The International Journal of Engineering and Science (IJES)
|| Volume || 10 || Issue || 8 || Series I || Pages || PP 15-23 || 2021 ||
ISSN (e): 2319-1813 ISSN (p): 20-24-1805

Forced Convection in Circular Pipe Flow for Newtonian Fluids with Viscous Dissipation Effect on Heat Transfer

M. U. UWAEZUOKE

Department of Mathematics, Imo State University, P.M.B 2000, Owerri, Nigeria.

ABSTRACT

In this study, an analysis of laminar forced convection in a pipe for a Newtonian fluid with constant properties is performed by taking the viscous dissipation into account. The study examines both hydrodynamically and thermally fully developed flow. Two different thermal boundary conditions are considered: constant heat flux (CHF) and constant wall temperature (CWT). Either wall heating (the fluid is heated) or wall cooling (the fluid is cooled) is considered. The temperature distribution and Nusselt numbers are analytically determined as a function of the Brinkman number.

Keywords: *Laminar forced convection; Viscous dissipation; Pipe; Constant heat flux; Constant wall temperature; Wall heating; Wall cooling*

Date of Submission: 17-07-2021 Date of Acceptance: 02-08-2021

Nomenclature

Br Brinkman number, Eq. (8)
Brq modified Brinkman number, Eq. (11)
c_p specific heat at constant pressure [J/kgK]
Nu Nusselt number
k thermal conductivity [W/mK]
Pr Prandtl number
q_w wall heat flux [W/m^2]
r radial coordinate [m]
r_0 radius of pipe [m]
R dimensionless radial coordinate
T temperature [K]
u velocity [m/s]
z axial direction [m]

Greek Symbols

α thermal diffusivity [m^2/s]
μ dynamic viscosity [Pa s]
ρ density [kg/m^3]
υ kinematic viscosity [m^2/s]
θ dimensionless temperature, Eq. (6)
θ_q dimensionless temperature modified, Eq. (12)

Subscripts

c Centreline
m Mean
vd viscous dissipation
w Wall

I. INTRODUCTION

Viscous dissipation changes the temperature distributions by playing a role like an energy source, which affects heat transfer rates. The merit of the effect of the viscous dissipation depends on whether the pipe is being cooled or heated.

Many studies involving pipe flows in the existing literature have neglected the effect of viscous dissipation. In fact, the shear stresses can induce a considerable power generation. However, in the existing convective heat transfer literature, this effect is usually regarded as important only in two cases of flow in capillary tubes and flow of very viscous fluids. The effects of viscous dissipation in laminar flows have not yet been deeply investigated. For liquids with high viscosity and low thermal conductivity, disregarding the viscous dissipation can cause appreciable errors.

The work of Brinkman [1] appears to be the first theoretical work dealing with viscous dissipation. The temperature distribution in the entrance region of a circular pipe at the wall of which was maintained at either a constant temperature as that of the entering fluid or constant heat flux was examined. The highest temperatures were, not surprisingly, discovered to be localized in the wall region.

Tyagi [2] performed a wide study on the effect of viscous dissipation on the fully developed laminar forced convection in cylindrical tubes with an arbitrary cross-section and uniform wall temperature.

Ou and Cheng [3] employed the separation of variables method to study the Graetz problem with finite viscous dissipation. They obtained the solution in the form of a series the eigenvalues and eigenfunctions of which satisfy the Sturm-Liouville system. The solution technique follows the same approach as that applicable to the classical Graetz problem and therefore suffers from the same weakness of poor convergence behavior near the entrance.

Lin et al. [4] showed that the effect of viscous dissipation was very relevant in the fully developed region if convective boundary conditions were considered. With these boundary conditions and if viscous dissipation was taken into account, the fully developed value of the Nusselt number was 48/5 for every value of the Biot number and of the other parameters. On the other hand, it is well known that, if a forced convection model with no viscous dissipation is employed, the fully developed value of the Nusselt number for convective boundary conditions depends on the value of the Biot number.

Basu and Roy [5] analyzed the Graetz problem by taking account of viscous dissipation but neglecting the effect of axial conduction. They showed that the effect of viscous dissipation could not be neglected when the wall temperature was uniform.

For the thermal condition, Liou and Wang [6] using the uniform wall heat flux, Berardi et al. [7] using the convection with an external isothermal fluid, Lawal and Mujumdar [8] and Dang [9] using the uniform wall temperature studied the effect of viscous dissipation in the thermal entrance region in a pipe.

The effect of viscous dissipation in the thermal entrance region of slug flow forced convection in a circular duct was studied by Barletta and Zanchini [10]. The temperature field and the local Nusselt number were determined analytically for any prescribed axial distribution of wall heat flux including uniform, linearly varying and exponentially varying heat fluxes.

Zanchini [11] analyzed the asymptotic behavior of laminar forced convection in a circular tube, for a Newtonian fluid at constant properties by taking into account the viscous dissipation effects. It was disclosed that particularly for the boundary conditions of uniform wall temperature and of heat transfer by convection to an external fluid yielded the same asymptotic behavior of the Nusselt number, namely $Nu = {}^{48}\!/_{5}$. And therefore, he obviously stated that, for these boundary conditions, when the wall heat flux $q_w(x)$ tended to zero, i.e., the local Brinkman number $Br(x)$ tended to infinity, it is completely wrong to neglect the effect of viscous dissipation on the asymptotic behavior of the forced convection problem.

Barletta [12] studied the asymptotic behavior of the temperature field for the laminar and hydrodynamically developed forced convection of a power-law fluid which flows in a circular duct taking the viscous dissipation into account. The asymptotic Nusselt number and the asymptotic temperature distribution were evaluated analytically in the cases of either the uniform wall temperature or convection with an external isothermal fluid.

Barletta and Rossi di Schio [13] investigated the laminar convection with viscous dissipation in a circular duct in the case of a sinusoidal axial variation of the wall heat flux.

Morini and Spiga [14] analytically determined the steady temperature distribution and the Nusselt numbers for a Newtonian incompressible fluid in a rectangular duct, in fully developed laminar flow with viscous dissipation, for any combination of heated and adiabatic sides of the duct.

Pinho and Oliveira [15] investigated the forced convection of Phan-Thien-Tanner fluid in laminar pipe and channel flows including the effects of viscous dissipation. It was shown that the beneficial effects of fluid elasticity were enhanced by viscous dissipation.

The effect of viscous dissipation on the laminar forced convection in a circular duct for a Bingham fluid under different thermal boundary conditions was studied by Vradis et al. [16], Min et al. [17] and Khatyr et al. [18]. The asymptotic temperature profile and the asymptotic Nusselt number were determined for various axial distributions of wall heat flux which yielded a thermally developed region.

The aim of the present work is to investigate the effect of viscous dissipation on both hydro-dynamically and thermally fully developed laminar forced convective flow in a pipe. The effect of Brinkman number on the temperature profile and the Nusselt number is obtained for the constant wall heat flux and the constant wall temperature thermal boundary conditions. Either the wall heating (the fluid is heated) case or the wall cooling (the fluid is cooled) case is considered.

II. ANALYSIS

The heat transfer parameter which has been popularly accepted as an indicator of the strength of viscous dissipation is the Brinkman number (Br). According to the definition adopted in this study, positive or negative values of Br represent heating or cooling, respectively.

2.1. Theoretical model

The flow is considered to be fully developed both thermally and hydrodynamically. Steady, laminar flow having constant properties (i.e. the thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature) is considered. The axial heat conduction in the fluid and in the wall is assumed to be negligible.

The well-known parabolic velocity profile (which is also called as Hagen-Poiseuille's velocity distribution) for fully developed laminar pipe flow is given as follows:

$$\frac{u}{u_c} = 1 - \left(\frac{r}{r_0}\right)^2 \tag{1}$$

The energy balance equation including the effect of the viscous dissipation is given by

$$u\frac{\partial T}{\partial z} = \frac{\upsilon}{\Pr}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial r}\right)^2 \tag{2}$$

where the second term on the right-hand side is the viscous dissipation term.

Due to axisymmetry at the center, the thermal boundary condition at $r = 0$ can be written as

$$\left.\frac{\partial T}{\partial r}\right|_{r_0} = 0 \tag{3}$$

Two kinds of thermal boundary condition at the wall are considered in this study, namely: constant wall heat flux (CHF) and constant wall temperature (CWT). They are treated separately in the following.

2.2 CHF case

For the constant heat flux at the wall, the thermal boundary condition can be written as

$$k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = q_w \tag{4}$$

where q_w is positive when its direction is to the fluid (wall heating), otherwise it is negative (wall cooling).

For the uniform wall heat flux case, the first term on the left-hand side of Eq. (1) is

$$\frac{\partial T}{\partial r} = \frac{dT_w}{dz} \tag{5}$$

By introducing the following non-dimensional quantities:

$$R = \frac{r}{r_0} \qquad \theta = \frac{T_w - T}{T_w - T_c} \tag{6}$$

Eq. (1) can be written as

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\theta}{dR}\right) = a\left(1 - R^2\right) - 4BrR^2 \tag{7}$$

where $a = \dfrac{u_c r_0^2}{\alpha\left(T_w - T_c\right)}\dfrac{dT_w}{dz}$ and Br is the Brinkman number given as

$$Br = \frac{\mu u_c^2}{k\left(T_w - T_c\right)} \tag{8}$$

For the solution of the dimensionless energy transport equation given in Eq. (7), the dimensionless boundary conditions are given as follows:

$$\theta = 1 \qquad \left.\frac{\partial \theta}{\partial R}\right|_{R=0} = 0 \quad \text{at} \quad R = 1 \qquad (9)$$

$$\theta = 0 \qquad \text{at} \quad R = 1$$

The solution of Eq. (7) under the thermal boundary conditions given in Eq. (9) is

$$\theta(R) = \left(1 - \frac{4}{3}R^2 + \frac{1}{3}R^4\right) + \frac{Br}{3}\left(R^3 - R^4\right) \qquad (10)$$

Since we have developed the above equation for the constant wall heat case, as it is usual in the existing literature, we can also use the modified Brinkman number which is in the following:

$$Br_q = \frac{\mu u_c^2}{r_0 q_w} \qquad (11)$$

In terms of the modified Brinkman number (based on the heat flux) given above, the temperature distribution is obtained as:

$$\theta_q = \frac{T - T_w}{\frac{r_0 q_w}{k}} = \left(\frac{3}{4} - R^2 + \frac{R^4}{4}\right) + Br_q\left(\frac{1}{2} - R^2 + \frac{R^4}{2}\right) \qquad (12)$$

In fully developed flow, it is usual to utilize the mean fluid temperature, T_m, rather than the center line temperature when definingthe Nusselt number. This mean or bulk temperature is given by:

$$T_m = \frac{\int \rho u T dA}{\int \rho u dA} \qquad (13)$$

The dimensionless mean temperature is obtained as

$$\theta_m = \frac{T_m - T_w}{T_c - T_w} = \frac{11}{18} + \frac{1}{18}Br \qquad (14)$$

In terms of Br_q defined in Eq. (11), the mean temperature is obtained as

$$\frac{T_m - T_w}{\frac{q_w r_0}{k}} = \frac{11}{24} + \frac{1}{4}Br_q \qquad (15)$$

2.3. CWT case

When the constant temperature is considered, since $\frac{dT_w}{dz} = 0$, the first term on the left-hand side of Eq. (1) is

$$\frac{\partial T}{\partial z} = \left(\frac{T_w - T}{T_w - T_c}\right)\frac{dT_c}{dz} \qquad (16)$$

Substituting this result into Eq. (2) and introducing the dimensionless quantities given in Eq. (6) gives the following dimensionless equation for the CWT case:

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\theta}{dR}\right) = b\theta\left(1 - R^2\right) - 4BrR^2 \qquad (17)$$

where $b = \frac{u_c r_0^2}{\alpha\left(T_w - T_c\right)}\frac{dT_c}{dz}$ and Br is the Brinkman number. The boundary conditions given in Eq. (9) is also valid for this case. Actually, no simple closed form solution θ can be obtained for this equation. However, the variation of d can be quite easily obtained to any required degree of accuracy by using an iterative procedure [19]. The temperature profile for the CHF case is used as the first approximation and Eq. (17) is then integrated to obtain the θ. This iterative procedure is repeated until an acceptable convergence is obtained.

From the practical viewpoint, it is important to determine the forced convective heat transfer coefficient, which will lead to the heat transfer rate, that is

$$h = \frac{-k\left(\frac{\partial T}{\partial r}\right)_{r=R}}{T_w - T_m}$$

which is obtained from Nusselt number that is defined as

$$Nu_D = \frac{q_w D}{\left(T_w - T_m\right)k} \tag{18}$$

where Nu_D is the Nusselt number based on the pipe diameter. After performing necessary substitutions, we obtain:

$$Nu_D = \frac{24\left(2 - Br\right)}{11 - Br} \tag{19}$$

In terms of the modified Brinkman number, Br_q

$$Nu_D = \frac{48}{11 + Br_q} \tag{20}$$

The above equation is previously shown in Refs. [11] and [15], which gives a credit to the validity of this study. Note that the Nusselt number definitions given in Eqs. (19) and (20) are obtained for the CHF case. For the CWT case the Nusselt numbers are found from the iterative procedure described above.

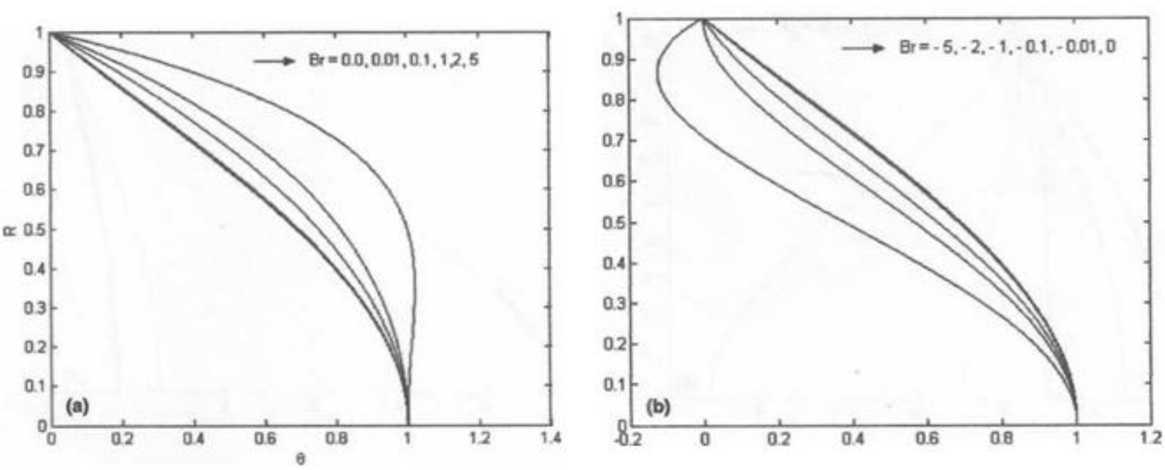

Fig. 1. Dimensionless temperature distribution in terms of Br for CHF case: (a) wall heating and (b) wall cooling.

III. RESULTS AND DISCUSSION

In the absence of viscous dissipation the solution is independent of whether there is wall heating or cooling. However, viscous dissipation always contributes to internal heating of the fluid, hence the solution will differ according to the process taking place. The Brinkman number is chosen as a criterion which shows the relative importance of viscous dissipation. As slated earlier that two different thermal boundary conditions have been considered for the pipe wall: the constant heat flux (CHF) and the constant wall temperature (CWT). For each boundary condition both wall heating or wall cooling case are examined. In the following they are treated separately.

For the CHF condition. Fig. la and b depicts the temperature distributions for different Brinkman numbers. Viscous dissipation, as an energy source, severely distorts the temperature profile. Remember positive values of Br correspond to wall heating (heat is being supplied across the walls into the fluid) case $\left(T_w > T_c\right)$, while the opposite is true for negative values of Br. Fig. la reveals the dimensionless temperature distribution for the wall heating case. As seen, it is an increasing function of Br. For lower values of $Br\left(Br = 0.01\right)$ the effect of viscous dissipation is found to be negligible. Since the highest shear rate occurs near the wall, the effect of viscous dissipation is most significant in this region. For the wall cooling case, the dimensionless wall temperature is a decreasing function of Br (Fig. lb).

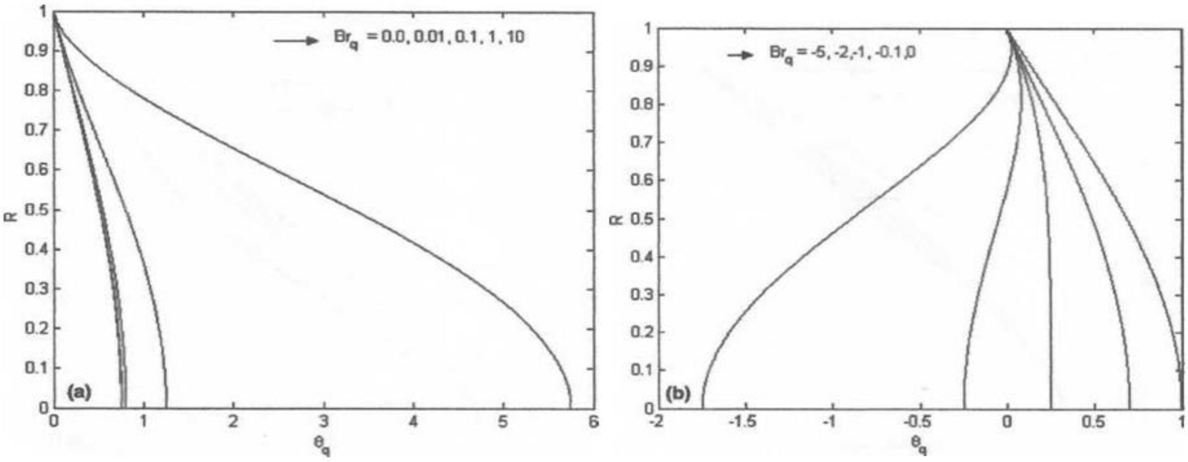

Fig. 2. Dimensionless temperature distribution in terms of Br_q for CHF case: (a) wall heating and (b) wall cooling.

The standard way of making temperature dimensionless based on Eq. (6) is not appropriate for the situation of imposed heat flux because the temperature scale $\Delta T = T_w - T_c$ varies with relevant parameters and may cause a misinterpretation of the corresponding variation of T . In fact, for a given $q_w, \Delta T$ is the unknown of the problem and it is more convenient to define a fixed temperature scale that we take as $\frac{q_w R}{k}$. For different values of the modified Brinkman number, Br_q , Fig. 2a and b shows the temperature profiles made dimensionless using this scale for wall heating and wall cooling cases, respectively. These plots make clear the aforementioned effects of increased dissipation. As expected, increasing dissipation increases the bulk temperature of the fluid due to internal heating of the fluid. For the wall heating case, this increase in the fluid temperature decreases the temperature difference between the wall and the fluid, as will be shown later, which is followed with a decrease in heat transfer (Fig. 2a). When wall cooling is applied, due to the internal heating effect of the viscous dissipation on the fluid temperature profile, temperature difference is increased with the increasing Br_q . In fact, wall cooling is applied to reduce the bulk temperature of the fluid, while the effect of the viscous dissipation is increasing the bulk temperature of the fluid. Therefore, the amount of viscous dissipation may change the overall heat balance. When the Br_q exceeds a certain limiting value, the heat generated internally by viscous dissipation process will overcome the effect of wall cooling.

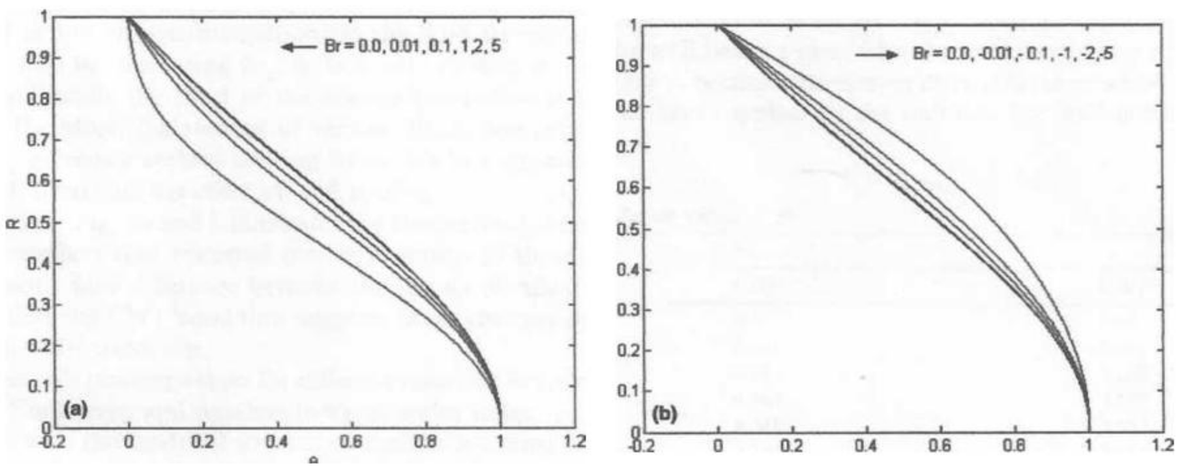

Fig. 3. Dimensionless temperature distribution in terms of Br for CWT case: (a) wall heating and (b) wall cooling.

For the CWT condition, Fig. 3a and b illustrates the temperature profile for different values of Br. The physical mechanisms that occurred are very similar to those occurring for the CHF condition. The most noticeable difference between the results obtained by the CHF and CWT conditions is the fact that the CWT condition suggests lower changes in the temperature profile when compared to the CHF condition.

Table 1: Nusselt number values for different values of Br

Br	Nu	
	CHF	CWT
-1	6	4.072
-0.1	4.541	3.701
-0.01	4.381	3.663
0	4.364	3.658
0.01	4.346	3.653
0.1	4.183	3.614
1	2.4	3.194

Table 2: Nusselt number values for different values of Br_q

Br_q (CHF)	Nu
-1	9.6
-0.1	4.615
-0.01	4.387
0	4.364
0.01	4.361
0.1	4.138
1	2.824

Table 1 shows the Nusselt number values for different values of Brinkman number for the CHF and CWT conditions. For brevity and standing in a reasonable range, $-1 < Br < 1$. The variation of the Nusselt number with the modified Brinkman number is shown in Table 2.

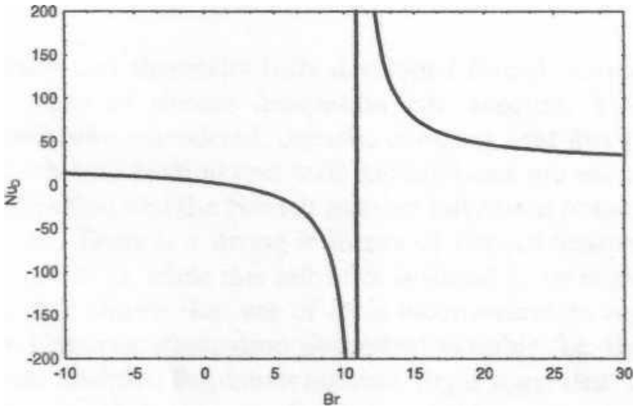

Fig. 4. Variation of Nu with Br for CHF case.

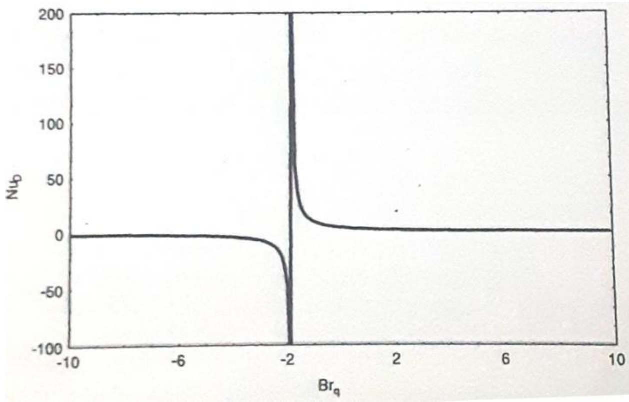

Fig. 5. Variation of Nu with Br_q for CHF case.

Fig. 4 represents the variation of Nusselt number with the Brinkman number for CHF case. As shown, a singularity is observed at $Br = 11$. Actually, this is an expected result, when Eq. (19) is closely examined. For the wall heating case, with the increasing value of Br, Nu decreases in the range of $0 < Br < 11$. This is because the temperature difference which drives the heat transfer decreases. At $Br = 11$, the heat supplied by the wall into the fluid is balanced with the internal heat generation due to the viscous heating. For $Br > 11$, the internally generated heat by the viscous dissipation overcomes the wall heat. When $Br \to +\infty$, Nu reaches an asymptotic value: $Br = 24$. When wall cooling $(Br < 0)$ is applied to reduce the bulk temperature of the fluid, as explained earlier, the amount of viscous dissipation may change the overall heat balance. With increasing value of Br in the negative direction, the Nusselt number reaches an asymptotic value (when $Br \to -\infty$, $Nu \to 24$). As noticed, when Br goes to infinity for either the wall heating or the wall cooling case, the Nusselt number reaches the same asymptotic value, $Nu = 24$. This is due to the fact that the heat generated internally by viscous dissipation processes will balance the effect of wall cooling. Fig. 5 illustrates the variation of Nu_D with Br_q. The behavior observed can be explained similarly to that for Br. For $Br_q = -11/6$, a singularity is observed, which is an expected result from Eq. (20).

IV. CONCLUSIONS

Both hydrodynamically and thermally fully developed forced convection in a pipe has been studied by taking the effect of viscous dissipation into account. Two types of wall thermal boundary condition have been considered, namely: constant heat flux (CHF) and constant wall temperature (CWT). Both wall heating and wall cooling cases are examined. The dimensionless radial temperature distribution and the Nusselt number have been obtained for different values of the Brinkman number, Br. There is a strong influence of viscous dissipation on the heat transfer for higher values of $Br\,(Br > 1)$, while this influence is found to be negligible for lower values of Br. For the CHF case, it is shown that use of Br is inconvenient to represent the viscous dissipation since it includes a viscous dissipation dependent variable (i.e. the centerline temperature, Tc). Instead the use of the modified Brinkman number, Br_q is suggested. For the wall heating case, with the increasing intensity of the viscous dissipation (with an increase at Br) the heat transfer decreases up to a critical value of Br. At this critical value, internally generated heat due to viscous dissipation (q_{vd}) balances the heat rate supplied by the wall (q_w). In the following, over this critical value, q_{vd} suppresses q_w. Similarly, for the wall cooling case, when the Br number exceeds a critical value, q_{vd} overcomes the heat removed at the wall (q_w) and the fluid heats up longitudinally. In the second part of this study, the effect of viscous dissipation under the condition examined here is investigated for the hydrodynamically fully developed but thermally developing flow.

REFERENCES

[1]. Brinkman HC. Heat effects in capillary flow I. Appl Sci Res 1951;A2:120-4.

[2]. Tyagi VP. Laminar forced convection of a dissipative fluid in a channel. J Heat Transfer 1996;88:161-9.

[3]. Ou WJ, Cheng KC. Viscous dissipation effects in the entrance region heat transfer in pipes with uniform heat flux. Appl Sci Res 1973;28:289-301.

[4]. Lin TF, Hawks KH, Leidenfrost W. Analysis of viscous dissipation effect on the thermal entrance heat transfer in laminar pipe flows with convective boundary conditions. Warm Stoffubertrag 1983;17:97-105.

[5]. Basu T, Roy DN. Laminar heat transfer in a tube with viscous dissipation. Int J Heat Mass Transfer 1985;28:699-701.

[6]. Liou CT, Wang FS. Solutions to the extended Graetz problem for a power-law fluid with viscous dissipation and different entrance boundary conditions. Numer Heat Transfer A 1990;17:91-108.

[7]. Berardi PG, Cuccurullo G, Aciemo D, Russo P. Viscous dissipation in duct flow of molten polymers. In: Proceedings of Eurotherm Seminar 46, Pisa, Italy; July 1995. p. 39-43.

[8]. Lawal A, Mujumdar AS. The effects of viscous dissipation on heat transfer to power law fluids in arbitrary cross- sectional ducts. Warm Stoffubertrag 199227:437—46.

[9]. Dang VD. Heat transfer of a power-law fluid at low Peclet number flow. J Heat Transfer 1983;105:542-9.

[10]. Barletta A, Zanchini E. Forced convection in the thermal entrance region of a circular duct with slug flow and viscous dissipation. Int J Heat Mass Transfer 1997;40(5):1181-90.

[11]. Zanchini E. Effect of viscous dissipation on the asymptotic behavior of laminar forced convection in circular tubes. Int J Heat Mass Transfer 1997;40:169-78.

[12]. Barletta A. Fully developed laminar forced convection in circular ducts for power-law fluids with viscous dissipation. Int J Heat Mass Transfer 1997:40< l):15-26.

[13]. Barletta A, Rossi di Schio E. Effects of viscous dissipation on laminar forced convection with axially periodic wall heat flux. Heat and Mass Transfer 1999;35:9-16.

[14]. Morini GL, Spiga M. Nusselt numbers in rectangular ducts with laminar viscous dissipation. ASME J Heat Transfer 1999;121:1083-7.

[15]. Pinho FT, Oliveira PJ. Analysis of forced convection in pipes and channels with the simplified Phan-Thien-Tanner fluid. Int J Heat Mass Transfer 2000;43:2273-87.

[16]. Vradis GC. Dougher J, Kumar S. Entrance pipe flow and heat transfer for a Bingham plastic. Int J Heat Mass Transfer 1993;36:543-52.

[17]. Min T. Yoo JY. Choi H. Laminar convective heat transfer of a Bingham plastic in a circular pipe-I. Analytical approach-thermally fully developed flow and thermally developing flow (the Graetz problem extended). Int J Heat Mass Transfer 1997;40:3025-37.

[18]. Khatvr R. Ouldhadda D. II Idrissi A. Viscous dissipation effects on the asymptotic behavior of laminar forced convection for Bingham plastics in circular ducts. Int J Heat Mass Transfer 2003;46:589-98.

[19]. Oosthuizen PH. Naylor D. Introduction to convective heat transfer analysis. New York: McGraw-Hill; 1999.

M. U. UWAEZUOKE. "Forced Convection in Circular Pipe Flow for Newtonian Fluids with Viscous Dissipation Effect on Heat Transfer." *The International Journal of Engineering and Science (IJES)*, 10(08), (2021): pp. 15-23.