &+ \frac{oa}{\delta} (\delta(T - \tau) - \log(1 + \delta(T - \tau))) \\
 &+ \frac{cai_p}{b+\theta} (\frac{(e^{(b+\theta)(\tau-M)} - 1)}{b+\theta} + (M - \tau)) \\
 &- Pi_e [ \frac{(a-1)\gamma^2}{2} - \frac{a}{b} (\gamma + \frac{(1-e^{b\gamma})}{b}) + \frac{a}{b+\theta} (\frac{\gamma+1}{b} - e^{(b+\theta)(\tau-\gamma)} (b\gamma - 1) + \frac{(e^{(b+\theta)(\tau-\gamma)} - e^{b\gamma})}{b^2} \\
 &+ a(1 + \frac{ab}{b+\theta}) \frac{(M^2 - \gamma^2)}{2} - \frac{ab}{(b+\theta)^3} \{ (M(b+\theta) - 1) e^{(b+\theta)(\tau-M)} - (\gamma(b+\theta) - 1) e^{(b+\theta)(\tau-\gamma)} \} ] ]
 \end{aligned}$$

**Case: 2 (A.P<sub>2</sub>)**

$$\begin{aligned}
 &= \frac{1}{T} [A + h \{ \frac{a}{b} (\frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r}) + (\frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma})(1 - e^{-(b+r)\gamma}) \\
 &+ \frac{a}{(b+\theta)} (\frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r}) ] \\
 &+ D\theta [ \frac{a}{b} (\frac{1}{(b+r)} (e^{b\gamma} - e^{-r\gamma}) + \frac{(e^{-r\gamma} - 1)}{r}) + (\frac{a}{(b+\theta)(b+r)} (e^{(b+\theta)\tau - \theta\gamma} - e^{-b\gamma})(1 - e^{-(b+r)\gamma}) \\
 &+ \frac{a}{(b+\theta)} (\frac{(e^{(b+\theta)\tau - (b+\theta+r)\gamma} - e^{-r\tau})}{(b+\theta+r)} + \frac{(e^{-r\tau} - e^{-r\gamma})}{r}) ] \\
 &- \frac{S\theta a}{\delta^2} \log(1 + \delta(T - \tau))
 \end{aligned}$$

$$\begin{aligned}
 &+ \frac{oa}{\delta} (\delta (T - \tau) - \log(1 + \delta (T - \tau))) \\
 &- P i_e \left[ \frac{(a-1)\gamma^2}{2} - \frac{a}{b} \left( \gamma + \frac{(1-e^{b\gamma})}{b} \right) + \frac{a}{b+\theta} \left( \frac{\gamma+1}{b} - e^{(b+\theta)(\tau-\gamma)} (b\gamma-1) + \frac{(e^{(b+\theta)(\tau-\gamma)} - e^{b\gamma})}{b^2} \right) \right. \\
 &\left. + a \left( 1 + \frac{ab}{b+\theta} \right) \frac{(\tau^2 - \gamma^2)}{2} - \frac{ab}{(b+\theta)^3} \{ (\tau(b+\theta) - 1) - (\gamma(b+\theta) - 1) e^{(b+\theta)(\tau-\gamma)} \} + (M - \tau) a \tau \right]
 \end{aligned}$$

### VII. NUMERICAL EXAMPLES

**Example:1**

The above given result are illustrated through the numerical examples. To illustrate the model we consider the following input data.

A = 260, h = 0.75, d = 1.56, s = 1.5, a = 1, b = 0.1, r = 0.15, c = 1.8, p = 1.5,  $\delta = 0.85$ ,  $\theta = 0.29$ ,  $\gamma = 0.4033$ ,  $i_p = 8$ ,  $i_e = 0.9$ ,

**Ans:** Applying solution process for case ‘1’. we get following results

A.P<sub>1</sub> = 41.3744, M = 0.0423810,  $\tau = 1.3341044$

**Example: 2**

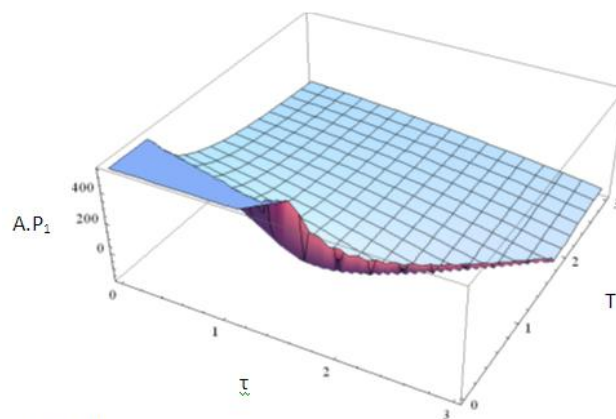
The above given result are illustrated through the numerical examples. To illustrate the model we consider the following input data.

A = 350, h = 0.75, d = 1.56, s = 1.5, a = 1, b = 0.1, r = 0.15, c = 1.8, p = 1.5,  $\delta = 0.85$ ,  $\theta = 0.29$ ,  $\gamma = 0.4033$ ,  $i_p = 8$ ,  $i_e = 0.9$ ,

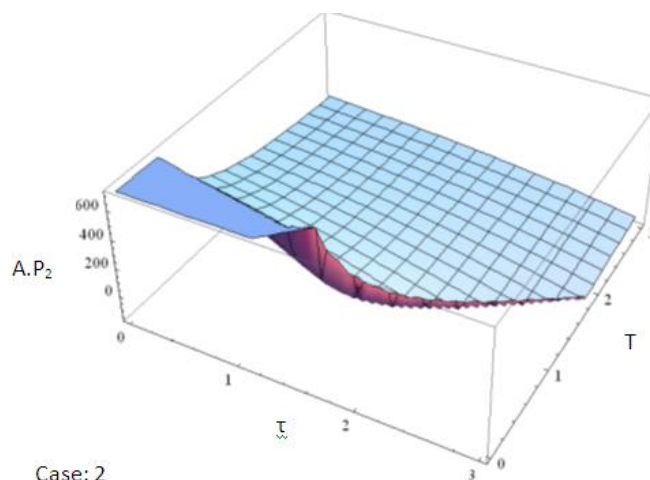
**Ans:** Applying solution process for case ‘2’. we get following results

A.P<sub>2</sub> = 8.32776, M = 1.98228,  $\tau = 1.91413$

Behavior of optimum cost function:



Case: 1



Case: 2



**VIII. SENSITIVITY ANALYSIS**

**Table : 1 Sensitivity analysis for parameters are as shown below-**

| Parameters | % value | M        | $\tau$  | $AP_1$   |
|------------|---------|----------|---------|----------|
| a          | -20%    | 0.16150  | 2.4630  | -28.128  |
|            | -10%    | 0.16151  | 2.4631  | -41.032  |
|            | 0%      | 0.04237  | 1.3341  | 41.374   |
|            | 10%     | 0.04237  | 1.3341  | 27.638   |
|            | 20%     | 0.04238  | 1.3342  | 13.902   |
| b          | -20%    | 0.042381 | 1.3342  | -27.1721 |
|            | -10%    | 0.042381 | 1.3342  | 12.4571  |
|            | 0%      | 0.042381 | 1.3341  | 41.3744  |
|            | 10%     | 0.161557 | 2.4631  | -31.664  |
|            | 20%     | 0.11933  | 8.1258  | -352.676 |
| h          | -20%    | 0.04231  | 1.33411 | 41.274   |
|            | -10%    | 0.04232  | 1.33412 | 41.324   |
|            | 0%      | 0.04231  | 1.33410 | 41.374   |
|            | 10%     | 0.16150  | 2.46310 | -53.854  |
|            | 20%     | 0.16151  | 2.46312 | -53.772  |
| d          | -20%    | 0.04231  | 1.3341  | 41.314   |
|            | -10%    | 0.04232  | 1.3340  | 41.344   |
|            | 0%      | 0.04231  | 1.3341  | 41.374   |
|            | 10%     | 0.16554  | 2.4639  | -53.886  |
|            | 20%     | 0.16551  | 2.4637  | -53.837  |
| s          | -20%    | 0.04231  | 1.33410 | 41.3764  |
|            | -10%    | 0.04232  | 1.33412 | 41.3805  |
|            | 0%      | 0.04233  | 1.33415 | 41.3744  |
|            | 10%     | 0.16151  | 2.46301 | -53.9463 |
|            | 20%     | 0.16152  | 2.46302 | -53.9570 |
| o          | -20%    | 0.04231  | 1.33415 | 41.36641 |
|            | -10%    | 0.04232  | 1.33412 | 41.37042 |
|            | 0%      | 0.04231  | 1.33412 | 41.37410 |
|            | 10%     | 0.16150  | 2.46302 | -53.8558 |
|            | 20%     | 0.16153  | 2.46302 | -53.7760 |
| $\delta$   | -20%    | 0.04231  | 1.33415 | 41.3562  |
|            | -10%    | 0.04231  | 1.33412 | 41.3659  |
|            | 0%      | 0.04231  | 1.33413 | 41.3744  |
|            | 10%     | 0.16153  | 2.46302 | -53.8651 |
|            | 20%     | 0.16154  | 2.46302 | -53.8075 |
| $\theta$   | -20%    | 0.04232  | 1.33414 | 36.489   |
|            | -10%    | 0.04233  | 1.33412 | 39.297   |
|            | 0%      | 0.04231  | 1.33413 | 41.374   |
|            | 10%     | 0.16150  | 2.46303 | -56.665  |
|            | 20%     | 0.16153  | 2.46302 | -60.898  |
| $\gamma$   | -20%    | 0.04235  | 1.33415 | 31.716   |
|            | -10%    | 0.04232  | 1.33412 | 36.041   |
|            | 0%      | 0.04231  | 1.33412 | 41.374   |
|            | 10%     | 0.16153  | 2.46303 | -51.192  |
|            | 20%     | 0.16153  | 2.46302 | -48.478  |
| r          | -20%    | 0.16152  | 2.46311 | -53.906  |
|            | -10%    | 0.16151  | 2.46312 | -53.921  |
|            | 0%      | 0.04233  | 1.33417 | 41.374   |
|            | 10%     | 0.04232  | 1.33412 | 41.369   |
|            | 20%     | 0.04230  | 1.33413 | 41.364   |

**Observations**

1. We remarked from table (1) that with the increases in demand rate parameter (a) results increases narrowly in length of time ( $\tau$ ), trade period (M) and decreases in total average cost  $AP_1$ . One sensitive point are located in table, in this point values of length of time ( $\tau$ ), trade period (M) are decrease hastily and the value of total average cost increases extremely. The table has same behaviour above and below from this sensitive point. That is, change in demand rate parameter (a) will cause positive change in length of time ( $\tau$ ), trade period (M) and negative change in total average cost ( $AP_1$ ), except the sensitive point.

2. It is clearly visible from table (1) that with the increment in backlogging cost (s), length of time ( $\tau$ ), trade period (M) and total average cost increases barely. Department of the table are similar upward and

downward from the sensitive point. One sensitive point are located in table. In this point, the values of length of time ( $\tau$ ), trade period (M) are quickly increases but the value of total average cost decreases intensely.

3. We perceived from Table (1) that as the backlogged parameter ( $\delta$ ), opportunity cost (o), unit cost (d), and holding cost (h) boosts the values of trade period (M), length of time ( $\tau$ ) boosts slightly and total average cost ( $AP_1$ ) is boosts. A place has located in table, which are sensitive because in this place the values of trade period (M), length of time ( $\tau$ ) are sharply escalations while the value of total average cost (T.A.C.) is extremely reduces. That is, change in backlogged parameter ( $\delta$ ), opportunity cost (o), unit cost (d), and holding cost (h) will cause positive change in trade period (M), length of time ( $\tau$ ) and total average cost (T.A.C.).

4. With the help of table (1), we examined that increases of deterioration rate ( $\theta$ ) and demand rate parameter (b) results slightly increments in trade period (M), length of time ( $\tau$ ) but increment in total average cost of the system. Table (8) has one sensitive point. In which the values of trade period (M), length of time ( $\tau$ ) and cycle time (T) are sharply increases while the value of total average cost is extremely decreases. Consequently, the value of deterioration rate ( $\theta$ ) and demand rate parameter (b) increases, then values of trade period (M), length of time ( $\tau$ ) are marginally increases but the value of total average cost is decreases.

5. We discerned that the values of trade period (M), length of time ( $\tau$ ) are barely increases while the value of total average cost is increases, with the increment in time ( $\gamma$ ) in the table (1). The swiftly increases in the values of trade period (M), length of time ( $\tau$ ) but the extremely decreases the value of total average cost of the system at sensitive point. This point are located in the table. After this point, the values of trade period (M), length of time ( $\tau$ ) are increases barely and total average cost is increases, when value increases in time ( $\gamma$ ).

6. From table (1), we observed that as inflation ( $r$ ) increases, the values of trade period (M), length of time ( $\tau$ ) and total average cost (T.A.C.) remains approximately constant. A sensitive point which are districted in table, in which the values of trade period (M), length of time ( $\tau$ ) are swiftly decreases while the value of total average cost (T.A.C.) is vastly increases. Aspects of the table are identical before and after from this sensitive point.

## IX. CONCLUSION

The present chapter revisited the classical (EOQ) inventory model for non-instantaneous deteriorating items proposed. A simple procedure for determining the optimal inventory policy which minimizes the total cost per unit time is suggested. This procedure exploited the fact that the objective function of the problem is differentiable on its domain of definition. It was shown that deterioration affects the optimal policy only if the time of the onset of deterioration, is less than the optimal time of the start of partial backlogging when deterioration is absent. Otherwise, the optimal policy corresponds to the (EOQ) of the inventory model with no deterioration. Finally, it is conjectured that the basic idea of exploiting the fact that the objective function of the present problem is differentiable to determine the optimal inventory is extended to inventory models with permissible delay in payment with inflation is has appropriate results and another extensions can be done in this model..

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