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Comparative Study of Time Series Models to Rainfall Data

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--***ABSTRACT***--
There are many time series models and some of them are Exponential models, regression models, ARIMA models, GARCH models, etc. For rainfall data we are fitted Holt-winter's seasonal model, Single exponential smoothing and Moving Average of order 4. For comparing these models we use mean square error (MSE) criteria. Upon comparing three models, we conclude that a model which having lowest MSE is the best for rainfall data.

Keywords: *Rainfall, Holt-Winter seasonal method, Single Exponential Smoothing, Moving average model, Mean Square Error criteria.*

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I. INTRODUCTION

Time series is an arrangement of observations according to time. Forecasting is Prediction of observations for the future. Time series and forecasting have wide number of applications, for instance, in atmospheric sciences, forecasting rainfall, temperature, humidity etc. In economics, the increase in production, banking growth of investments, Financial plans, cash position receivables, credit policy needs for funds, in companies objectives such as profits, returns on investments, market share, product leadership, marketing plans, advertising and sales effort, prices, product features or new products inventories, National Economy, Trend and cycle of general business, expectations and confidence competing products in other industries; In Customers, number of customers, characteristics of customers, income cash, line of credit customer's stocks of goods and rate of usage, preference for industry products; In competitors and own company, advertising and Sales effort prices, distribution, availability of products, new products, production levels, inventories and capacity, Company distributors, competitive effectiveness inventories, efficiency of channels used in company production, organization capacity, current inputs, current employment, inventories, production methods and costs, new products scheduled; In company financial organizations, objectives such as cash position, Receivables credit policy, needs for funds; In executive office activities such as company goals; profits, return on investment, market share, growth, product leadership.

There are wide variety of models for time series analysis and forecasting methods. The simple and the easiest among many are moving averages. Moving averages methods are used for reduction of errors and smoothes the data. The effect of this smoothing is to eliminate randomness, so this pattern can be projected into the future and used for forecast. Arithmetic mean generally used for moving average procedure. Moving average procedure especially for measurement of trend by smoothing out the fluctuations of the data. Arithmetic mean is affected by extreme values. In time series data, extreme values are affected due to random fluctuations. Geometric mean is not affected as Arithmetic mean due to extreme observations. A simple moving average is simple and easiest of all forecasting models. Average of time series observations for particular period gives moving average and proceeds till last time series observations. Average for simple moving averages gives double moving averages. Average for averages of averages for number of times gives multiple moving averages.

Exponential smoothing[1-8] is a special case of moving averages, Weights for moving averages are equal and these weights are exponentially decayed according to time. Exponential smoothing is also simple exponential smoothing, double exponential smoothing, triple exponential smoothing, adaptive smoothing, etc. Simple exponential smoothing for time 't+1' is sum of time series 't' and forecast constant multiply with error. Single exponential smoothing for exponential smoothing gives double exponential smoothing. Simple exponential smoothing with double exponential smoothing gives triple exponential smoothing.

Forecasting seasonals and Trends by Exponentially weighted moving Averages' published by Charles C. Holt [9]. J. W. Taylor[10] explains different exponentially weighted models for time series data. Granger and yongil Jeon[11] gave an research paper on A Time – distance criterion for Evaluating forecasting models. Forecasting functional time series was given by Han Lin Shang and Hyndman R.J.[12].

II. METHODOLOGY

We fitted Holt Winter Seasonal method, Simple Exponential Smoothing Method and Moving Averages method.

Holt-Winter's Seasonal Method: Holt's Method was extended by Holt and Winter by adding Seasonality. Holt-winter Seasonal Method consists of Forecast Equation and three Smoothing Equations i.e l_t for level, b_t for Trend and S_t for Seasonal Component and their corresponding Smoothing parameters are α, β and ϒ. We use m to denote frequency of the Seasonality i.e the number of seasons in a year i.e for monthly data m=12, for quarterly data m=4. There are two methods for Holt Winter Seasonality, Holt-Winter's additive model and Holt-Winter's Multiplicative method.

Holt Winter's Additive Model

The component form for the Additive method is

$$\hat{Y}_{t+h/t} = l_t + hb_t + s_{t+h-m(k+1)}$$

For level l_t $= \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$

For Trend b_t $= \beta(l_t - l_{t-1}) + (1-\beta^*)b_{t-1}$

For seasonality s_t $= \Upsilon(y_t - l_{t-1} - b_{t-1}) + (1-\Upsilon)s_{t-m}$

Holt Winter's Multiplicative Method

The Component form for the Multiplicative Method is

$$\hat{Y}_{t+h/t} = (l_t + hb_t)\, s_{t+h-m(k+1)}$$

For level l_t $= \alpha(y_t / s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$

For Trend b_t $= \beta(l_t - l_{t-1}) + (1-\beta^*)b_{t-1}$

For seasonality s_t $= \Upsilon(y_t/(l_{t-1} - b_{t-1})) + (1-\Upsilon)s_{t-m}$

Single Exponential Smoothing

Exponential smoothing is the special type of moving average method in forecasting. In simple moving average the mean of the past k observations that mean the weight of k time series observations having equal value 1/k. Where as in exponential smoothing, if observations get older weights are also exponentially decreasing. The most recent observations will usually provide the highest weight value and observation is getting.

Single exponential smoothing is also called a simple exponential smoothing. The parameter in simple exponential smoothing is 'α'. If we want to estimate forecast value of some 't+1' point then equation becomes

$F_{t+1} = F_t + \alpha\ (Y_t - F_t)$

Where F_{t+1} = forecast for time point 't+1'

F_t =Forecast for time point 't'

Y_t= our time series observation at time 't'

α = constant

α lies between 0 & 1

$\alpha + \beta = 1$

The forecast for time 't+1' is simply the old forecast for time 't' plus an adjustment for the error with constant.

$$F_{t+1} = F_t + \alpha (Y_t - F_t)$$

$$= F_t + \alpha Y_t - \alpha F_t$$

$$= \alpha Y_t + (1 - \alpha) F_t$$

$$= \alpha Y_t + \beta F_t$$

If this expansion of the equation is

$F_{t+1} = F_t + \alpha (Y_t - F_t)$

$= F_t + \alpha Y_t - \alpha F_t$

$= \alpha Y_t + (1 - \alpha) F_t$

$= \alpha Y_t + \beta F_t$

$F_{t+1} = \alpha Y_t + (1 - \alpha)[\alpha Y_{t-1} + (1 - \alpha) F_{t-1}]$

$= \alpha Y_t + \alpha (1 - \alpha) Y_{t-1} + (1 - \alpha)^2 F_{t-1}$

$= \alpha Y_t + \alpha (1 - \alpha) Y_{t-1} + (1 - \alpha)^2 [\alpha Y_{t-2} + (1 - \alpha) F_{t-2}]$

$= \alpha Y_t + \alpha (1 - \alpha) Y_{t-1} + \alpha (1 - \alpha)^2 Y_{t-2} + (1 - \alpha)^3 F_{t-2}$

If this substitution process is repeated by replacing F_{t-2} by its components F_{t-3} by its components and so on the results

$$F_{t+1} = \alpha Y_t + \alpha (1 - \alpha) Y_{t-1} + \alpha (1 - \alpha)^2 Y_{t-2} + \alpha (1 - \alpha)^3 Y_{t-3} + \alpha (1 - \alpha)^4 Y_{t-2} + \ldots\ldots\ldots + \alpha (1 - \alpha)^{t-1} Y_1 + (1 - \alpha)^t F_1$$

$$F_t^{(i)} = (1 - \beta) \sum_{j=0}^{T-1} \beta^j Y_{T-j}$$

Moving Average Method: Moving averages smooths the past data simple moving averages. Generally if we have data of type

t: 1 2 3 … n

u_t: u_1 u_2 u_3 …. u_n

In the simple moving averages model, the average of the time series observations and put at middle of k observations

$$M_T = (u_1 + u_2 + \ldots.. + u_k)/k$$
$$= \left(\sum_{i=1}^{k} u_i\right)/k$$

Remaining moving averages are obtained by adding new term i.e, u_{k+1} term to the above equation and deducting first term i.e, u_1 term from above formulae.

$$M_{T+1} = \frac{u2+u3+\cdots.+uk+1}{k}$$
$$= \frac{u1+u2+u3+\cdots.+uk+uk+1-u1}{k}$$
$$= \frac{u1+u2+\cdots.+uk}{k} + \frac{uk+1-uk}{k}$$
$$= M_T + \frac{Uk+1-u1}{k}$$
$$M_{T+1} = M_T + (u_{k+1} - u_1)/k$$

By using the above Recurrance formulae of moving averages, we estimate mean for last 'n' terms. The moving averages of $\frac{k-1}{2}$ of upper values and $\frac{k-1}{2}$ terms of lower values are not estimated using moving averages method.

Time point	time series value	moving average
1	u_1	
2	u_2	
.	.	
.	.	
.	.	
k	u_k	M_T
k+1	u_{k+1}	M_{T+1}

k+2	u_{k+2}	M_{T+2}
.	.	.
.	.	.
.	.	.
n	u_n	M_k

If length of moving average is odd then moving average value corresponds to$(\frac{k+1}{2})^{th}$ term. If length of moving average is even then moving average value corresponds to middle of $(n/2)^{th}$ term and$(\frac{n+2}{2})^{th}$ terms.

III. EMPERICAL INVESTIGATION

Generally the single exponential smoothing model is

$$S_t = \alpha Y_t + \beta S_{t-1}$$

$$= 0.2Y_t + 0.8S_{t-1}$$

Where α is parameter of the smoothing model and it lies between 0 and 1.

Holt Winter's Additive Model:

$$\hat{Y}_{t+h/t} = l_t + hb_t + s_{t+h-m(k+1)}$$

For level l_t $= \alpha(y_t-s_{t-m})+(1-\alpha)(l_{t-1}+b_{t-1})$

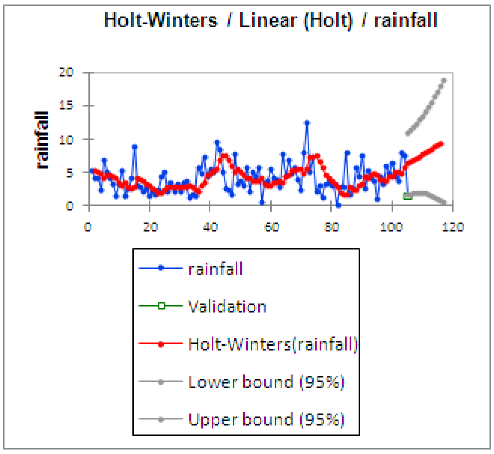
For Trend b_t $= \beta(l_t-l_{t-1})+(1-\beta^*)b_{t-1}$

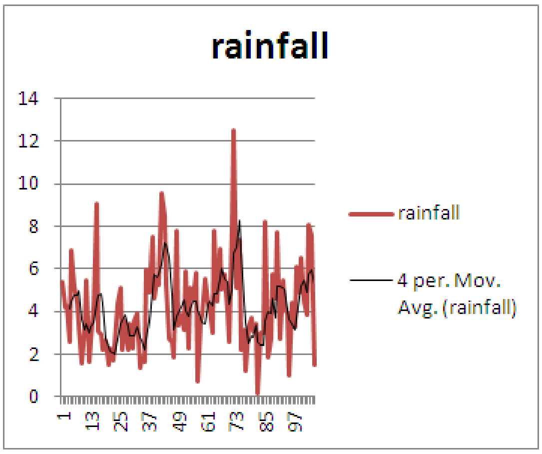
For seasonality s_t $= \Upsilon(y_t-l_{t-1}-b_{t-1}) + (1-\Upsilon)s_{t-m}$

Series before and after smoothing (rainfall)

	Rainfall	Exponential(rainfall)	Holt-Winter model	Moving Average(4)
1	5.380			
2	4.170	5.380	5.380	
3	4.240	5.138	5.090	
4	2.560	4.958	4.837	
5	6.870	4.479	4.208	4.0875
6	5.210	4.957	4.674	4.46
7	4.370	5.008	4.735	4.72
8	3.320	4.880	4.602	4.7525
9	1.600	4.568	4.234	4.9425
10	3.330	3.974	3.491	3.625
11	5.480	3.846	3.235	3.155
12	1.620	4.172	3.551	3.4325
13	2.630	3.662	2.954	3.0075
14	4.250	3.456	2.665	3.265
15	9.060	3.614	2.822	3.495
16	3.060	4.704	4.159	4.39
17	2.930	4.375	3.984	4.75
18	2.200	4.086	3.777	4.825
19	2.670	3.709	3.401	4.3125
20	1.510	3.501	3.166	2.715
21	2.300	3.103	2.679	2.3275
22	1.700	2.942	2.433	2.17
23	2.490	2.694	2.086	2.045
24	4.430	2.653	1.983	2
25	5.140	3.008	2.387	2.73
26	2.230	3.435	2.962	3.44
27	3.520	3.194	2.810	3.5725
28	2.230	3.259	2.976	3.83
29	3.410	3.053	2.820	3.28
. .. ..				
101	4.450	4.776	5.128	5.19
102	3.860	4.710	5.122	5.4875
103	8.040	4.540	4.949	4.925

104	7.590	5.240	5.770	5.72
105	1.480	5.710	6.409	5.985
MSE		**4.796**	**5.644**	**5.5004**





The plots for original data and exponential smoothing, Holt-Winter model and MA(4) are drawn separately by taking time ‘t’ on X-axis and time series values on Y-axis.

IV. CONCLUSIONS

The three fitted models with their MSE are shown in the following table

Model	Equation	MSE
SES	$S_t=0.2Y_t+0.8S_{t-1}$	4.796
Holt-Winter	$\hat{Y}_{t+h/t} = l_t + h\, b_t + s_{t+h-m(k+1)}$ For level $l_t = \alpha(y_t-s_{t-m})+(1-\alpha)(l_{t-1}+b_{t-1})$ For Trend $b_t = \beta(l_t-l_{t-1})+(1-\beta^*)b_{t-1}$ For seasonality $s_t = \Upsilon(y_t-l_{t-1}-b_{t-1}) + (1-\Upsilon)s_{t-m}$	5.644
MA(4)	$U_{t+4}=\sum_{t=1}^{4} u_t/4$	5.5004

Upon observing MSE values, the best model for Rainfall data is Single Exponential Smoothing other than Holt-Winter Smoothing model and Moving Average model.

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