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Numerical Assessment of Plotting Position Formulae on Generalized Extreme Value(GEV), Pearson Type(P3), and Log – Pearson Type(LP3) distributions.

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ABSTRACT

The probabilistic modelling of extreme flood data is crucial to decision-making process in hydrological and hydraulic projects. Consequences, it is necessary to find an unbiased plotting positions which will accurately model probability distribution function. The appropriate plotting position for GEV, LP3, PR3 were evaluated using statistical goodness-of-test. The PPF were evaluated based on optimum value of the goodness-of-fit-test; Maximum absolute error (MAE), Nash-Sutcliffe efficiency (NSE), Percent bias (PBAIS) and Root-Mean Square error (RMSE). On the basis of the selected goodness-of-fit tests, 8 unbaised plotting position formula recommended of GEV, 7 for PR3, 6 for LP3 were evaluated with annual maximum series of Niger River at Baro, Kouroussa and Shinkatu. The results of the study show that Weibull, is the best plotting position formula for GEV, PR3 and LP3 distributions with overall percentage scores of 18.17%, 18.24% and 22.91% respectively. The study also indicates that Chegodayev plotting position Formula is second with 17.18% for LP3 and 17.4 for PR3 distributions. While Beard plotting position formula is second for GEV distribution with a score of 17.76%. The result of the study will complement the development of a standardised flood estimation manual for flood frequency analysis in Nigeria.

KEYWORDS: Plotting position formulae, GEV, PR3, LP3, Goodness – of- fit tests, Parameter estimation.

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I. INTRODUCTION

Floods are responsible for 20 – 30 % of economic losses caused by natural hazards globally and also responsible for more than 50 % of all fatalities due to natural disasters [1]. Furthermore, floods cause deaths, sickness, stress, anxiety, and reduced environmental quality which cannot be quantified in monetary terms [2]. Consequently, large investments are expended in flood related engineering interventions in river basins. Due to the enormous cost to societies in combating floods, there is need for accurate estimation of design flood and exceedance probabilities for optimum design of infrastructural projects, flood risk estimation, and decision making in water resources engineering. Thus the design flood estimation must be accurate as possible to avoid severe economic implications, damages and loss of human life. The cumulative probability of non – exceedance given as $F(Q_T) = P(Q_T \leq q) = 1- P(Q_T > q) = (1-1/T)$, is the basis for estimating the magnitude of Q_T, given its exceedance probability or its inverse the return period (T). The cumulative distribution function (CDF) may be expressed in terms of the plotting position formulae (PPF), from it the exceedance probability and the return period are computed.

The use of plotting position formulae has had a long history and research work on the subject is still continuing [3]. Consequently, several plotting position formulae have been developed. Reviews of related works include; [4] to [17]. Analysis of the reviewed literatures revealed numerous claims, counter claims and recommendations among researchers concerning the use plotting position formulae. For example, [13] and [14] studies the various plotting position methods on the criteria of unbiasedness and minimum variance, and concluded that Weibull's formula is biased and plots the largest values of a sample at too small return periods. Also [17] studies estimation of plotting position for flood frequency analysis and proposed that plotting positions are examined in more details and advocated collaboration amount researchers. Furthermore [15] has shown that an underestimation of the Probability of extreme events has resulted from the use unbiased plotting positions, and that this has had an adverse impact on building codes and other means for optimum design against extreme – weather events. The various PPF predict different quantile values when extrapolated to extreme return periods of exceedance probabilities.

In spite of significant progress made in development of plotting position formulae, the selection of an appropriate plotting position from the plethora of plotting position formulae is still a subject of continuing research investigation [16].. [18] evaluated flood flow probability distribution models on Niger and Benue river basins in Nigeria and found GEV, LP3 and PR3 the best distribution. In this study, unbiased plotting position formulae for GEV, PR3, and LP3 distributions are evaluated using the numerical indices of MAE, PBIAS, NSE and RMSE against eight unbiased plotting position formulae for GEV, seven for PR3 and six for LP3 using data from three hydrological stations on the Niger River in Nigeria. Presently, there is no recommended probability distribution model(s) for flood frequency analysis and probability distribution are applied inconsistently across Nigeria. The selection of appropriate plotting position formulae for GEV, PR3 and LP3 distributions will supplement flood frequency analysis standardisation in Nigeria.

III. MATERIALS AND METHODS

3.1. Catchment and Hydrologic Characteristics

The data of three hydrological stations on the Niger river basin at Baro, Kouroussa and Shintaku were employed in this study. The stations catchment attributes and descriptive statistics are shown in Table 1. The Niger River basin covers a total area of approximately 2,156,000km^2, only about 1,270,000km^2 actively contribute to runoff and river discharge. The whole basin is spread over the territory of ten countries. In Table 1, the coefficient of variation shows that the flow is moderately variable. Secondly, the annual maximum discharges are generally skewed and normal distribution will not be a suitable probability distribution model. Figure 1 is map of the Niger River Basin showing the hydrological stations.

Table 1: Catchment Characteristics and Descriptive Statistics

Parameter	Hydrological Stations		
	Baro	Kourassou	Shintaku
Latitude	08° 35′	08° 51′	07° 10′
Longitude	08° 23′	10° 47′	06° 45′
Minimum Flow	423 m^3/sec	275 m^3/sec	7730.59 m^3/sec
Maximum Flow	1150 m^3/sec	1185 m^3/sec	16480 m^3/sec
Mean Flow	716.34 m^3/sec	713.98 m^3/sec	13, 320.8 m^3/sec
Standard Deviation	188.71 m^3/sec	223.643 m^3/sec	2015.43 m^3/sec
Coefficient of Variation	0.264	0.313	0.151
Skewness	0.122	0.242	-0.671
Data length	1948 – 2000 (53 years)	1950 – 2000 (51 years)	1957 – 2014 (58 years)

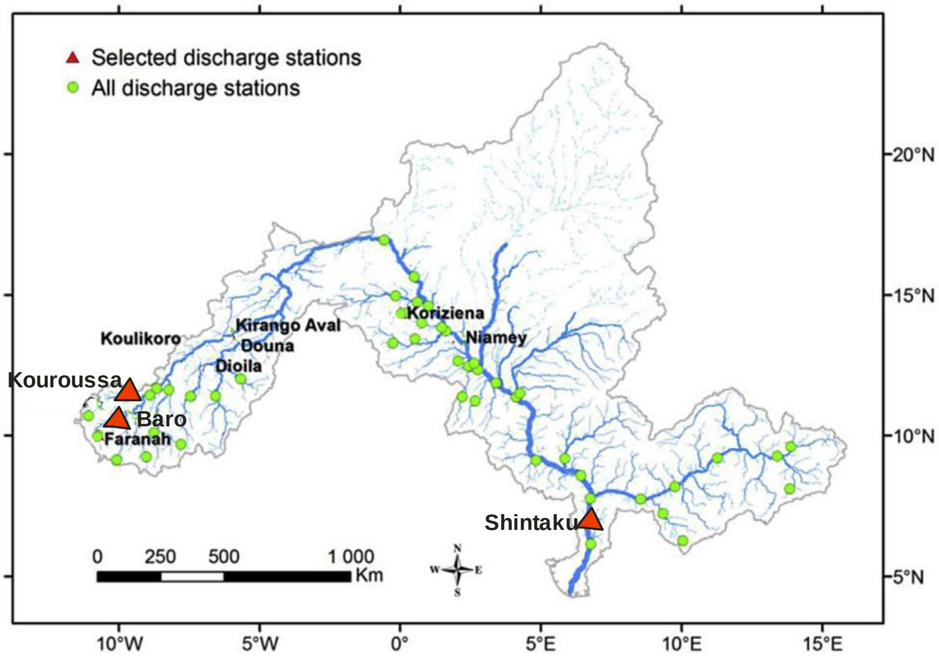

Figure 1:Map of Niger River Basin showing hydrologic stations

3.2 Probability Distributions

Parameter estimation for GEV distribution was performed using probability –weighted moments and L-moments. While the method of moments (MOM) was used for LP3 and PR3. The steps followed to derive the quantile relations for GEV, PR3 and LP3 distributions are presented for each distribution in subsections 3.1 to 3.4 respectively.

3.1.1 LOG PEARSON TYPE III (LP3)

The LP3 is a member of Gamma distribution family. It uses 3 parameters: location (μ), scale (α) and shape (β). If the logarithm of a variable (LNQ) obeys Pearson Type III distribution, then Q can be described according to the log. Pearson Type III (LP3) distribution [3] and [19]. The steps using Method of Moment (MoM) estimators to compute Q_T are:

i) Transform the Annual Maximum Series AMS) to LNQ_1, LNQ_2, ……, LNQ_n

ii) Compute the mean ($\overline{Q}$), variance (σ_Q^2) and skew (Cs) of the log-transformed series.

iii) Estimation of LP3 distribution parameters (α β and u) by the Method of Moment (MoM); the sample moments (mean standard deviation and skew coefficient).

iv) Determine the MoM estimators: LP3 (α, β and u)

v) Calculate the frequency factor K_T is calculated as follows:

i) $$W = \left[LN\left(\frac{1}{P^2}\right)\right]^{1/2} \quad (1)$$

ii) $$z = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3} \quad (2)$$

when p > 0.5, 1 – p is substituted for p in Equation 2 and the value of z computed by Equation 2 is given a negative sign.

iii) K_T is approximated by:

$$K_T = z + (z^2 - 1)k + \tfrac{1}{3}(z^3 - 6z)k^2 - (z^2 - 1)k^3 + zk^4 + \tfrac{1}{3}k^5 \quad (3)$$

where k = Cs/6

vi) The T-year flood quantile is obtained by Equations 4 and 5.

$$Z_T = LNQ_T = \mu_z + K_T\sigma_z \quad (4)$$

$$Q_T = e^{ZT} \quad (5)$$

3.1.2 Pearson Type III: Distribution

PT-III is a three parameter distribution (α, β and u), therefore, three sample moments (mean (μ), variance (σ_y^2), and skewness are required, from the sample data to compute the population parameters. The sample moments (E(Q), var(Q) and Cs are estimated from their relations to the distribution parameters, which in turn, are estimated by MoM. See [18] for detailed steps taken to compute quantile relation given in Equation 6:

$$Q_T = \alpha + \beta + u + K * \sqrt{\hat{\ }\alpha^2 + \beta} \quad (6)$$

3.1.3 Probability Weighted Moments/L-Moments

The L-Moments are easily calculated in terms of probability weighted moments. Sample probability weighted moments, computed from data values $Q_{1:n}$, $Q_{2:n}$, … $Q_{n:n}$, arranged in ascending order [20]. :

The 4L-Moments (λ_1, λ_2, λ_3 and λ_4) were derived using the 4 PWMS. The procedure for fitting GEV distribution using the PMW method are: i) arrange the observed annual maximum series in ascending order, ii) Calculate the 4L-Moments (λ_1, λ_2, λ_3 and λ_4), iii) Calculate the 4 PWMs (b_0, b_1, b_2 and b_3), iv) Calculate the three parameters: location (ζ), scale (α), and shape (k). Using a specified recurrence interval, fit all the parameters to the quantile relations and estimate the magnitudes.

The L-moments ratios are L-cv(τ_2), L-Skewness (τ_3) and L-Kurtoses (τ_4), and defined as:

$$L - CV = \tau_2 = \frac{\lambda_2}{\lambda_1} \quad (7)$$

$$L - Skewness = \tau_3 = \frac{\lambda_3}{\lambda_2} \ldots \quad (8)$$

$$L - kurtosis = \tau_4 = \frac{\lambda_4}{\lambda_3} \quad (9)$$

The parameters of GEV distribution are define according to [21] as:

k = 7.8590c + 2.9554c^2 (10)

$$\alpha = \frac{\lambda_2 k}{(1-2-k)\Gamma(1+k)} \quad (11)$$

$$\xi = \lambda_1 - \frac{\alpha}{k}\{1-\Gamma(1+k)\} \quad (12)$$

$$C = \frac{2}{3+\tau_3} - \frac{\log 2}{\log 3} \quad (13)$$

Where Γ is a gamma function

The calculation proceed first with the calculation of the parameter "c" in Equation 13 using the L-skewness estimator τ_3, then the shape parameter, k using Equation 10, followed by the scale parameter, α, according to Equation 11. Finally, the estimate of the location parameter, ξ, using Equation 12, and sample mean $\bar{Q}$ given by λ_1 (or b_0). Once all parameters have been estimated, the quartile relation for GEV distribution calculated as shown in Equation 14 see [21].:

$$Q_T = \zeta + \left\{\frac{\alpha}{k}\right\}\left[1-(-\log(\frac{T-1}{T}))^K\right] \quad (14)$$

3.2 Numerical Evaluation of Plotting Position Formulae

Quantitative error assessment reporting the degree the simulated series matches with the observed series are use to evaluate model performance. Model performance is high when the error is low, and low when the model error is high. In this study, the error indices; NSE, RMSE, RSR, and PBIAS. presented mathematically in subsections 3.4.1 to 3.4.4 respectively have been used to assess the accuracy of the estimates [22] and [23].

3.2.1 Nash-Sutcliffe efficiency (NSE)

$$\text{NSE}= 1-\left[\frac{\sum_{i=1}^{n}(Q_{i,obs}-Q_{i,sim})^2}{\sum_{i=1}^{n}(Q_{i,obs}-Q_{obs})^2}\right] 0‹\text{NSE}‹1.0 \quad (15)$$

3.2.2 RMSE – Observation Standard Deviation Ratio (RSR)

$$RSR = \frac{RMSE}{STDEV_{obs}} = \frac{\left[\sqrt{\sum_{i=1}^{n}(Q_{i.obs}-Q_{i,sim})^2}\right]}{\sqrt{\sum_{i=1}^{n}\left(Q_{i.obs}-\bar{\bar{Q}}_{sim}\right)^2}} \ldots \quad (16)$$

3.2.3 MAE = $\frac{1}{N}\sum_{i=1}^{N}|Q_i - P_i|$ - ∞≤ MAE ≤ 1.0 (17)

3.2.4 Percent Bias (PBIAS)

$$\text{PBIAS} = \left[\frac{\sum_{i=1}^{n}(Q_{i.obs}-Q_{i,sim})^2 * 100}{\sqrt{\sum_{i=1}^{n}(Q_{i.obs})}}\right] \quad (18)$$

3.3 Plotting position Formulae

The plotting position formula provides an estimate of the probability so that the data series can be plotted, magnitude against probability. The plotting position formula depends on the assumed distribution and can be expressed in a general form [11]:

$$\text{Pi} = \frac{i-a}{n+1-2a} \ldots \quad (19)$$

Where "a" varies from 0 to 0.5; Pi is the plotting probability and i is the rank in ordered observation with i = 1 for the smallest observation in data series. When a = 0.375, Equation becomes Blom Plotting position formula which gives unbiased quantiles for the normal distribution . For a = 0.44, the Gringorten formula which gives unbiased quantiles for the Gumbel distribution. With the appropriate coefficient; Cunnane, Weibull and Bear provide quantile – unbiased plotting position for a range of distributions as shown in Table 2

Table 2 Plotting Position Formulae

S/N	Class	Plotting Position Formula (PPF) Fi = ( )	Recommended Probability Distribution
1	Blom (1958)	$\frac{i-0.375}{N+0.25}$	Normal Gamma, Log normal, log-Pearson type III
2.	Gringorten (1963)	$\frac{i-0.44}{N+0.12}$	Gumbel, Weibull, GEV
3.	Cunnane (1978)	$\frac{i-0.4}{N+0.2}$	GEV, log-Gumbel, PR3 LP3
4.	Chegodayev	$\frac{i-0.3}{N+0.4}$	Russia and Eastern Europe standard.
5.	IN-na and Nguyen (1989)	$\frac{N-i+0.05Cs+0.65}{N-0.08Cs+0.38}$	GEV
6.	Goel and De (1993)	$\frac{i+0.02Cs+0.32}{N-0.04Cs+0.36}$	GEV
7.	Kim et al. (2012)	$\frac{i-0.3200}{N-0.0149C^2s+0.1364Cs\ 0.3225}$	GEV
8.	Weibull (1939)	$\frac{i}{N+1}$	All Distributions
9.	Nguyen et al., (1989)	$\frac{1-0.42}{N+0.3Cs+0.05}$	Pearson Types 3(PR3) -3≤Cs≤; 5≤N≤100
10.	Bear (1945)	$\frac{i-0.3175}{N+0.365}$	All Distributions
11.	In-na (1988)	$\frac{1-0.53+0.3}{N+0.05+0.3}$	PR3
12.	Hosking (1990)	$\frac{i-0.35}{N}$	Some 3-parameter Distribution

IV. RESULTS AND DISCUSSION

4.1 Results

All calculations and graphical plots were performed using MS Excel software (2010 version). The MS Excel has built – in functions which implemented applied routines.

4.1.1: **Quantile Relations.**

Distribution fitting is a critical component of frequency analysis wherein a probability distribution that best fit geophysical data selected and used to established a quantile relation , which in-turn used foe extrapolation to higher return periods. Tables 3, 4. And 5 show the quantile relations for GEV, PR3 and LP3 distributions which were used to evaluate the best plotting position formula for each distribution.

Table 3: Quantile Relations for GEV Distribution.

Station	Q_T – T Model
Baro	Q_T = 645.97 + 746.29 * {1 – (-LN $\left\{1-(-LN(1-\frac{1}{T}))^{0.25343}\right\}$

Kourassou

$$Q_T = 623.48 + 1195.05 * \{1 - (-LN \left\{ 1-\left(-LN\left(1-\frac{1}{T}\right)\right)^{0.1786} \right\}$$

Shintaku

$$Q_T = 12875.51 + 4015.57 * \{1 - (-LN \left\{ 1-\left(-LN\left(1-\frac{1}{T}\right)\right)^{0.5532} \right\}$$

Table 4: Quantile Relations Pearson Type 3 Distribution

Station	Q_T – T Model	β	α	γ
Baro	$Q_T = 716.34 + 188.71K_T$	268.56	11.52	-2376.23
Kouroussa	$Q_T = 713.98 + 223.64K_T$	68.65	26.99	-1139.05
Shintaku	$Q_T = 13320.80 + 2015.43K_T$	8.90	675.72	7309.44

Table 5: Quantile Relations for Log – Pearson Type 3 Distribution

Station	Q_T – T Model: $LNQ_T = \mu_z + K_T\sigma_z$	Cs	σ_z	μ_z
Baro	$LNQ_T = 6.541 + 0.274K_T$	0.51	0.274	6.541
Kouroussa	$LNQ_T = 6.52 + 0.33K_T$	0.449	0.334	6.520
Shintaku	$LNQ_T = 9.495 + 0.164K_T$	0.151	0.164	9.485

Table 6: Percent Scores of PPF for GEV Quantile Relations.

PPF	Hydrological Stations(GEV)				Hydrological Stations(PR3)			
	Baro	**Kourous**	**Shintak**	**TOTL**	**Baro**	**Kourou**	**Shintaku**	**Total**
Cunnane	10.81	07.91	15.85	11.54	15.03	14.77	11.81	13.87
Weibull	16.89	20.53	17.07	18.17	15.03	17.45	22.22	18.24
Gringorten	08.11	04.64	13.42	08.72	17.65	16.78	18.06	17.49
Beard	17.57	19.87	15.85	17.76	16.99	14.77	15.28	15.68
IN-NA & Ngugea	16.22	15.23	07.32	12.92	09.15	08.54	10.42	09.21
Geol & De	16.22	15.23	13.42	14.96	09.15	08.73	06.94	08.27
Kim et al.	10.14	10.60	07.32	09.35	09.15	11.41	09.28	09.87
Hosking	04.06	05.96	09.76	06.59	07.84	08.54	06.25	07.38

PPF: Plotting Position Formulae

Table 7: Percent Scores of PPF for LP3 Quantile Relations

PPF	Hydrological Stations (LP3)			
	Baro	**Kourous**	**Shintak**	**TOTAL**
Cunnane	15.07	18.29	11.11	14.98
Weibull	19.18	17.07	33.33	22.91
Blom	12.33	15.85	13.89	14.10
Beard	17.81	17.07	16.67	17.18
Chegodayev	16.44	15.85	19.44	17.18
Ngugen	19.18	15.85	05.56	13.66

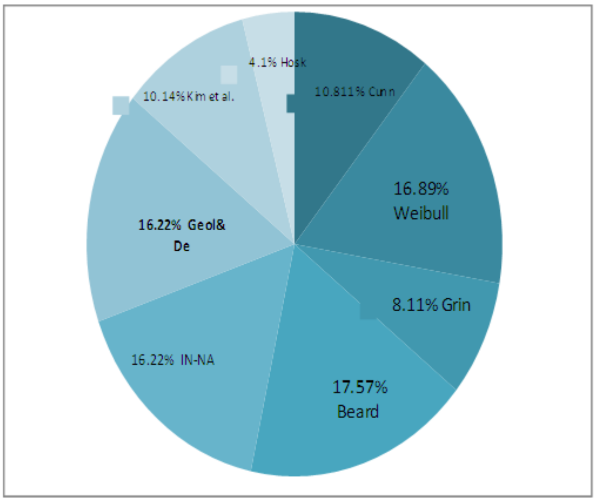

Fig. 2: Percent Scores of PPF for Baro (GEV)

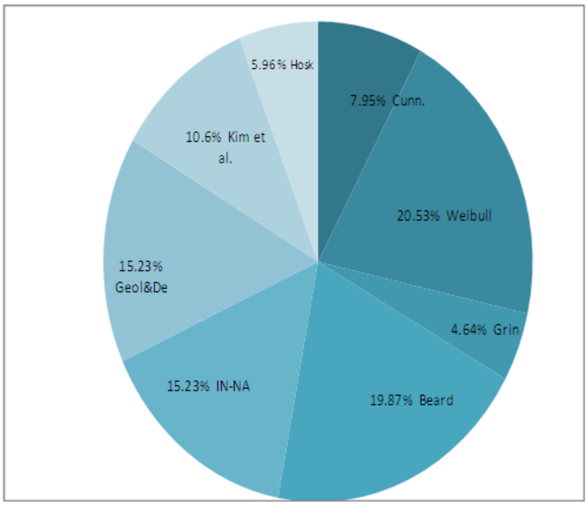

Fig. 3: Percent Scores of PPF for Kouroussa (GEV)

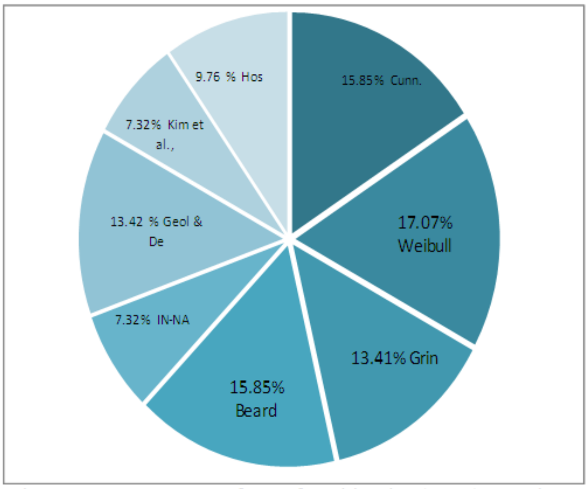

Fig. 4: Percent Scores of PPF for Shintaku (GEV)

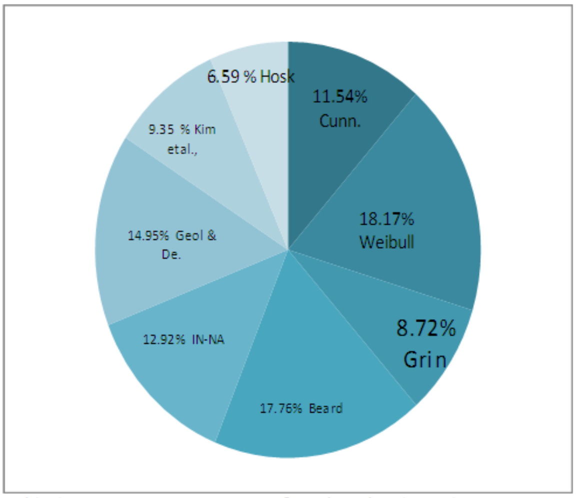

Fig. 5: Total(%) Scores Baro, Kour.& for Shntaku (GEV)

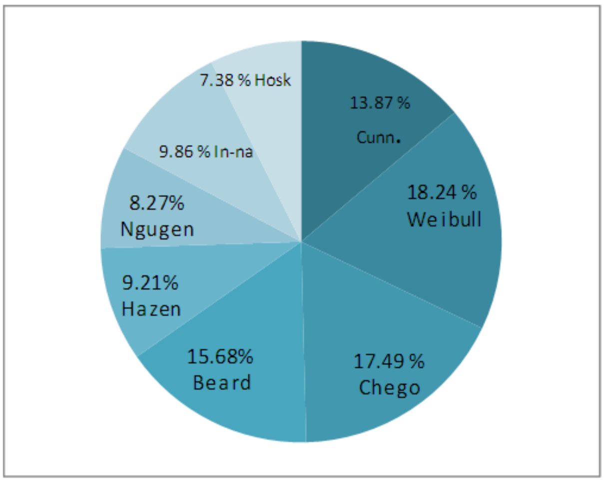

Fig. 6: Total(%) Scores Baro,Kour.& for Shntaku (PR3)

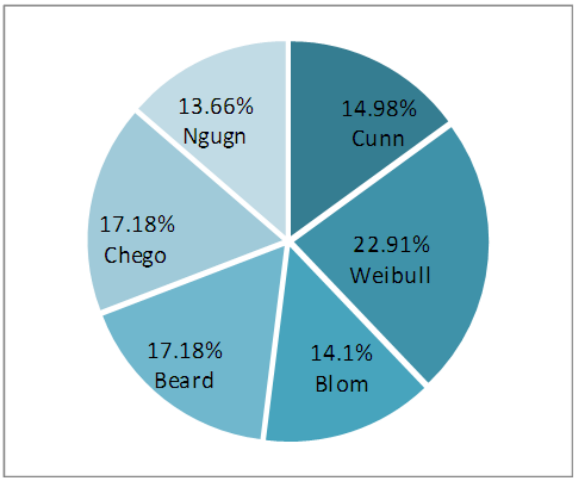

Fig. 7: Total(%) Scores Baro,Kour.& for Shntaku (LP3)

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4.1.2: Evaluation of Plotting Position Formulae

Pie charts

Pie charts were constructed for graphical assessment and visualization of numerical evaluation of the selected plotting position formulae. Figures 2-7 show reults of the selected PPF for GEV, LP3 and PR3 at Baro, Kouroussa and Shinkatu respectively and so on. The predicted series of each plotting position formula model was obtained by inputing the exceedance probability or its inverse, the return period (T) into the respective quantile models in Tables 3- 5. The predicted and observed series were inputed into the numerical assessment indices to compute MAE, NSE, PBIAS, and RMSE.

The computed values by the numerical indices were ranked according to the number of PPF involved. For example, for GEV distribution, eight PPF were used in the comparative study, seven for Pearson type 3, and six for LP3 distributions.

Ranking scores are assigned to each plotting position formulae according to the optimal value of the test statistic. For examples, a plotting position formula with MAE, RMSE and PBIAS values, of zero and NSE value of 1.0 is scored 8. In case of a tie, equal scores are given to the contending plotting position formulae. The plotting position formulae were ranked on the scale of 1 to 8 for GEV distribution with a score of “8” being the best and 1, the least.

For PR3 distribution, the PPF were ranked on a scale of 1 to 7, a score of 7 is ranked the best and 1 the least, while for LP3, the PPF were ranked on a scale of 1 to 6, a score of 6 being the best and 1the least. Finally, for each Distribution, all the PPF across each hydrological station, the total scores for each plotting position formulae is obtained by summing the individual scores across the goodness-of-fit-tests.

Tables 5 and 6 show the percentage scores of each PP formula across the stations. These values have been translated into pie charts for graphical assessment and visualization of the performances of the PPF for the distributions across the sStudy stations.

The pie charts in Figures 2-4 indicate the percent scores of all the plotting position formulae across the stations for GEV distribution. Figure 5 indicates the overall performances of the unbiased plotting position formulae for GEV across the hydrological stations. Figures 6 and 7 show the overall scores of all plotting position formulae for both Pearson type 3and Log – Pearson type 3distributions across the stations. Figures 5-7, indicate that Weibull is the best plotting position formula for GEV distribution with a percentage score of 18.17%, seconded by Beard with 17.76%. Also for PR3 distribution, Weibull is the best with percentage score of 18.24%, seconded by Chegodayev with 17.49% . Similarly for LP3, Weibull scored 22.91%, seconded by both Chegodayev 17.18% and Beard 17.18%.

This study is corroborated with similar studies, for example; [9] studied the ranking of plotting position formulae for South India; [24] compared plotting position formulae for the Pearson Type 3 distribution for Malaysian Peninsula; Also, Adeboye and Alastise (2007) [25] fitted the normal distribution to peak flow discharges of two rivers in Nigeria, all found the Weibull plotting position formula most suitable for frequency analysisof flood and rainfall data. Furthermore, the study agrees with [26] who found Weibull plotting position most appropriate with Pearson Type 3 (and LP3) distributions. Finally studies recommending different PPF for GEV, LP3 and PR3 are rejected.

V. CONCLUSION

The selection of appropriate plotting position formulae for Generalized Extreme Value, Pearson Type 3 and Log-Pearson Type 3 distributions have been studied in this paper. Eight unbiased plotting position formulae recommended for GEV distribution were evaluated. Also, seven recommended for Pearson Type 3 and six for Log-Pearson distributions were also evaluated.

Parameter estimation for GEV distribution was executed using Probability-Weighted Moments/ L-moments. While MOM was applied to PR3 and LP3 distributions. The best plotting position formula is identified based on the highest total rank score of goodness-of-fit tests.

The results indicate Weibull is the best PP Formula for GEV, LP3 and PR3 distributions, seconded by Chegodayev. The results found in this research study is useful for water resources management and design hydraulic structures in the Niger River basin.

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