The International Journal of Engineering and Science (IJES)
|| Volume || 10 || Issue || 5 || Series I || Pages || PP 15-25|| 2021 ||
ISSN (e): 2319-1813 ISSN (p): 20-24-1805

Mathematical Model for the Deflection of Rectangular Stiff Plate on Elastic Foundation Using Improved Finite Difference Method

Umeonyiagu I.E [1], Ohanyere. G. C.[2], Onyike. I.P[3].
1. Department of Civil Engineering, Chukwuemeka Odumegwu Ojukwu University, Uli, Nigeria.
2. Department of Civil Engineering, Chukwuemeka Odumegwu Ojukwu University, Uli, Nigeria.
3. Federal University of Technology, Owerri, Nigeria.

ABSTRACT

Numerical solutions to various types of plate structures are indispensible in engineering since it provides approximate solutions to mathematically expressed equation governing a plate. On the other hand, analytical solution provides exact solution to plate bending problems but has restrictions in areas of practical interest. In this study, an improved finite difference method (IFDM) which is a numerical method was used to transform the governing differential equation of a rectangular stiff plate on elastic foundation into improved finite difference coefficients using the central difference method on the discretized plate on elastic foundation. The coefficients obtained from the numerical method were applied into the governing differential equation of the rectangular stiff plate on elastic foundation to obtain the mathematical model for deflection of the plate. Thereafter, 25 interior nodal points of the plate were considered, and the improved finite difference coefficients were evaluated at nodal points to obtain a set of simultaneous linear equations using the boundary conditions for an all edge clamped plate CCCC. The set of simultaneous linear equations were presented in matrix form and solved in a MAT-LAB environment to obtain the unknown deflections at nodal points. The non-dimensional central deflections for the improved finite difference method were compared with that of analytical solution and other numerical solutions from literature, for various sub-grade reactions, ranging from 0 to 6 $(0 \leq K_s \leq 6)$*, and the result were found to be very close. The improved finite difference solutions have an average percentage difference of 0.000076% to Ozgan and Daloglu, 0.000069% to Mishra and Chakrabarti and 0.000068% to Ogunjiofor and Nwoji. Hence, the mathematical model for deflection developed can be used for the analysis of plates on elastic foundation.*

Key Words: *Numerical Solution; Rectangular Stiff Plate; Elastic Foundation; Subgrade Reaction; Central Deflection; Improved finite difference method; mathematical model; boundary condition.*

Date of Submission: 28-04-2021 Date of Acceptance: 12-05-2021

I. INTRODUCTION

Plates are straight, plane, two dimensional structural components of which one dimension referred to as thickness, h, (the distance between the plane faces) is smaller than the other dimensions referred to, as length and width [1]. Plates resist transverse loads by means of bending, and therefore, its flexural rigidity depends significantly on its thickness [1]. In engineering, plates have widely been used by different researchers to solve the problem of bending in different areas such as offshore and port foundations, railway structures, pressure pipes used as liquid and gas pipes and bio-mechanics this is because they combine light weight with a high load carrying capacity to provide technological effectiveness and economic advantage. Hence Szilard stated that the analyses of plates are in three ways which are static analysis, dynamic analysis and stability analysis.

Foundation on the other hand is that part of the structure which transmits the loads from the superstructure to the surrounding soil in which it is in contact with. The response of structures in contact with bearing soils depends greatly on the soil and foundation properties.

The key issue in the analyses of plates in contact with soil is modeling the contact between the structural

Element (plate) and the soil [1]. Since the main task is the analysis of the plate in contact with foundation soil and not the soil itself, the foundation is made elastic foundation by replacing it with simple models, usually spring elements which are closely spaced and discretized [1]. The stiffness of the spring describes the behavior of the elastic foundation [2]. Since plate bending refers to the deflection of a plate perpendicular to the plane under the action of external forces and moments, hence bending analysis of rectangular stiff plate on elastic foundation refers to the determination of deflection of the rectangular stiff plate resting on an elastic foundation under the action of external forces and moments. The amount of deflection is determined by solving the governing differential equation of the stiff plate on elastic foundation. The stresses in the plate can be calculated from the deflections [2]. For plates on elastic foundation, the most commonly used modal for the analysis of such plate is that Winker model because of its simplicity.

Winkler idealization assumes that the soil medium is a system of identical but mutually independent, closely spaced, discrete, linearly elastic springs. In 1867, winker assumed that the vertical displacement of a point on the elastic foundation is proportional to the pressure outside the loaded regions, winker model is a vital tool used for the analysis of most structures in contact with the soil. But, it does not give a realistic representation of the practical soil accurately, since displacement discontinuity does not occur between the loaded and unloaded regions of the foundation [2]. Winker foundation deficiencies was overcome by connecting them to other elements such as flexural elements (beams in one dimensions, plates in 2-D); shear layers; deformed layers and pretensioned membranes [3]. Finalenko (1940), restored continuity between the individual spring elements in the Winkler model by connecting them to a thin elastic membrane under a constant tension, T. Pasternak (1954), connected the individual spring elements in the winker model to shear layers which deforms in transverse shear only. Hetenyi (1950), incorporated an elastic beam or plate which undergoes flexural deformation, D. long-chyuan etal, [3] developed an edge function approach using furrier series on boundary value problem on polygonal domains. They solved the governing differential equation for a polygonal plate with a convex domain and obtained a levy type solution for each edge which serves as fundamental functions. Mama et al, [3], solved the governing differential equation of a rectangular Kirchhoff plate on Winkler foundation using the finite furrier series transform method and obtained solutions for deflection for the case of point load applied at any point (x, y) on the plate surface, and for the uniformly distributed load applied over the entire plate domain. They compared their results with that of Navier series solutions that yielded exact results and their results were identical. Ozgan and Daloglu,[3], used a computer program based on finite element method to analyze thin and thick plates on elastic foundations. They considered a four-noded plate bending quadrilateral (PBQ4) and an eight-noded quadrilateral (PBQ8) based on Mindlin plate theory for the analyses of the plate on Winkler foundation. Mishra and Chakrabarti,[3], worked on shear and attachment effects on the behaviour of rectangular plates resting on tensionless elastic foundation using finite element techniques. In their analyses, a nine-noded Mindlin element was used to account for transverse shear effects. Ogunjiofor and Nwoji [3] used characteristic orthogonal polynomial to obtain solutions for deflection for the case of uniformly distributed load applied on an all clapped isotropic rectangular plate on elastic foundation. They compared their results with results from Ozan and Dalglu; Mishra & Chakrabarti and they obtained satisfactory results for different values of subgrade reaction K ranging from 0-10 ($0 \leq k \leq 10$).

Hence, this study presents an understandable and easy to use mathematical model for deflection of all clamped rectangular stiff plate on elastic foundation. In this paper, an improved finite difference method (IFDM) was used to solve the differential equation of a rectangular stiff plate on elastic foundation and a mathematical model was used to obtain set of simultaneous linear equations which is transformed in matrix form. The analysis was carried out in mat lab environment using dimension less parameter in both axes for various values of subgrade reaction Ks ranging from 0 to 6 ($0 \leq Ks \leq 6$) .

Of 49 nodes and the deflection at various nodes is W_0 to W_{48}.

The governing differential equation of the plate on elastic foundation is given as;

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{K_s w}{D} = \frac{q}{D} \quad (1)$$

$$D = \frac{E_p}{12}\frac{h^3}{(1-\mu_p)^2} \quad (2)$$

D is the flexural rigidity of the plate, Ep is the young's modules of elasticity of the plate, hp is the plate's thickness, v is the Poisson's ratio of the plate, w is the deflection and q is the uniformly distributed load applied over the entire plate to main. Considering Figure 1 and taking w_0 as the pivotal point, the improved finite difference expressions along the x-axis Taylor's series expansion are. 1.4

$$w_4 = w_0 + hw_0' + \left(\frac{h^2}{2}\right)w_0'' + \left(\frac{h^3}{6}\right)w_0''' + \left(\frac{h^4}{24}\right)w_0^{iv} \quad (3)$$

$$w_1 = w_0 - hw_0' + \left(\frac{h^2}{2}\right)w_0'' - \left(\frac{h^3}{6}\right)w_0''' + \left(\frac{h^4}{24}\right)w_0^{iv} \quad (4)$$

The above equation (3) and (4) are the forward difference and backward difference of w_0 respectively for mesh size $\Delta x = h$.

Taking differences between nodes w_0 and w_5; and nodes w_O and w_2

$$w_5 = w_0 + 2hw_0' + (2h^2)w_0'' + \left(\frac{4h^3}{3}\right)w_0''' + \left(\frac{2h^4}{3}\right)w_0^{iv} \quad (5)$$

$$w_2 = w_0 - 2hw_0' + (2h^2)w_0'' - \left(\frac{4h^3}{3}\right)w_0''' + \left(\frac{2h^4}{3}\right)w_0^{iv} \quad (6)$$

Equations (3), (4), (5) and (6) transforms into:

$$w_0' = \frac{1}{12h}(-8w_1 + 8w_4 + w_2 - w_5) \quad (7)$$

$w_0'' = \frac{1}{12h^2}(-w_2 + 16w_4 + 16w_1 - 30w_0 - w_5)$ (8)

$w_0''' = \frac{1}{24h^3}(29w_1 - 16w_2 + w_3 - 29w_4 + 16w_5 - w_6)$ (9)

$w_0^{iv} = \frac{1}{12h^2}(-w_3 + 18w_2 - 63w_1 + 92w_0 - 63w_4 + 18w_5 - w_6)$
(10)

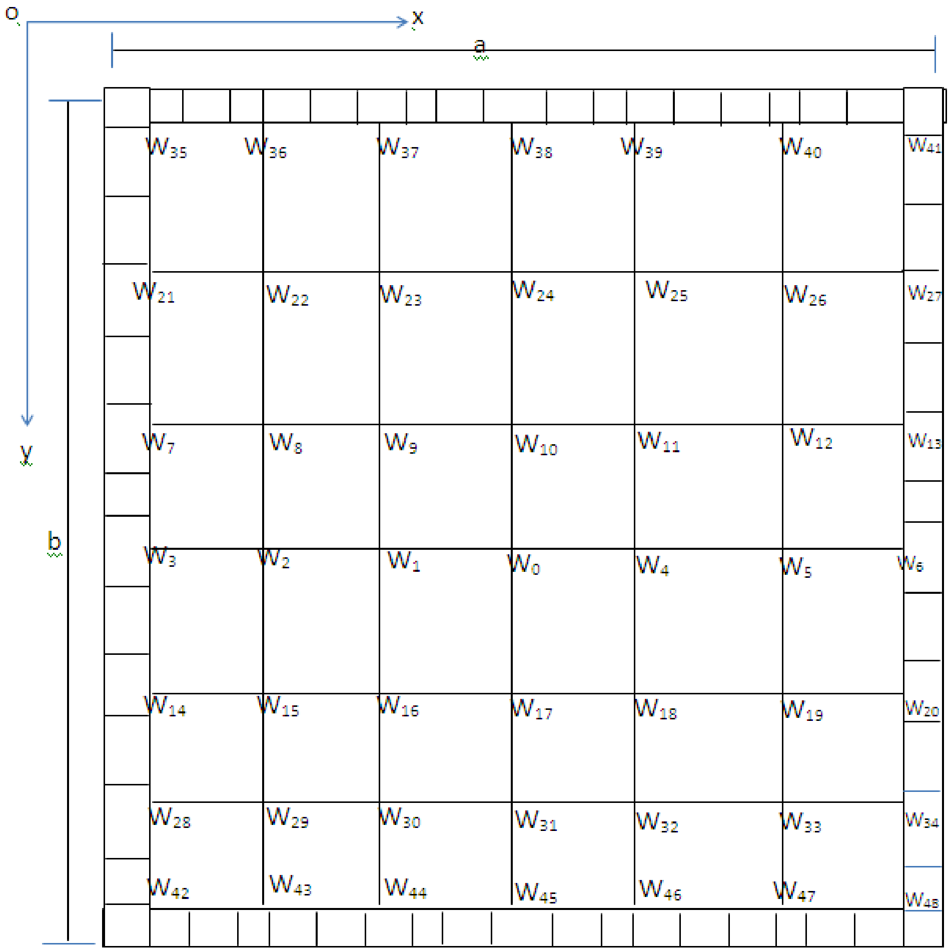

Figure 1: CCCC discretized rectangular stiff plate on Elastic Foundation.

$w_0 = \frac{1}{12h^0}(8w_1 - 2w_2 + 8w_4 - 2w_5)$ (11)

Also, the improved finite difference expressions along the y-axis using Taylor's series expansion are:

$w_{17} = w_0 + hw_0' + \left(\frac{h^2}{2}\right)w_0'' + \left(\frac{h^3}{6}\right)w_0''' + \left(\frac{h^4}{24}\right)w_0^{iv}$ (12)

$w_{10} = w_0 - hw_0' + \left(\frac{h^2}{2}\right)w_0'' - \left(\frac{h^3}{6}\right)w_0''' + \left(\frac{h^4}{24}\right)w_0^{iv}$ (13)

Equations (12) and (13) shows the forward difference and backward difference of w_0 respectively for mesh size $\Delta y = h$

Taking difference between nodes w_0 and w_{31}, and nodes w_0 and w_{24}, the forward difference and backward difference of w_0 are given in Equations (14) and (15) respectively as:

$w_{31} = w_0 + 2hw_0' + (2h^2)w_0'' + \left(\frac{4h^3}{3}\right)w_0''' + \left(\frac{2h^4}{3}\right)w_0^{iv}$ (14)

$w_{24} = w_0 - 2hw_0' + (2h^2)w_0'' - \left(\frac{4h^3}{3}\right)w_0''' + \left(\frac{2h^4}{3}\right)w_0^{iv}$ (15)

Equations (14) and (15) transforms into:

$w_0' = \frac{1}{12h}(-8w_{10} + 8w_{17} + w_{24} - w_{31})$ (16)

$w_0'' = \frac{1}{12h^2}(-w_{24} + 16w_{17} + 16w_{10} - 30w_0 - w_{31})$ (17)

$$w_0''' = \frac{1}{24h^3}\begin{pmatrix} 29w_{10} - 16w_{24} + w_{38} - 29w_{17} \\ +16w_{31} - w_{45} \end{pmatrix} \qquad (18)$$

$$w_0^{iv} = \frac{1}{12h^2}(-w_{38} + 18w_{24} - 63w_{10} + 92w_0 - 63w_{17} + 18w_{31} - w_{45}) \qquad (19)$$

$$w_0 = \frac{1}{12h^0}(8w_{10} - 2w_{24} + 8w_{17} - 2w_{31}) \qquad (20)$$

II. MATHEMATICAL MODEL OF IMPROVED FINITE DIFFERENCE FOR DEFLECTION

The improved finite difference for deflection is obtained by substituting Equations (7) to (11) and Equations (16) to (20) into equation (1) to get Equation (21)

$$\left[-\frac{w_3}{12h^4} + \frac{18w_3}{12h^4} - \frac{63w_1}{12h^4} + \frac{92w_0}{12h^4} - \frac{63w_4}{12h^4} + \frac{18w_5}{12h^4} - \frac{w_6}{12h^4}\right] + \left[\frac{900w_0}{72h^4} - \frac{480w_{10}}{72h^4} - \frac{480w_{17}}{72h^4} + \frac{30w_{24}}{72h^4} + \frac{30w_{31}}{72h^4} - \frac{480w_4}{72h^4} + \right.$$

256w1172h4+ 256w1872h4− 16w2572h4− 16w3272h4− 480w172h4+256w972h4+256w1672h4−16w2372h4−16w3072h4+30w572h4−16w1272h4−16w1972h4+w2672h4+w3372h4+30w272h4−16 w872h4−16w1572h4+w2272h4+w2972h4+−
w4512h4+18w3112h4−63w1712h4+92w012h4−63w1012h4+18w2412h4− w3812h4+KsD
8w112h0− 2w212h0+ 8w412h0− 2w512h0+8w1012h0− 2w2412h0+ 8w1712h0− 2w3112h0=qx,yD
(21)

When considering a rectangular mesh, (Szilard, 2004),

$$\left.\begin{aligned} \Delta x &= \propto(\Delta y) \\ \frac{m}{n} &= \propto\frac{1}{\propto} \end{aligned}\right\} \qquad (22)$$

Where Δx = distance along the x – axis
Δy = distance along the y-axis
M=length per mesh, $\propto$ = width
Per mesh and ∝= aspect ratio
Substituting for Δx=∝ Δy into Equation (21) and simplifying gives:

$$\frac{46w_0D+75\propto^2w_0D+46\propto^4w_0D}{6h^4} - \frac{w_3D}{12h^4} + \frac{18w_2D+5\propto^2w_2D-2K_Sw_2h^4}{12h^4} - \frac{63w_1\mathrm{D}-80\propto^2w_1D+8K_Sw_1h^4}{12h^4} - \frac{63w_4D-80\propto^2w_4D+8K_Sw_4h^4}{12h^4} + \frac{18w_5D+5\propto^2w_5D-2K_Sw_5h^4}{12h^4} - \frac{w_6D}{12h^4} - \frac{80\propto^2w_{10}D-63\propto^4w_{10}D+8K_Sw_{10}h^4}{12h^4} - \frac{80\propto^2w_{17}D-63\propto^4w_{17}D+8K_Sw_{17}h^4}{12h^4} + \frac{5\propto^2w_{24}D+18\propto^4w_{24}D-2K_Sw_{24}h^4}{12h^4} + \frac{5\propto^2w_{31}D+18\propto^4w_{31}D-2K_Sw_{31}h^4}{12h^4} + \frac{32\propto^2w_{16}D}{9h^4} - \frac{2\propto^2w_{23}D}{9h^4} - \frac{2\propto^2w_{30}D}{9h^4} - \frac{2\propto^2w_{12}D}{9h^4} - \frac{2\propto^2w_{19}D}{9h^4} + \frac{\propto w_{26}D}{72h^4} + \frac{\propto^2w_{33}D}{72h^4} - \frac{2\propto^2w_8D}{9h^4} - \frac{2\propto^2w_{15}D}{9h^4} + \frac{\propto^2w_{22}D}{72h^4} + \frac{\propto^2w_{29}D}{72h^4} - \frac{\propto^4w_{45}D}{12h^4} - \frac{\propto^4w_{38}D}{12h^4} = q(x,y) \qquad (23)$$

The mathematical model for deflection developed from Equation (23) is shown in Fig 2

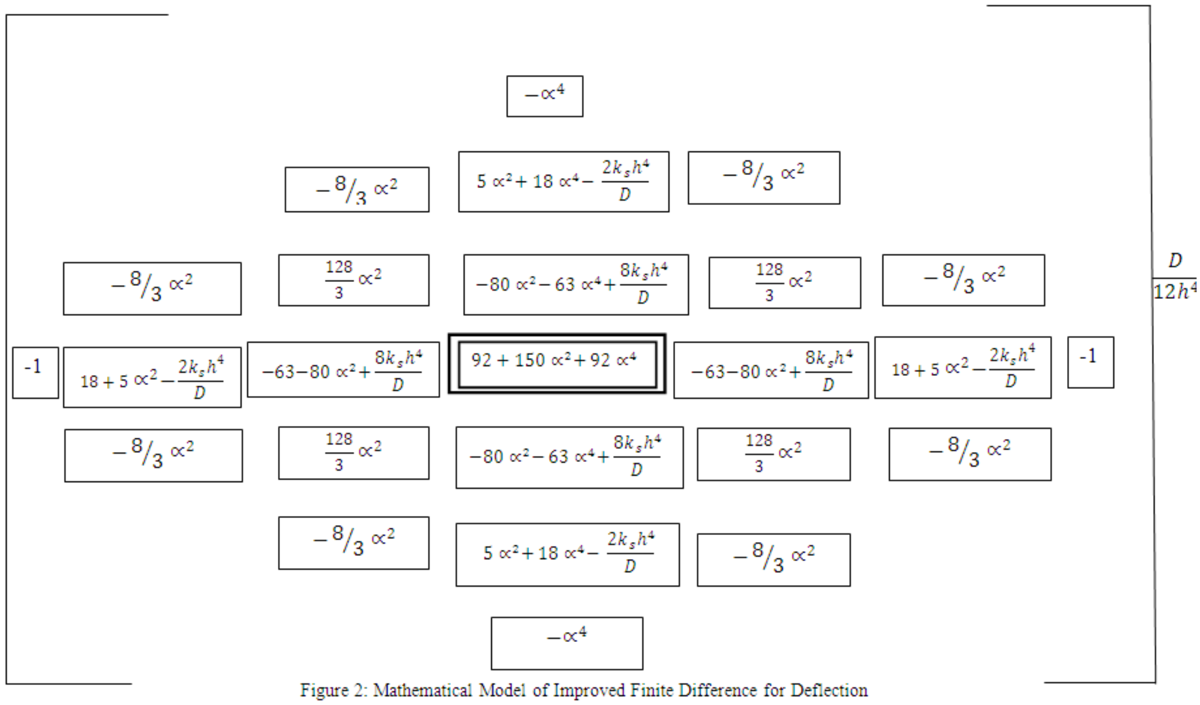

Figure 2: Mathematical Model of Improved Finite Difference for Deflection

2.1 The Improved Boundary Conditions

For a stiff rectangular plate on elastic foundation with all edge clamped, and having an edge length, a, and edge width, b, the prescribed boundary conditions are;

$$\left.\begin{array}{r} (w)_x = 0 \\ \left(\frac{dw}{dx}\right)_{x=0,a=0} = 0 \\ (w)_y = 0 \\ \left(\frac{dw}{dy}\right)_{y=0,b=0} = 0 \end{array}\right\} \qquad (24)$$

2.1.1 Improved boundary conditions along x-axis

$$\left.\begin{array}{l} (w)_x = 0 = \frac{1}{12h^0}\,(8w_1 - 2w_2 + 8w_4 - 2w_5) \\ \left(\frac{dw}{dx}\right)_x = 0 = \frac{1}{12h}\,(w_2 - 8w_1 + 8w_4 - w_5) \end{array}\right\} \qquad (25)$$

Balancing equation (17) gives,

$$\left.\begin{array}{l} (8w_4 - 2w_5) = +(2w_2 - 8w_1) \\ (w_2 - 8w_1) = +(w_5 - 8w_4) \end{array}\right\} \qquad (26)$$

Hence, along X-axis the improve boundary condition can be expressed as:

$$w_{x+1} = +w_{x-1} \qquad (27)$$

2.1.2 Improved boundary conditions along y-axis

The improved boundary conditions along y-axis are

$$\left.\begin{array}{l} (w)_y = 0 = \frac{1}{12h^0}\,(-2w_{24} + 8w_{10} + 8w_{17} - 2w_{31}) \\ \left(\frac{dw}{dy}\right)_y = 0 = \frac{1}{12h}(w_{31} - 8w_{17} + 8w_{10} - w_{24}) \end{array}\right\} (28)$$

Balancing Equation (20) gives

$$\left.\begin{array}{l} (8w_{10} - 2w_{24}) = +(2w_{31} - 8w_{17}) \\ (w_{31} - 8w_{17}) = +(w_{24} - 8w_{10}) \end{array}\right\} \qquad (29)$$

From Equation (21), the improved boundary condition along y-axis can be expressed as:

$$w_{y+1} = w_{y-1} \qquad (24)$$

2.2 Numerical Analysis

An all edge clamped static/elastic, isotropic and homogeneous rectangular stiff plate on elastic foundation as shown in Figure 3, is subjected to a uniformly distributed load, q. Using the mathematical model of the improved finite difference for deflection in Figure 2, the deflection of the plate can be calculated by applying the developed model for deflection on the rectangular stiff plate on elastic foundation with equally spaced 25 interior nodal points. The plate has the following non dimensional parameters.

i. Length of plate = a
ii. Width of plate = b
iii. Thickness of plate= h_p
iv. Modulus of elasticity of plate = E_p = 51.6667 Gpa
v. Poisson's ratio of plate = μ_p = 0.15
vi. Aspect ratio =∝= 1.2
vii. Flexural rigidity of plate =D=0.0257

2.2.1 Description of Problem

Considering the plate in figure 3 with 25 interior nodal points and taking a step size.

$$\left.\begin{aligned} \Delta L = \Delta a = \frac{a}{6} = h \\ \Delta L = \Delta b = \frac{b}{6} = h \end{aligned}\right\} \qquad 23$$

There are total of 25 nodes on the plate. All the nodes along the clamped edges have zero deflection and are marked with zero value. Due to symmetric loading and support, all nodal points with the same deflection components are marked with the same deflection notation. Hence there are 9 unknown deflections.

2.2.2 Application of the Improved Finite Difference Equation at the Nodal Points

Applying the improved finite difference model for deflection at node O; and substituting Ks=0; and the non-dimensional parameters D, h and ∝**,** we have

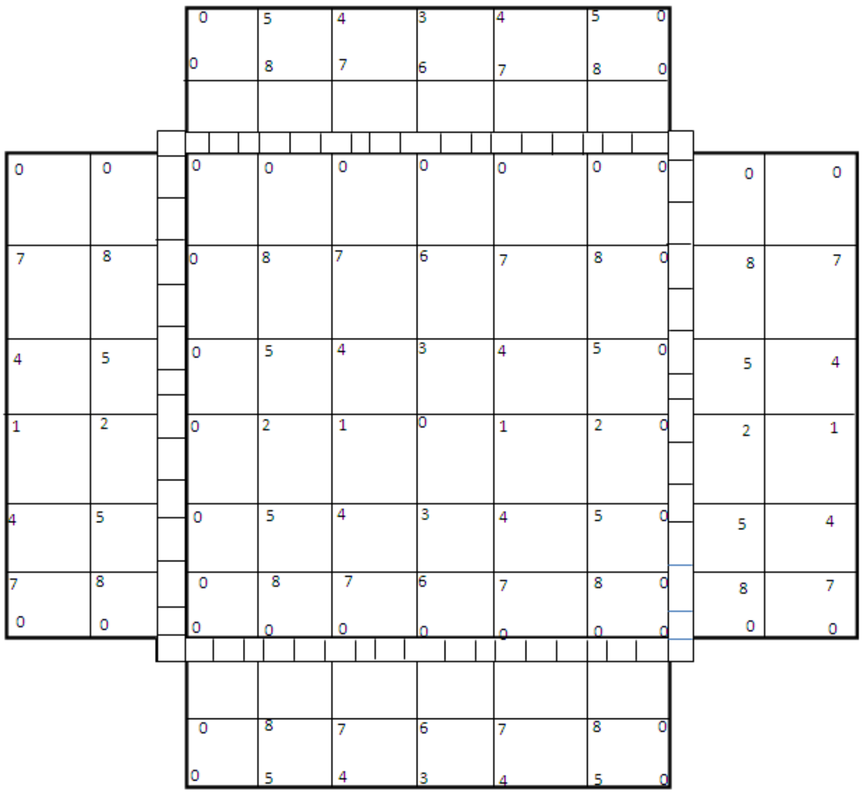

Figure 3: Plate with equally spaced 25 nodal points having all edges clamped

$$(498.7712\,W_0 - 356.4\,W_1 + 50.4 - 491.6736\,W_3 + 245.76\,W_4 - 15.36\,W_5 + 89.0496\,W_6 - 15.36\,W_7 + 0W8 = 12h4\,q\Delta l4D \qquad \text{(A)}$$

Applying the improved finite difference model for deflection at node 1

$(-178.2w_0 + 523.9712\, w_1 - 180.2\, w_2 + 122.88\, w_3 - 499.3536w_4 + 122.88w_5 - 7.68w_6 + 89.0496\, w_7 -$ *7.68w8 =12h4 qΔl4D* (B)

Applying the improved finite difference model for deflection at node 2

$(25.2w_0 - 180.2w_1 + 523.9712w_2 - 7.68w_3 + 122.88w_4 - 499.3536w_5 + 0w_6 - 7.68w_7 + 89.0496w_8) = 12h^4\ \frac{q(\Delta l)^4}{D}$ (C)

Applying the improved finite difference model for deflection at node 3

$(-245.8368\, w_0 + 122.88w_1 - 7.68\, w_2 + 543.296\, w_3 - 364.08w_4 + 50.4\, w_5 - 249.984\, w_6 + 122.88w_7 -$ *7.68w8=12h4 qΔl4D* (D)

Applying the improved finite difference model for deflection at node 4

$(61.44\, w_0 - 249.6768\, w_1 + 61.44\, w_2 - 182.04\, w_3 + 523.9712\, w_4 - 180.2\, w_5 + 61.44\, w_6 - 253.824w_7 -$ *61.44w8=12h4 qΔl4D* (E)

Applying the improved finite difference model for deflection at node 5

$(-3.84\, w_0 + 61.44\, w_1 - 249.67678\, w_2 + 86.64\, w_3 - 183.04\, w_4 + 568.496\, w_5 - 3.84\, w_6 + 61.44\, w_7 -$ *253.824w8=12h4 qΔl4D* (F)

Applying the improved finite difference model for deflection at node 6

$(44.5248w_0 - \ 7.68\, w_1 + 0\, w_2 - 249.984w_3 + 122.88w_4 - 7.68\, w_5 + 543.296w_6 - 364.08w_7 +$ *50.04w8 =12h4 qΔl4D* (G)

Applying the improved finite difference model for deflection at node 7

$(-3.84\, w_0 + 44.5248\, w_1 - 3.84\, w_2 + 61.44w_3 - 253.824\, w_4 + 61.44\, w_5 - 182.04\, w_6 + 568.496w_7 -$ *184.04w8=12h4 qΔl4D* (H)

Applying the improved finite difference model for deflection at node 8

$(0\, w_0 - 3.84w_1 + 44.5248w_2 - 3\,.84w_3 + 61.44\, w_4 - 253.824w_5 + 25.2\, w_6 - 184.04w_7 + 568.496w_8) = 12h^4\ \frac{q(\Delta l)^4}{D}$ (I)

Solving equations (A) to (I) with MAT LAB program, we get

w_0 = 0.1351
w_1 = 0.1073
w_2 = 0.0251
w_3 = 0.1124
w_4 = 0.0906
w_5 = 0.0139
w_6 = 0.0496
w_7 = 0.0398
w_8 = 0.0084

These Mat lab values are then multiplied by $12x(\Delta L)^4$x100 to get the actual deflections at various nodal points as shown below.

w_0 = 0.1251
w_1 = 0.0994
w_2 = 0.0232
w_3 = 0.10407
w_4 = 0.0839
w_5 = 0.0129
w_6 = 0.0459
w_7 = 0.0369
w_8 = 0.0078

The process is repeated for Ks= 1, 2, 3, 4, 5 and 6; and the results are shown in Tables 1 to 10.

Table 1: Numerical Result for Aspect Ratio 1.2 and Ks=0

Nodal point (distance)	Mat lab Value	Multiplied Mat-lab Value by $12(\Delta l)^4$ (deflection)
w_0	0.1351	0.00125
w_1	0.1073	0.00099
w_2	0.0251	0.00023
w_3	0.1124	0.00104
w_4	0.0906	0.00084

w_5	0.0139	0.00013
w_6	0.0496	0.00046
w_7	0.0398	0.00037
w_8	0.0084	0.00008

Table 2: Numerical Result for Aspect Ratio 1.2 and Ks=1

Nodal point (Distance)	Mat lab Value	Multiplied Mat-lab Value by $12(\Delta l)^4$
w_0	0.1312	0.00121
w_1	0.1051	0.00097
w_2	0.0263	0.00024
w_3	0.1092	0.00101
w_4	0.0886	0.00082
w_5	0.0146	0.00014
w_6	0.0482	0.00045
w_7	0.0390	0.00036
w_8	0.0086	0.00008

Table 3: Numerical Result for Aspect Ratio 1.2 and Ks=2

Nodal point (Distance)	Mat lab Value	Multiplied Mat-lab Value by $12(\Delta l)^4$
w_0	0.1215	0.00113
w_1	0.0970	0.00090
w_2	0.0234	0.0002
w_3	0.1013	0.00094
w_4	0.0821	0.00076
w_5	0.0133	0.00012
w_6	0.0450	0.00042
w_7	0.0363	0.00034
w_8	0.081	0.00008

Table 4: Numerical Result for Aspect Ratio 1.2 and Ks=3

Nodal point (Distance)	Mat lab Value	Multiplied Mat-lab Value by $12(\Delta l)^4$
w_0	0.1178	0.00109
w_1	0.0946	0.00088`
w_2	0.0241	0.00022
w_3	0.0982	0.00091
w_4	0.0800	0.00074
w_5	0.0138	0.00013
w_6	0.0436	0.00040
w_7	0.0354	0.00033
w_8	0.0082	0.00008

Table 5: Numerical Result for Aspect Ratio 1.2 and Ks=4

Nodal point (Distance)	Mat lab Value	Multiplied Mat-lab Value by $12(\Delta l)^4$
w_0	0.1087	0.00101
w_1	0.0874	0.00081
w_2	0.0219	0.00020
w_3	0.0906	0.00084
w_4	0.0740	0.00069
w_5	0.0127	0.00012
w_6	0.0399	0.00037
w_7	0.0328	0.00030
w_8	0.0078	0.00007

Table 6: Numerical Result for Aspect Ratio 1.2 and Ks=5

Nodal point (Distance)	Mat lab Value	Multiplied Mat-lab Value by $12(\Delta l)^4$
w_0	0.1053	0.00098
w_1	0.0846	0.00078
w_2	0.0212	0.00020
w_3	0.0880	0.00081
w_4	0.0718	0.00066
w_5	0.0124	0.00011
w_6	0.0394	0.00036
w_7	0.0321	0.00030
w_8	0.0076	0.00007

Table 7: Numerical Result for Aspect Ratio 1.2 and Ks=6

Nodal point (Distance)	Mat lab Value	Multiplied Mat-lab Value by $12(\Delta l)^4$
w_0	0.0833	0.00077
w_1	0.0640	0.00059
w_2	0.0097	0.00009
w_3	0.0698	0.00065
w_4	0.0540	0.00050
w_5	0.0022	0.00002
w_6	0.0320	0.00030
w_7	0.0250	0.00023
w_8	0.0037	0.00003

III. COMPARISON WITH PREVIOUS WORKS

To check the validity of the mathematical model for deflection, the non dimensional central deflection of the improved finite difference model from the present study for various non dimensional subgrade reactions is compared with the non dimensional central deflection from characteristic orthogonal polynomials by Ogunjiofor and Nwoji, (2017); a computer coded program based on finite element method by Ozgan and Daloglu (2007) and finite element techniques by Mishra and Chakrabarti, (1997), (Table 8).

Table 8: non-dimensional central defections for the clamped plate with uniformly distributed load

$100\ \frac{q}{D}$							
K_s	Ozgan and Daloglu (2007)	Mishra and Chakrabarti (1997)	Ogunjiofor and Nwoji (2017)	Present study	% difference with O & D	% difference with M & C	% difference with O & N
0	0.1369	0.1360	0.1327	0.1250	0.000119	0.00011	0.000077
1	0.1367	0.1350	0.1315	0.1250	0.000157	0.00014	0.000105
2	0.1350	0.1340	0.1307	0.1130	0.00022	0.00021	0.0000177
3	0.1277	0.1270	0.1288	0.1090	0.000187	0.00018	0.000198
4	0.1114	0.1110	0.1182	0.1010	0.000104	0.00001	0.000172
5	0.0874	0.0870	0.0873	0.0980	-0.000106	-0.000011	-0.000107
6	0.0622	0.0620	0.0623	0.0770	-0.000148	-0.00015	-0.000147

LEGEND: O & D = Ozgan and Daloglu;
M & C= Mishra and Chakrabarti;
O & N = Ogunjiofor and Nwoji

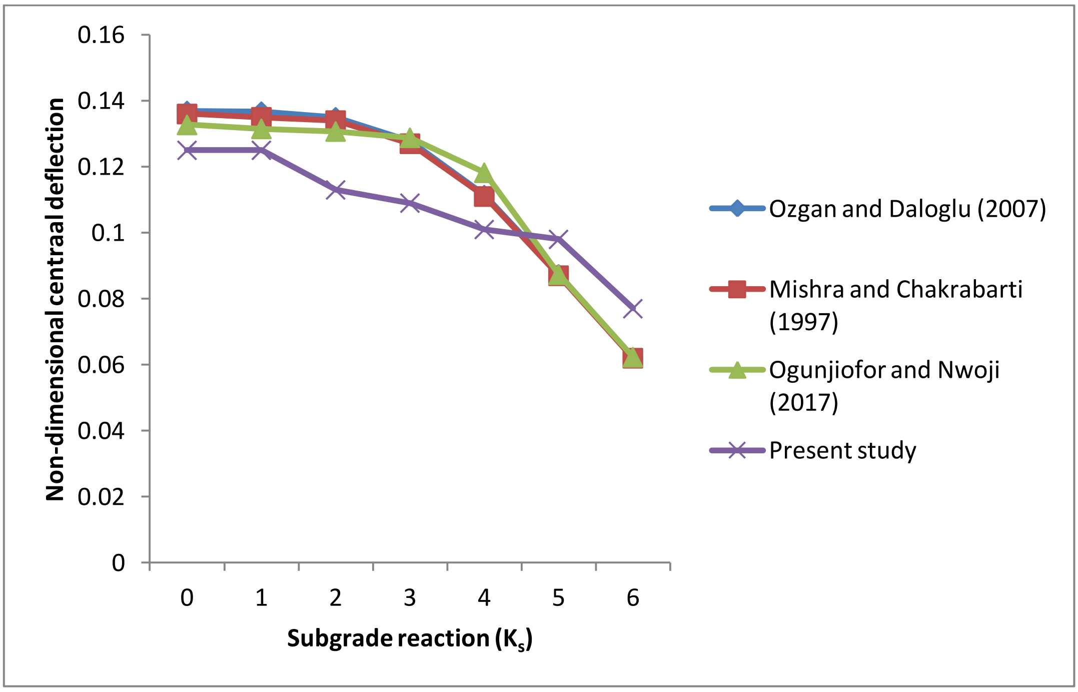

Figure 4: Variation of non-dimensional central deflection of the clamped plate with different K_s values subjected to uniformly distributed load.

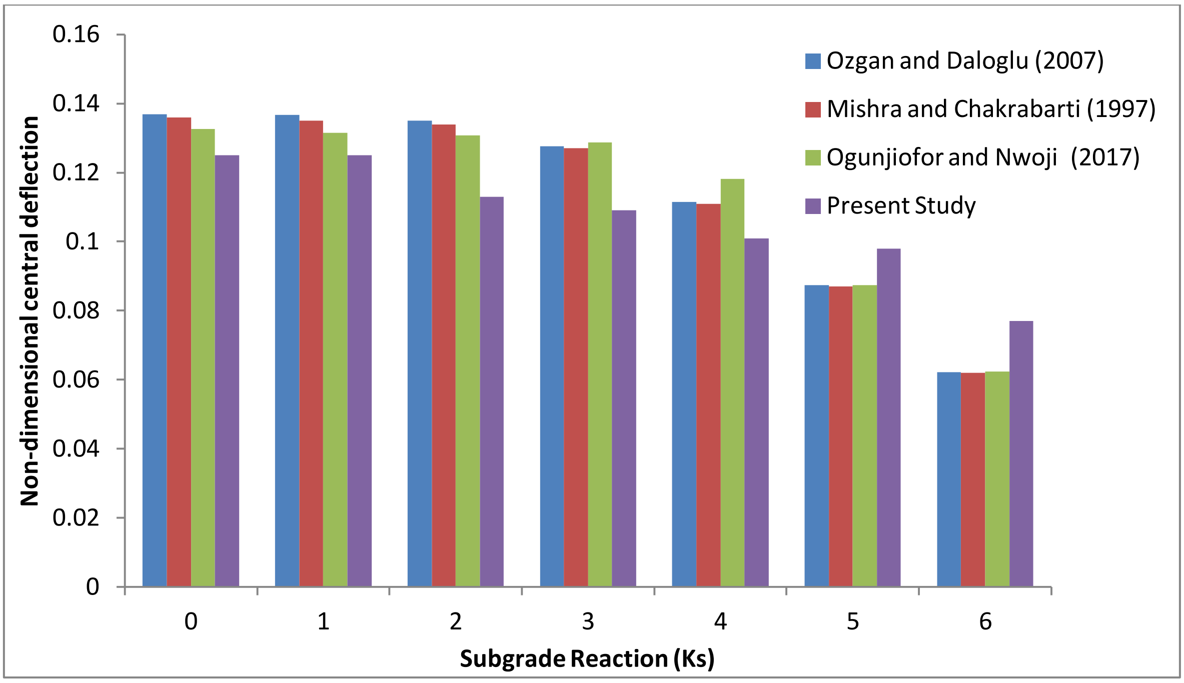

Figure 5: Bar chart showing variation of non-dimensional central deflection of the clamped plate subjected to uniformly distributed load with different K_s values

IV. CONCLUSIONS

In this study, a 49 noded rectangular stiff plate resting on elastic foundation was considered for the deflection analysis of the plate. The improved finite difference expression along the x and y axes was formulated by obtaining the displacement of each node in figure 3.1 using central difference method, and the fourth-order expansion for the deflection was obtained using Taylor series with step size $(\Delta x) = (\Delta y)$=h. Thereafter, the mathematical model of improved finite difference for deflection was developed. The developed model for deflection was used to obtain coefficient matrix of the unknown deflections at nodal points by considering 25 interior nodal points and applying the appropriate boundary conditions of an all edged clamped plate. The coefficient matrix was solved in mat lab environment to obtain the unknown deflections which gives satisfactory results comparing with solutions available from literature for subgrade reactions $0 \leq k_s \leq 6$. Hence, the mathematical model of improved finite difference developed for the deflection analysis of rectangular stiff plate on elastic foundation can be used for the analysis of plates on elastic foundation.

ACKNOLEDGEMENT

The authors acknowledge that there were no financial supports from any organization.

Conflict of Interest

“No competing interest are at stake and there is no conflict of interest’’ with other people or organization that would influence the content of the paper.

REFERRENCES

[1]. Mishra, R. C and Chakrabarti, S. K. Shear and Attachment effects on the Behaviour of Rectangular Plates Resting on Tensionless Elastic foundation, Eng. Struct, 19(7), 551-567. 1997.

[2]. Ogunjiofor E. I. and Nwoji C. U. (July, 2017). Analytical Bending Solution of all Clamped Isotropic Rectangular Plate on Winkler’s Foundation Using Characteristic Orthogonal Polynomial. Nigerian Journal of Technology. Vol. 36, pp.724-728.

[3]. Okafor F.O and Udeh O.T. (2015). Direct method of analysis of an isotropic rectangular plate using characteristic orthogonal polynomials. Nigerian journal of technology. Vol 34. No 2. Pp 232-239.

[4]. Ozgan , K. O. and Daloglu, A. T., Alternative Plate Finite Elements for the Analysis of Thick Plates on Elastic Foundations, Structural Engineering and Mechanics Journal , 26(1). 69-86. 2007.

[5]. Ozgan, K., (2007) Finite Element Analysis of Thick Plates on Elastic Foundation Using Modified Vlasov Model and Determination of The Effective Depth of Soil Stratum, Ph. D. Thesis, Karadeniz Technical University. Trabzon, Turkey (in Turkish),

[6]. Timoshenko S and Woinowsky-Krieger S . (1987). Theory of Plates and Shells. 2nd Edition, McGraw-Hill Book.

[7]. Umeonyiagu, I. E & Onyike, I. P (April- September, 2018) Bending Moment Analysis of Rectangular Thin Plate Using Improved Finite Difference Method, International Journal of Mechanical and Industrial Technology, Vol 6., Issue 1, pp. 1-7.

[8]. Umeonyiagu, I. E & Onyike, I. P (April- September, 2019) Deflection of Thin Rectangular Plate Using Improved Finite Difference, Lap Lambert Academic Publishing, 2019.