

Evaluation of Four SWCCs Models' Flexibility for Selected Reconstituted Tropical Red Earth Soils

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ABSTRACT

Soil water characteristic curve (SWCC) is a significant property of unsaturated soil which includes tropical red earth soil. This is due to the fact that other soils' properties which include shear strength, permeability function and compressibility can be related to it. Selected reconstituted tropical red soil samples were prepared to simulate tropical soils characteristics with plasticity properties ranging from low to high configuration. The soil's matric potential was obtained from the filter paper technique. Gravimetric water content was utilized in the computation of the SWCC. Four models were used to estimate the SWCCs of the soils investigated and MSSR, ARE, and R² values were utilized in the determination of the order of suitability of the models' capability to predicting the SWCCs. The models utilized were: Fredlund and Xing (1994); Van Genuchten (1980), Brooks and Corey (1964) and Kosugi (1996). It was observed that some models perform better for certain class of tropical red earth and the implication is that by simply identifying the fines content of a tropical soil, the class of the soil can be inferred and ultimately the SWCC can be predicted using the most suitable model for the particular class of tropical red earth.

Keywords: Tropical soil, SWCC, matric suction, filter paper

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I. INTRODUCTION

Soil-Water Characteristic Curve (SWCC) is a curve that shows the relationship between the water content in a soil and its suction. It also can be seen as a continuous sigmoid function which describes the water storage capacity of a soil as it is subjected to various soil suctions (Matlan, Mukhlisin, & Taha, 2014). SWCC is a very significant property of unsaturated soil because other soil properties such as shear strength, permeability function, compressibility, moduli and fluid flow can be related to it (Zhai & Rahardjo, 2013); (Houston, Dye, Zapata, Perera, & Harraz, 2006); (Choudhury & Bharat, 2014).

Different models have been developed in the field SWCC over the last five decades and some of the most commonly used ones include Brooks and Corey (1964), van Genuchten (1980), Fredlund and Xing (1994), Kosugi (1999), Omuto (2009), Krishnapillai and Ravichandran (2012). (Taban, Sadeghi, & Rowshanzamir, 2018).

Fredlund and Xing (1994)'s equation

Fredlund and Xing (1994)'s equation with the correction factor $C(\Psi)$ can be expressed as shown in equation 1.

$$\theta = C(\Psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} = \left[1 - \frac{\ln \left(1 + \frac{\psi}{c_r} \right)}{\ln \left(1 + \frac{10^6}{c_r} \right)} \right] \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} \quad (1)$$

Where

θ = Volumetric water content; θ_s = saturated volumetric water content

a, n, m: are fitting parameters

c_r = Parameter related to residual suction, often assigned a value of 1500

Van Genuchten (1980) model

Van Genuchten (1980) SWCC model is widely used for the description of the SWCC of various soils. It's among the most widely used model observed to be suitable for use for a wide range of both disturbed and undisturbed soils ranging from fine grained to coarse grained soils (Taban, Sadeghi, & Rowshanzamir, 2018).

The van Genuchten model presents the relationship between the normalized water content and suction as stated in equation 2 below:

$$S = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)} = \frac{1}{[1 + (a\psi)^n]^{1/m}}, \quad (2)$$

Where $m = 1 - \frac{1}{n}$ (2b)

Where S is the normalized water content (a dimensionless parameter)

Θ is the volumetric water content, the indices r and s in θ_r and θ_s symbolize the residual and the saturated volumetric water contents respectively, ψ is the suction (unit is kPa), while “a” and “n” are the model fitting parameters. Parameters “a” is related to the air entry value while “n” is the value related to the pores size distribution parameters m is the value related to the asymmetry of the model (Taban, Sadeghi, & Rowshanzamir, 2018).

Brooks and Corey (1964) model

Brooks and Corey (1964) equation is the form of power law relationship and it's expressed as:

$$\Theta = \left(\frac{\psi_b}{\psi}\right)^\lambda \tag{3}$$

Where :

Θ is the normalized water content which is expressed as:

$$\Theta = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)} \tag{4}$$

Where θ is the volumetric water content, also the indices r and s in θ_r and θ_s symbolize the residual and the saturated volumetric water contents respectively, ψ is the suction and ψ_b is the air entry value while λ is the poresize distribution index. This model is relatively simple and thus widely used although the model doesn't provide a continuous mathematical function for the entire SWCC (Matlan, Mukhlisin, & Taha, 2014)

Kosugi(1996)'s model

Kosugi(1996)'s models, the most recent model of the four models evaluated in this study is reported to be among the three commonly used models, the other two are the Brooks and Corey(1964) and van Genuchten(1980). (Mavimbela & Rensburg, 2012). The model was developed by applying a lognormal distribution law and its parameters are directly related to the soil pores radius distribution (Matlan, Mukhlisin, & Taha, 2014). Kosugi's model is expressed as stated below:

$$\Theta = Q \left[\frac{\ln \psi / h_m}{\sigma} \right] \tag{5}$$

Where Q is related to the complementary error function (erfc), and it's defined as

$$Q(x) = \text{erfc} \left(\frac{x}{\sqrt{2}} \right) \tag{6}$$

Where h_m and σ are the fitting parameters. h_m is a capillary pressure head and is related to the median pore radius while σ is a dimensionless parameter related to the width of the pores radius distribution.

While commenting on the theoretical basis of the shape of the soil-water characteristic curve, (Fredlund & Xing, 1994), stated that the equations proposed in the research literature are empirical in nature. He further opined that each equation appears to apply for a particular group of soils. It is therefore reasonable to evaluate any given soil against some selected popular SWCC models to determine which most closely describes its WCC. It is for this reason that the four models were chosen for the evaluation of selected tropical residual reearth SWCC. The existence of a suitable template for easy determination of the SWCC from basic index tests would spur researchers and practitioners to harness the numerous benefits of utilizing SWCCs in their routine geotechnical assessments of the soil.

II. MATERIALS AND METHODS

The filter paper method of determination of SWCC in the laboratory was adopted in this study owing to its simplicity and adaptability to a wide range of suction values. Whatman No. 42 filter paper was utilized in the study.

Sample Preparation

Red earth residual soils were obtained and separated into coarse fraction and fines (silt/clay fractions) using 75µm sieve aperture and they were marked as A and B subsamples respectively. The subsamples were reconstituted back into one soil specimen by partial blending of coarse (sand) sub samples in 10 percent increments from 10% (90A+10B) fines to 100% fines (0A+100B). Each fraction was reconstituted at roughly the OMC and the reconstituted specimen subjected to soil water characteristics curve evaluation using filter paper method. Other samples were prepared at water content less than optimum and also at water content greater than optimum in order to capture a broad range of the matrix suction characteristics of the soil. All reconstituted soil samples were allowed to cure for a minimum of 20 hours. The cured samples were thereafter extruded and prepared for matrix suction test .

Matric Suction Sample Preparation

The matric suction test was executed using the filter paper technique. The procedures include placing a portion of the filter paper in-between two protective filter papers which are in contact with the soil sample in such a way that suction equilibrium is established between them. Although the filter paper and the soil have different water content, they're subjected to the same matric potential (Lucas de Almeida, Teixeira, Filho, Raimundo, & Raimundo, 2015). In this work Whatman No.42 filter paper was used.

Each matric suction test was performed on a sample of 42 mm diameter and 30 mm height. The samples were carefully extruded and cut to ensure that the surface is planar and smooth to enable good contact surface between the filter papers and the soil. Each set of three filter papers (2 No. protective filter papers with diameter 42 mm and 1 No. 38 mm diameter Whatman No.42 filter paper) were placed between two soil samples. The choice of the central filter paper being made smaller in diameter is to prevent soil samples from polluting the central filter paper. The joint was thereafter sealed with an electrical tape and the sealed sample placed in an airtight plastic container with the cover again sealed with the electrical tape to prevent moisture loss from the soil. The whole assembly was kept in a well-insulated container for suction equilibrium. After a minimum of three days of suction equilibration, the assembly was opened and both the soil samples and the central filter paper were weighed with a 0.0001 g precision balance. Weighing was carried out within 20 seconds to avoid possible evaporation. Finally suction was calculated from the computed filter paper water content using the appropriate calibration curve depending on the initial state of the filter paper, whether wet or dry.

Matric suction values were computed using calibration equations provided by (Kim, Prezzi, & Salgado, 2017). The equations adopted in this study are stated below:

$$\log_{10} S = 5.327 - 0.0779(w_{c_{fp}}) \quad (7)$$

For $w_{c_{fp}} < 45.26\%$

And

$$\log_{10} S = 2.412 - 0.0135(w_{c_{fp}}) \quad (8)$$

For $w_{c_{fp}} > 45.26\%$

Evaluation Criteria

In the evaluation, three criteria were used for comparison and understanding the descriptive and predictive capabilities of the four models. The first criterion is the degree of curve match. The closer the difference between the predicted curve and the measured data the better the descriptive capability of the model. (Guan, Rahardjo, & Choon, 2010). These can be evaluated using Average Relative Error (ARE) computed from equation (9) shown below:

$$ARE = \frac{1}{N} \sum_{i=1}^N \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100 \quad (9)$$

Where y_i is the actual value of i th data; \hat{y}_i is the predicted value of the i th data; and N is the total number of data available.

In this report agreement of predicted with actual will be fashioned along the same line as those of (Guan, Rahardjo, & Choon, 2010) where "agreement" was defined as ARE being smaller than or equal to 20%, and "discrepancy" was defined as ARE being larger than 20%.

The second criterion used in this study is the normalized sum of square error (SSE_{norm}). In this evaluation, the smaller the value of SSE_{norm} the better the predictive capability of the model. The SSE_{norm} is defined as:

$$SSE = \sum_{i=1}^N \left(\frac{y_i - \hat{y}_i}{y_i} \right)^2 \quad (10)$$

The parameters needed were obtained using minimization algorithm for SSE_{norm} , which implies least number of parameters in the equations being able to provide the minimum SSE_{norm} (MSSE) for all selected data sets also known as the residual error.

The third criterion is the coefficient of determination, r^2 . This is the percentage of variance in one variable that is accounted for by the variance in the other variable. It's the square of the correlation coefficient. The sum of squares of the deviation from the mean, \bar{y} in the y direction is given as $\sum_{i=1}^N \left(\frac{y_i - \bar{y}}{1} \right)^2$ the coefficient of determination is therefore the fraction of this sum of squares which is explained by the linear relation between \hat{y} and x given by the regression of y on x . thus the coefficient is given by the ratio of $\sum_{i=1}^N \left(\frac{y_i - \hat{y}_i}{1} \right)^2$ to $\sum_{i=1}^N \left(\frac{y_i - \bar{y}}{1} \right)^2$. i.e.

$$r^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \quad (11)$$

If the coefficient of determination becomes larger for the same algebraic forms, it indicates the relationship between the variables has become stronger.

III. RESULTS

Soil-water characteristics curve of a reconstituted A-3 tropical red earth

Reconstituted A-3 tropical earth was investigated to determine the matric suction at different gravimetric water content. The results gotten from the SWCC using the four models of FX, VG, BC and K are presented in Figures 1 to 4. The performance of the models is shown in Table 1.

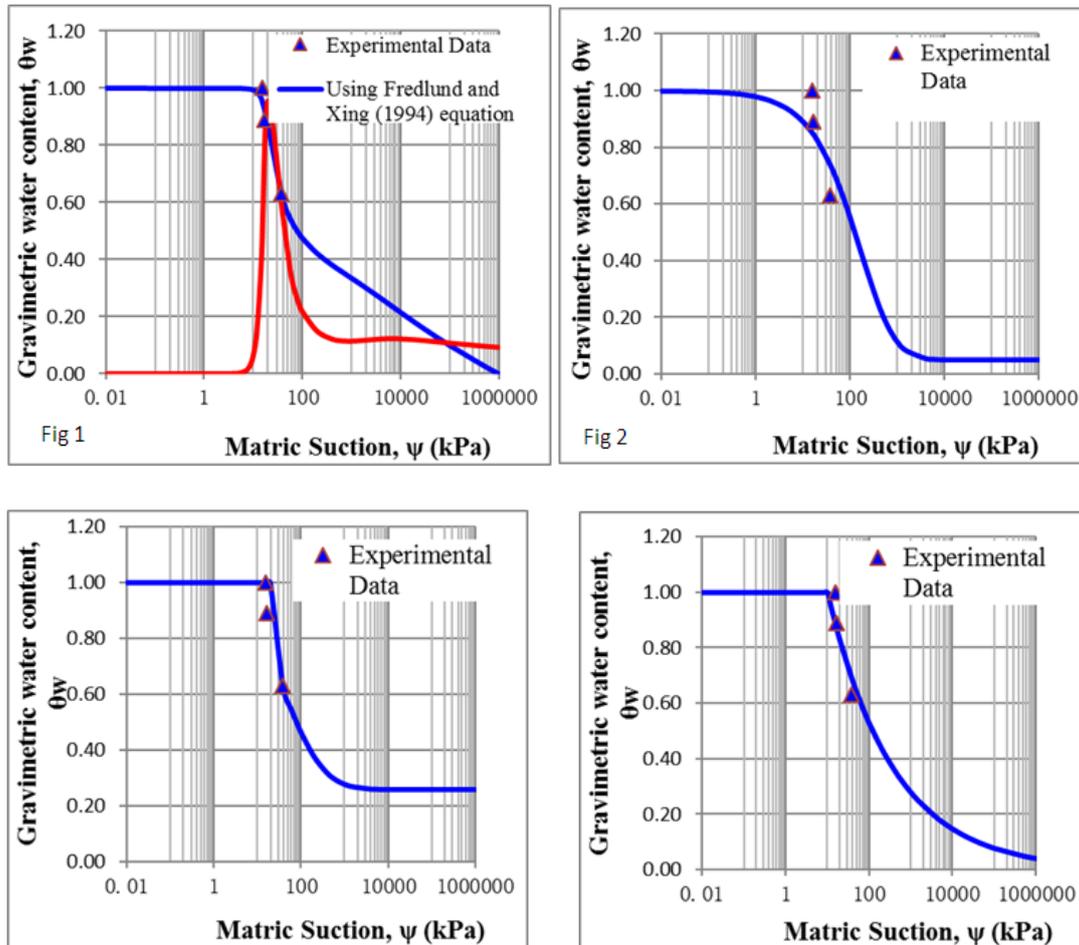
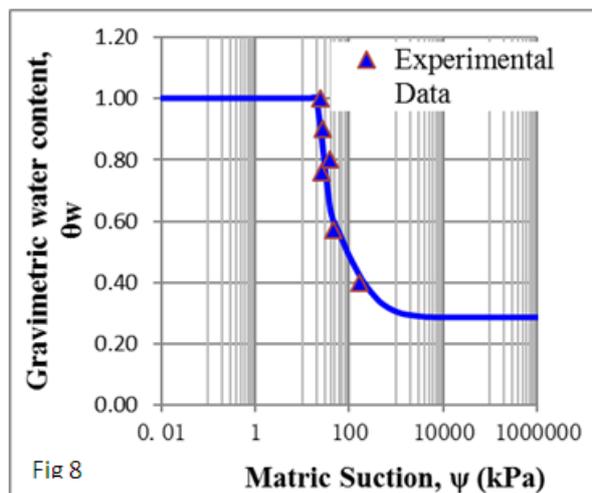
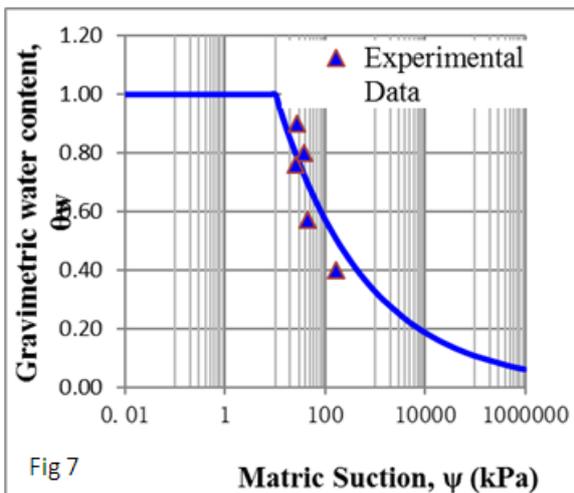
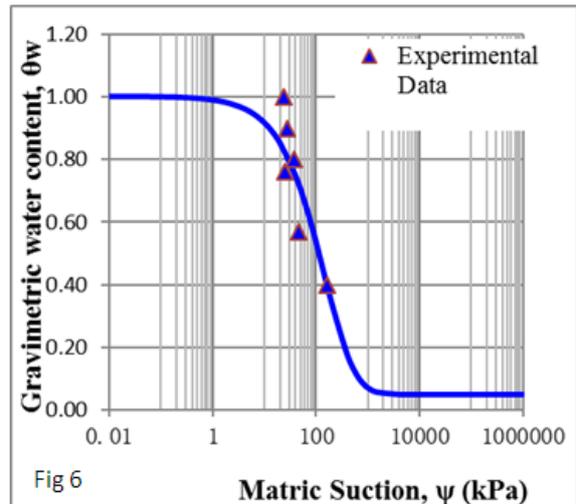
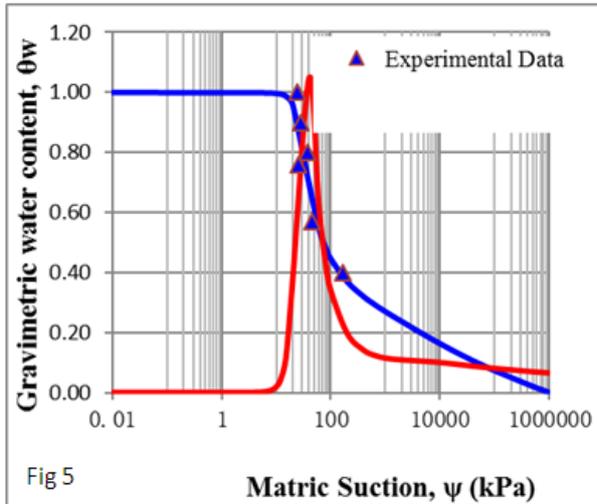


Figure 1-4: .SWCC of anA-3 tropical red earth soil using FX, VG, BC and K model respectively

Soil water characteristics curve of an A-7-6 reconstituted tropicalred earth

A-7-6 reconstituted tropical red earth, also classified as SC using USCS classification was evaluated for matric suction using gravimetric water content. The results gotten from the SWCC using the four models of FX, VG, BC and K are presented in Figures 5 to 8 while the summary of the data are presented in Tables 1.

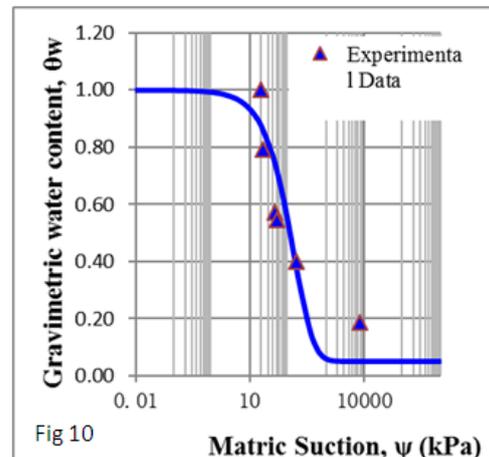
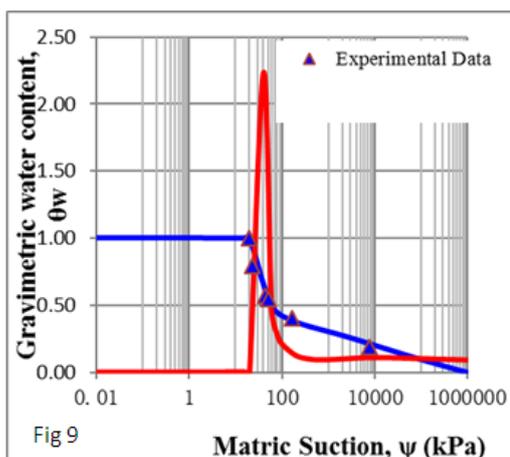


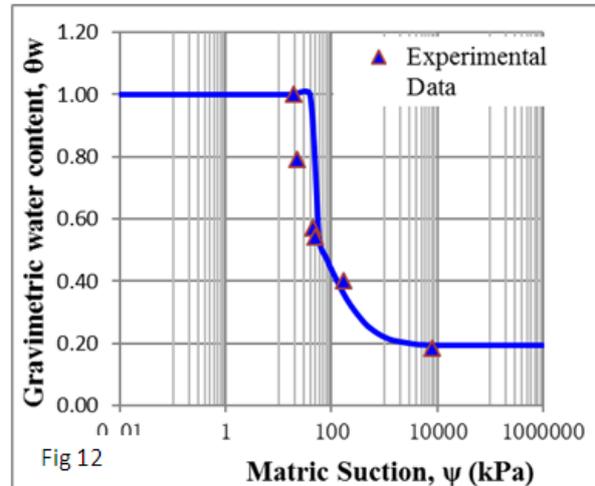
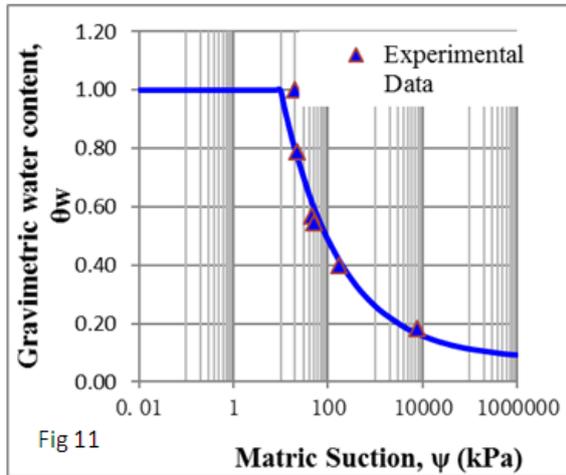
Figures 5-8. SWCC of an A-7-6 tropical red earth using FX, VG, BC and K models respectively

Soil water characteristics curve of an A-7-5(5) tropical red earth soil

The results gotten for samples of the A-7-5(5) tropical red earth soil using the chosen four equations proposed for determining SWCC of various soils are presented in Figures 9 to 12 while the summary of the results are shown in Tables 1-4.

Figure 9. SWCC of an the A-7-5(5) tropical red earth soil using FX-model

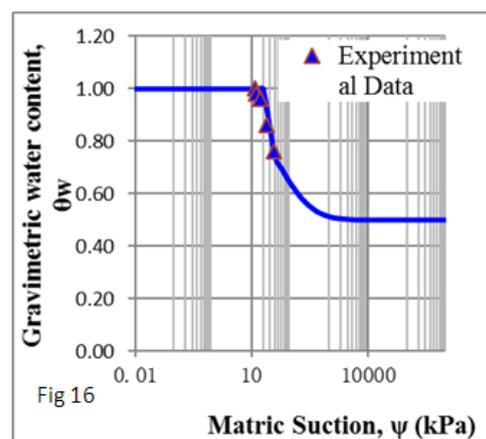
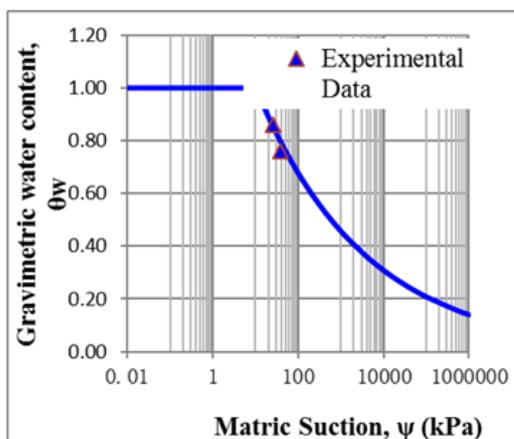
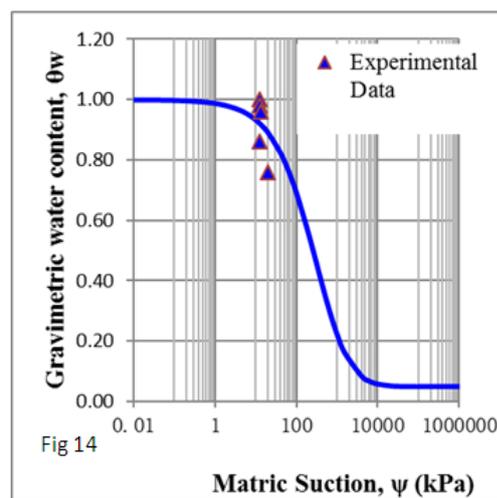
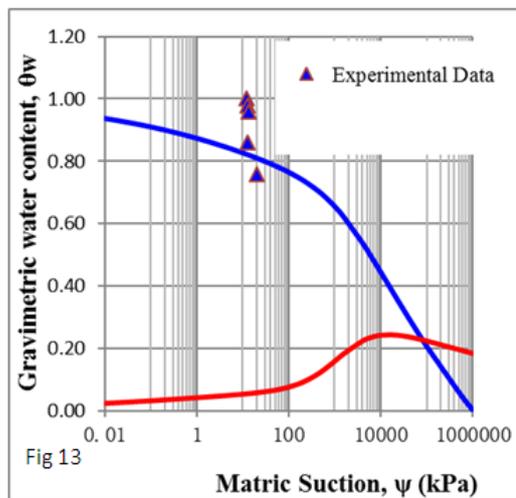




Figures 9-12.SWCC of an the A-7-5(5) tropical red earth soil using FX, VG, BC and K models respectively

Soil water characteristic curve of reconstituted A-7-5(20) tropical red earth soil

Reconstituted A-7-5(20) tropical soil is examined to determine its water characteristic curve. The results gotten from the SWCC using the four models of FX, VG, BC and K are presented in Figures 13 to 16 while the summary of the data are presented in Table 1.



Figures 13-16.SWCC of A-7-5(20) tropical red earth soil using FX, VG, BC and K models respectively

Determination of Models' Suitability for Selected Tropical Red Earth Soils

In order to determine the flexibility of the four models evaluated in this study to adequately predict the SWCC of selected classes of tropical soils cutting across low to high plasticity properties of the soils, Table 1 were prepared. The ranking on each table section provides a quick guide to the type of soil most suitable for evaluation using the given model.

Table 1. Summary of models' fitting parameters for selected tropical red earth soils

Soil Class	a	n	m	R ² (%)	MSSE (%)	ARE (%)
Fredlund and Xing						
A-3	18.012	5.497	0.326	94.21 (1)	0.16	3.33[A]
A-7-5(5)	34.459	44.658	0.222	89.21(2)	1.57	8.26[A]
A-7-6(15)	26.262	4.761	0.430	83.42 (3)	1.18	9.49[A]
A-7-5(20)	33.548	50	0.206	-91.96 (4)	2.52	10.70[A]
Van Genuchten						
A-7-5(5)	0.001	0.981	6.58	93.84(4)	12.82	5.64[A]
A-3	0.001	0.881	5.48	97.04 (3)	2.04	2.43 [A]
A-7-6(15)	0.001	0.760	2.48	98.88(1)	0.91	1.52[A]
A-7-5(20)	0.001	0.760	1.380	98.88(2)	0.91	1.52[A]
Brooks and Corey						
A-7-5(5)	10.000	0.349	0	97.37(3)	1.18	1.98[A]
A-3	10.000	0.277	0	98.96(4)	0.80	0.93[A]
A-7-6(15)	10.000	0.243	0	96.30(4)	3.03	3.27[A]
A-7-5(20)	10.000	0.170	0	99.81(1)	0.13	0.59[A]
Kosugi						
A-7-5(5)	1.049	2.868	1.761	97.28(3)	1.60	1.42[A]
A-3	1.000	2.967	1.711	99.30(1)	0.51%	0.48[A]
A-7-6(15)	1.000	2.959	1.711	95.89(4)	2.65	2.58[A]
A-7-5(20)	1.000	2.966	1.711	99.24(2)	0.58	0.85[A]

Note: MSSE value is between zero infinity, the smaller the value, the better the model's predicting capability. The value in parentheses at every row is the best-prediction ranking for models evaluated. Value of 1 indicates the best prediction for the data using the model. The letters [A] and [D] signifies "agreement" and "discrepancy", respectively.

As shown in the first section of Table 1, FX model performs best with predicting the SWCC of an A-3 tropical red earth soil with a coefficient of determination of about 94%. A-7-5(5) tropical red earth soil came second in the ranking with a coefficient of determination of slightly under 90%. A-7-6(15) reconstituted tropical soil came third in the ranking of soils suitable for SWCC determination using FX equation. A-7-5(20) reconstituted tropical soil however gave a negative coefficient of determination apparently indicating the data are not explained by the FX model indicating none suitability of FX model for predicting the SWCC of the soil.

Table 1 also shows the summary of Van Genuchten model fitting parameters for selected tropical red earth soils. The suitability of VG model for predicting the soil water characteristic curve of selected tropical soils with varying plasticity characteristic is shown in the table. VG model performs best with predicting the SWCC of A-7-6(15) and A-7-5(20) tropical red earth soils with R² value of almost 99%. A-3 tropical red earth soil came third in the ranking with a coefficient of determination of about 97%. A-7-5(5) came last in the ranking of the soils with a coefficient of determination of slightly above 94%. It is noteworthy however that across all the soils' variations the coefficient of determination is over 94%, an indication of the relative strong correlation between the experimental data and the VG model. It can therefore be concluded that the VG model can be used for predicting the SWCC of virtually all classes of tropical soils with a relatively strong conviction in the results to be obtained using the model. The significance of the relationship is further confirmed by the ARE value, all of which are under 6% much less than the 20% criterion of significance of any correlation.

Also found in Table 1 is the summary of Brooks and Corey model fitting parameters for selected tropical red earth soil the BC model and its suitability for predicting the SWCC across a broad range of plasticity characteristics of tropical red earth soil ranging from low to high plasticity. With the fitting parameters stated in Table 1, the model performs wonderfully well in predicting the SWCC of virtually all the various classes of tropical soils investigated. A-7-5(20) and A-3-soils have high coefficient of determination of over 98%, thus rank top in the list. A-7-5(5) tropical red earth with coefficient of determination of approximately 97% came third on the ranking. A-7-6(15) reconstituted tropical soil came last in the ranking of the soils although its coefficient of determination is also high with a value of approximately 96% indicating the reliability of BC model to predicting the SWCC of tropical soils with different values of fines content reflecting in different plasticity behaviour.

Kosugi's model and its predicting capability for various classes of tropical soil according to their plasticity are further presented in Table 1. At a glance through the coefficient of determination column it can be seen that the model is suitable for predicting the SWCC of virtually all the groups of the tropical soils irrespective of the fines content or plasticity characteristics. This conclusion was reached as the coefficient of determination range from roughly 96 to 99%. A-3 tropical red earth soil ranked first in the list implying it's the most suitable soil for the model's predicting capability, for the coefficient of determination obtained for it was over 99%. A-7-5(20) tropical red earth soil came second also with coefficient of determination of over 99%. A-7-5(5) tropical soil came third with over 97% coefficient of determination while A-7-6(15) ranked last in the list although its coefficient of determination is also high with a value of approximately 96%, thus indicating the capability of the K model to also predict the SWCC of tropical soils irrespective of its plasticity properties or fines content.

Proposed Easy Template for Utilising SWCC In Routine Engineering Procedures

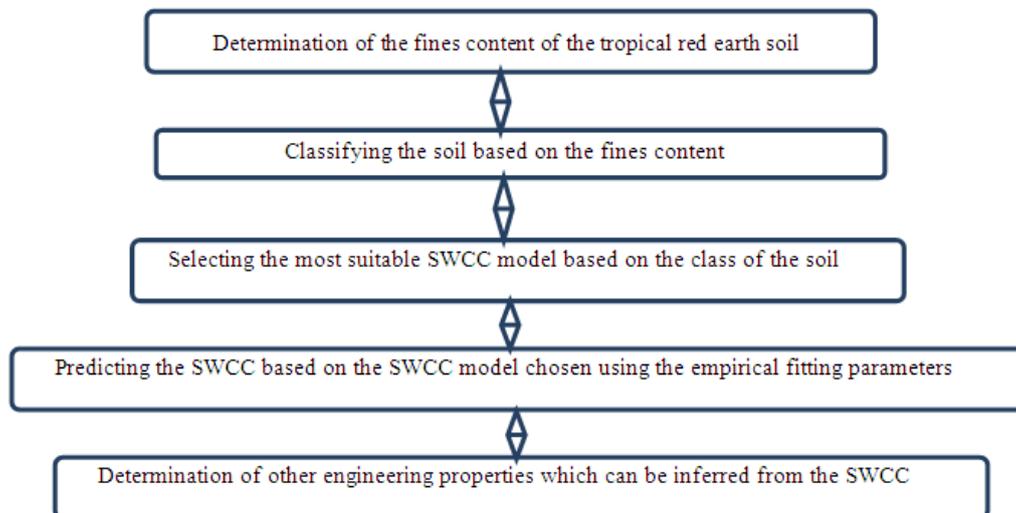


Figure 17 Flow chart for utilising SWCC in tropical red soils in routine engineering procedures

Armed with the information in Table 1, it becomes rational to propose an attractive yet easy template for incorporating SWCC determination into routine geotechnical engineering laboratory procedure. Beginning with the identification of the fines content of the tropical red earth soil other index properties of the soil can be predicted and the soil classified. Using the information on Table 1, the appropriate SWCC parameters can be selected and the SWCC determined. Following the choice of the SWCC, other engineering properties of the soil can thereafter be determined as illustrated in the flow chart shown in Figure 17.

IV. CONCLUSION

In conclusion it can be said that of the four models evaluated in this study viz: Fredlund and Xing (1994); Van Genuchten (1980), Brooks and Corey (1964) and Kosugi (1996), the BC model performed best with the estimation of the SWCC of tropical red earth soils irrespective of the fines content or plasticity characteristics as the least coefficient of determination recorded was over 96%. Closely following the BC model was the K-model which also had significant high values of r^2 across all classes of tropical soils investigated. The VG model came third on the list of general performance while FX model came last on the list. Generally, all models had ARE values significantly less than 20%, indicating agreement exist between the models and the experimental data. FX model performed fairly with various classes of tropical soils except with that of high

plasticity characteristics(A-7-5(20)) where the results indicated no correlation between experimental data and the model. It should however be noted that a sizeable database of soil water characteristic curve data is required in order for ideal correlations to be drawn between the fitted model parameters and the soil properties under consideration (Sillers & Fredlund, 2001). This implies that the results presented and evaluated are open for further refinement should a more rigorous testing scheme highlight the need for such. Recent researches also highlight the need for interpreting gravimetric soil water characteristic parameters in conjunction with shrinkage properties of the soil (Zhai, Rahardjo, Satyanaga, Dai, & Zhuang, 2020)

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