

## Analyzing of an ECG Signal Mathematically By Generating Synthetic-ECG

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### -----ABSTRACT-----

ECG is a non-invasive technique is used as a primary diagnostic tool for cardiovascular disease. The proposed method will help to acquire an ECG signal by generating a synthetic ECG from a patient and to analyze and detect the anomalies which are also called as abnormalities. Abstract Data obtained from electrocardiogram (ECG) signals provides invaluable tools for diagnosing cardiac disorders. However, ECG signals recorded from electrocardiograph are usually corrupted by noise attributed to several factors. Finite Impulse Response Filtering Techniques will eliminate noise in the signal to some extent, but the resultant output signal has a noise component. To help solve these problems, we simulated a simple but inexpensive and easy-to-implement MATLAB model that generates ECG signals and gives us mathematical control over the ECG signal. Our model fuses Mathematical functions in MATLAB with physiological data.

**Keywords** – ECG, Fourier Transform, QRS, SynECG

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### I. INTRODUCTION

Electrocardiogram (ECG) is a diagnosis tool that reported the electrical activity of heart recorded by skin electrode. The morphology and heart rate reflects the cardiac health of human heart beat. It is a non-invasive technique that means this signal is measured on the surface of human body, which is used in identification of the heart diseases. Any disorder of heart rate or rhythm, or change in the morphological pattern, is an indication of cardiac arrhythmia, which could be detected by analysis of the recorded ECG waveform. The amplitude and duration of the P-QRS-T wave contains useful information about the nature of disease afflicting the heart. The electrical wave is due to depolarization and re polarization of  $\text{Na}^+$  and  $\text{K}^+$  ions in the blood. The ECG signal provides the following information of a human heart.

A typical ECG signal shows the oscillations between cardiac contractions (systole) and relaxations (diastole) states as reflected in a heart rate (HR). Thus the ECG signal determines the number of heart beats per minute.

A number of important events characterize cardiac functions. Atrial and ventricular depolarization/re-polarization takes place for each heart beat. The cardiac cycle is associated with portions of the heart becoming positively charged, while the remaining parts become negatively charged interchangeably. This potential difference generated initiates the flow of current. A typical ECG signal depicts a series of waveforms which occur in a repetitive order. The waveforms are initiated from the isometric line, from which a deflection indicates electrical activity. One normal heart beat is represented by a set of three recognizable waveforms that start with the P-wave, followed by the QRS complex and ends with the T-wave. The relatively small P-wave is initiated by the depolarization of the atrial muscles and is related to their contraction.

The large QRS-wave complex, made up of three waves, is caused by the depolarization of the ventricles and is connected to their contraction. Atrial re-polarization happens during the depolarization of the ventricles but its weak signal is undetected on an ECG. The T-wave is caused by currents flowing during the repolarization of the ventricles. A normal cardiac cycle of an individual at rest consisting of all waveforms (from P –T waves) spans 0.8 seconds.

ECG signals are generated using electrocardiographs. Such signals are usually vitiated by several sources of noise which include (i) electrical interference from surrounding equipment (e.g. effect of the electrical mains supply), (ii) measurement (or electrode contact) noise, (iii) electromyographic (muscle contraction), (iv) movement artifacts, (v) baseline drift and respiratory artifacts and (vi) instrumentation noise (such as artifacts from the analogue to digital conversion process). One method of dealing with corrupt ECG signals is through the use of signal filtration systems.

## II. METHODOLOGIES AND MATHEMATICAL EQUATIONS

In order to eliminate the perennial problem of dealing with noise from the tainted ECG signals, dynamic functions are usually developed for the generation of artificial synthetic ECG signals. Such functions are usually equation based. The work of McSharry and Clifford relies on three coupled differential equations to generate ECG signals. We summarize their method below. McSharry and Clifford captured the spectral characteristics of beat-to-beat RR intervals or RR tachogram, including both the oscillation in the RR tachogram resulting from parasympathetic activity, in synchrony with respiration (Respiratory Sinus Arrhythmia), and the waves in arterial blood pressure (Mayer waves), using a bi-modal spectrum made up of the sum of two Gaussian functions.

This is given by:

$$S(f) = \frac{\sigma_1^2}{\sqrt{2\pi c_1^2}} \exp\left(-\frac{(f-f_1)^2}{2c_1^2}\right) + \frac{\sigma_2^2}{\sqrt{2\pi c_2^2}} \exp\left(-\frac{(f-f_2)^2}{2c_2^2}\right) \text{----- 1}$$

Where  $f_1$  and  $f_2$  are the two means, and  $c_1$  and  $c_2$  are the corresponding standard deviations. In spectral analysis of the RR tachogram, two critical frequency bands, usually referred to as the low-frequency (LF) band (0.04 to 0.15 Hz) and high-frequency (HF) band (0.15 to 0.4 Hz) are considered.

The power in the LF and HF bands are denoted by  $\sigma_1^2$  and  $\sigma_2^2$  respectively and the variance is represented by

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 \text{----- 2}$$

Consequently, the LF/HF ratio is given by  $\sigma_1^2/\sigma_2^2 \text{----- 3}$

The function  $S(f)$  gives the spectrum of a time series denoted by  $T(t)$ . In order to obtain  $T(t)$ , the inverse Fourier transform is applied on a sequence of complex numbers whose amplitudes are given by  $\sqrt{S(f)}$ , such that the phases are randomly distributed between the interval 0 and  $2\pi$ . Next, an appropriate scaling constant is chosen to multiply the resulting time series and then an offset value is added. This makes it possible to assign any required mean and variance to the resulting time series  $T(t)$  based on the initial series.

It also specifies different realizations of the random phases by simply changing the seed of the random number generator. The resulting series will inherit the same temporal and spectral properties of the original. To complete the process, a dynamical model is then used to obtain the quasi-periodicity property of the ECG signal. This is guaranteed by making sure that the model has an attracting limit cycle. One complete revolution around the limit cycle in the  $x$ - $y$  plane mimics a heart-beat. The PQRST peaks and troughs of the ECG signal are captured by a series of exponential functions inherent in the model, while the PQRST are the extremes of the signal are specified by five angles correspondingly denoted by  $\theta_p, \theta_q, \theta_r, \theta_s$ , and  $\theta_t$ . The mathematical model is represented by the following system of differential equations.

$$\dot{x} = \alpha x - \omega y \text{----- 4}$$

$$\dot{y} = \alpha y + \omega x \text{----- 5}$$

$$\dot{z} = -\sum_{i \in \{P, Q, R, S, T\}} \alpha_i \Delta \theta_i \exp(-\Delta \theta_i / 2b_2) - (z - z_0) \text{----- 5}$$

Where,  $\alpha = 1 - \sqrt{(x^2 - y^2)}$ ,  $\Delta \theta_i = (\theta - \theta_i) \bmod (2\pi)$ ,  $\theta = \text{atan2}(y, x)$  is the four quadrant arctangents of the elements of  $x$  and  $y$ , ranging over  $[-\pi, \pi]$  and  $\omega$  is the angular frequency of the trajectory in its motion around the limit cycle and is related to the heart beat rate as  $2\pi f$  [9]. MATLAB Is used to develop and test “SynECG”, the artificial ECG generating function.

The five inbuilt MATLAB functions used to develop “SyECG” are “sgolayfilt”, “kron” “ones”, “round”, and “linspace”. These functions are described briefly below. Function `sgolayfilt()` is known as the Savitzky-Golay Filter. `y = sgolayfilt(x, k, f)` applies a Savitzky-Golay FIR smoothing filter to the data in vector  $x$ . If  $x$  is a matrix, `sgolayfilt` operates on each column. The polynomial order  $k$  must be less than the frame size,  $f$ , which must be odd. If  $k = f - 1$ , the filter produces no smoothing. Savitzky-Golay smoothing filters (also called digital smoothing polynomial filters or least-squares smoothing filters) are typically used to “smooth out” a noisy signal whose frequency span (without noise) is large. It also helps preserve the peaks and valleys of ECG signals better than a standard FIR filter.

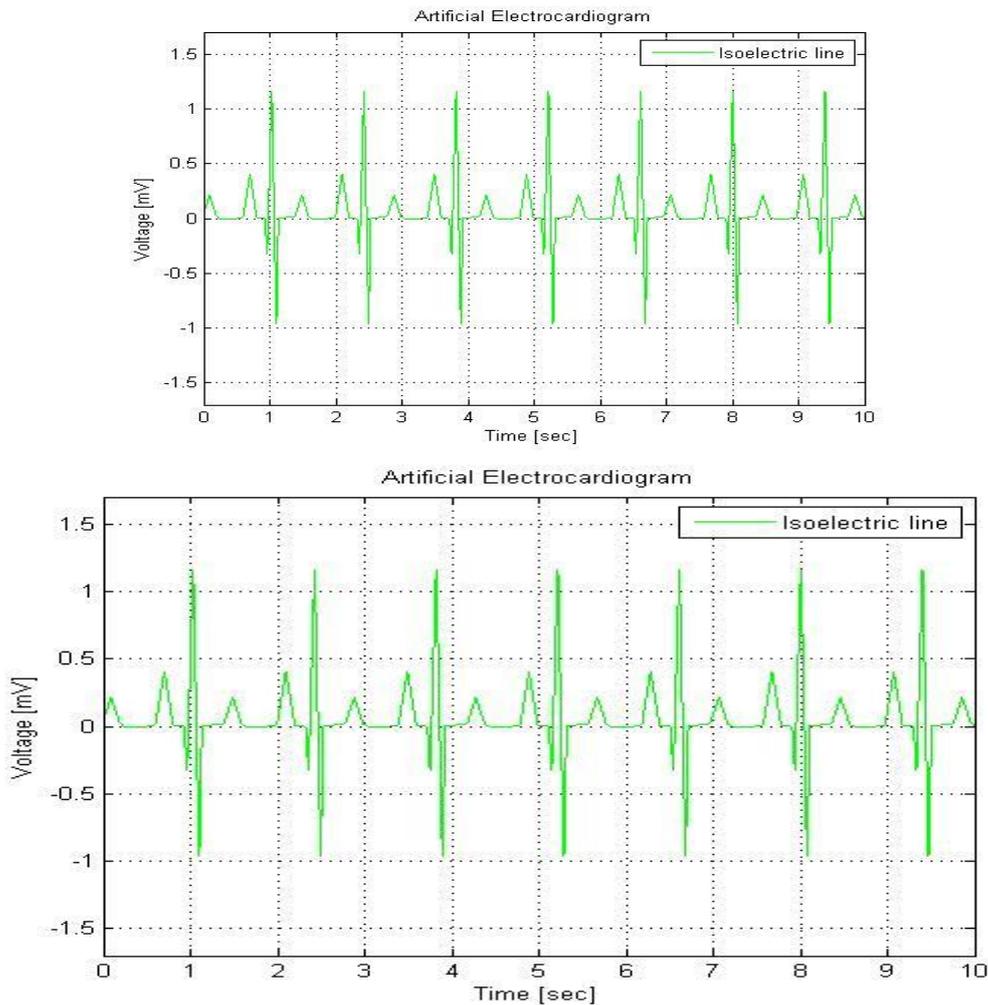
## III. SIMULATION RESULTS

We use the SynECG to generate 10-seconds synthetic ECGs for a number of heart rates of a resting individual with a peak voltage of 1.2mV. Figure 1.2 Illustrates how the function is summoned in the MATLAB command window; Figures 2 through 4 show the artificial ECGs generated for 40bpm, 50bpm, 60bpm, 70bpm, 80bpm, 90bpm, 100bpm and 110bpm heart rates at a desired peak voltage of 1.2mV.

We note that heart beats in a regular rhythm is usually between 60 and 100, that is when the signal-to-noise ratio is usually quite good in a person at rest. Basically, the ECG is a piecewise continuous graph of potential difference (in mV) against time (in seconds). The Synthetic ECG generated for 95bpm displays approximately 17 cardiac cycles in 10 seconds.

With the SynECG, we have been able to generate a 10-seconds artificial ECG for some selected heart rates. The 60bpm generated 10-seconds ECG displays an approximate 10 cardiac cycles.

As heart rate increases, the number of approximate cardiac cycles in the 10-seconds ECG also increases. Variation in the heart rates time intervals can be monitored with this increase in heart rate. A careful observation of the 60bpm ECG generated by SynECG reveals that one cardiac cycle takes 0.8 seconds as human physiology suggests. We observe from the generated ECGs that as heart rate increases; the time taken for a cardiac cycle reduces. Similar observations were made for 80bpm and 95bpm. The various waveforms which constitute a cardiac cycle vary in length as heart rates change from time to time for the resting individual.



**Figure 2: Synthetic ECG signal generated from SYNECG using 40bpm. This displays approximately 10 cardiac cycles in ten seconds.**

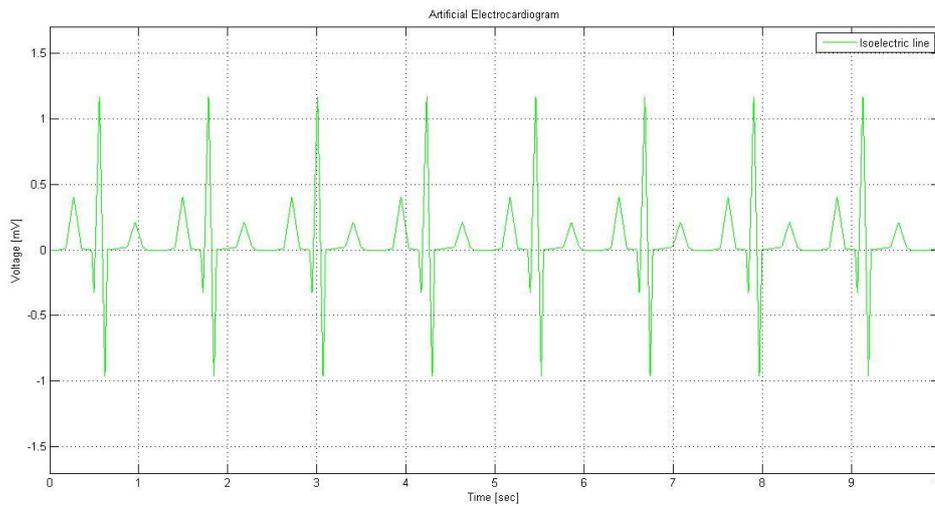


Figure 2: Synthetic ECG signal generated from SYNECG using 50bpm. This displays approximately 10 cardiac cycles in ten seconds.

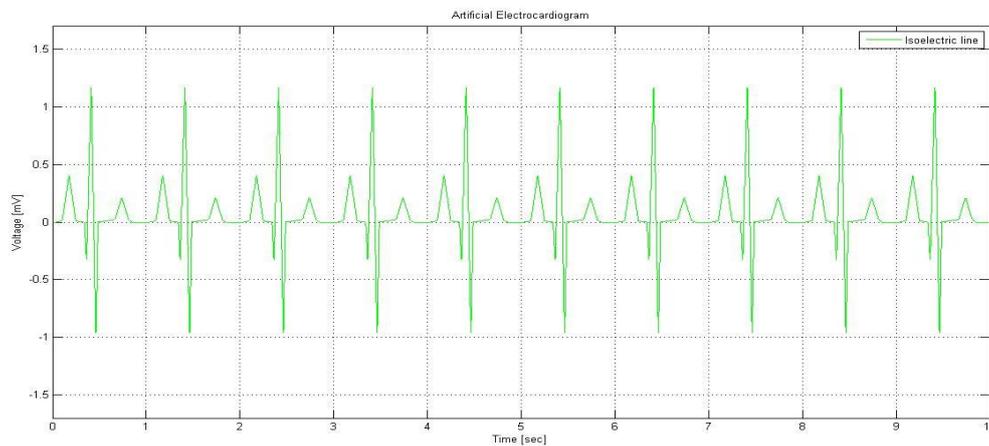


Figure 3: Synthetic ECG signal generated from SYNECG for 60bpm. This displays approximately 13 cardiac cycles in ten seconds.

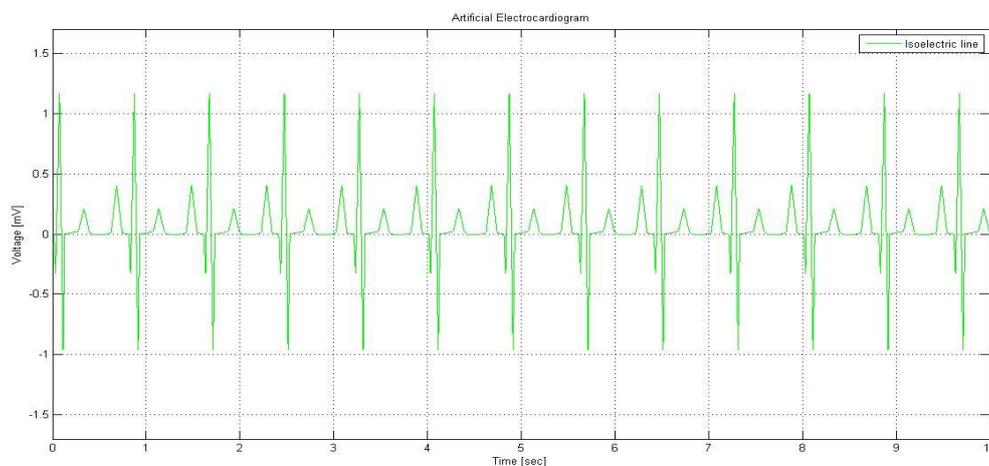


Figure 4: Synthetic ECG signal generated from SYNECG using 70bpm. The graph displays approximately 17 cardiac cycles in ten seconds.

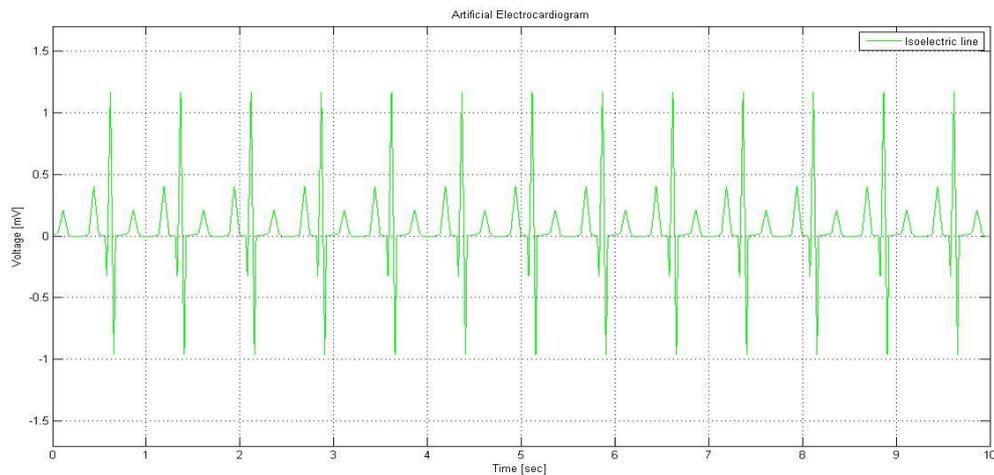


Figure 5: Synthetic ECG signal generated from SYNECG for 80bpm. This displays approximately 13 cardiac cycles in ten seconds.

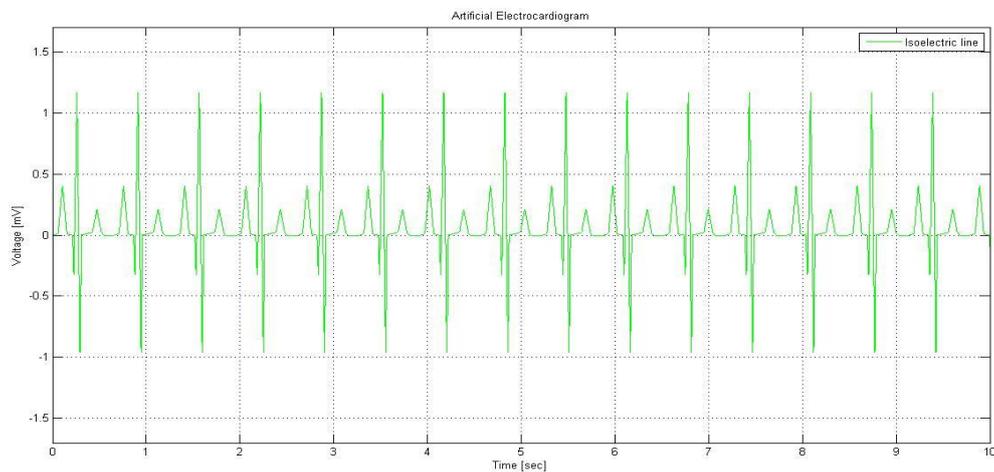


Figure 6: Synthetic ECG signal generated from SYNECG for 90bpm. This displays approximately 13 cardiac cycles in ten seconds.

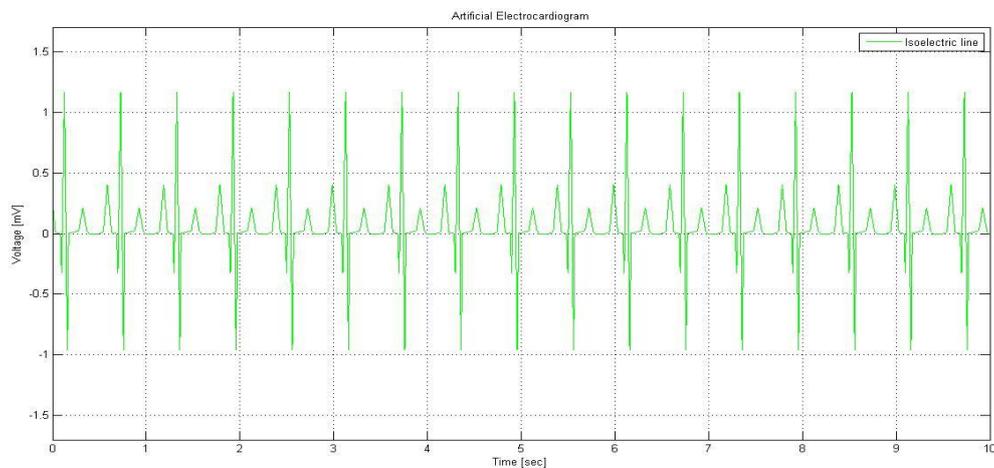


Figure 7: Synthetic ECG signal generated from SYNECG for 100bpm. This displays approximately 13 cardiac cycles in ten seconds.

#### IV. CONCLUSION

Finally we have also worked with the SynECG, with which we have been able to generate a 10-seconds artificial ECG for some selected heart rates. The 50bpm generated 10-seconds ECG displays an approximate 10 cardiac cycles. As heart rate increases, the number of approximate cardiac cycles in the 10-seconds ECG also increases. Variation in the heart rates time intervals can be monitored with this increase in heart rate. A careful observation of the 60bpm ECG generated by SynECG reveals that one cardiac cycle takes 0.8 seconds as human physiology suggests. We observe from the generated ECGs that as heart rate increases; the time taken for a cardiac cycle reduces. Similar observations were made for 80bpm and 95bpm. The various waveforms which constitute a cardiac cycle vary in length as heart rates change from time to time for the resting individual.

With the SynECG function, synthetic ECG signals of various heart rates can be generated at a desired peak voltage for off-line study of the electrical activity of the heart. Our function made use of the function "sgolay" which is specifically geared towards ECG noise reduction and uses Savitzky-Golay polynomial filtering which is well-suited to smoothing of ECG data and preserve the peaks and valleys of the ECG signals better than a standard FIR filter. We have been able to show that our function generates synthetic ECG for 10 seconds by inputting heart rate (per minute) and the desired peak in millivolts.

#### ACKNOWLEDGEMENTS

SynECG can be used to further develop a complete dynamic signal monitoring software for the electrical activity of the heart. This is an example of how Mathematical principles can be used in the dynamic modelling of biological rhythms.

#### REFERENCES

- [1] P.de Chazal, M.O. Duyer, and R.B. Reilly, "Automatic classification of heartbeat using ECG morphology and heart beat interval features," *IEEE Trans. Biomed. Eng.* vol. 51, pp. 1196-1206, 2004.
- [2] T.Ince, S. Kiranyaz, and M. Gabbouj, "A generic and robust system for automated patient-specific classification of ECG signals," *IEEE Trans. Biomed. Eng.* vol. 56, pp. 1415-1426, 2009
- [3] R. Sameni, M. B. Shamsollahi, and C. Jutten, Filtering Electrocardiogram Signals Using the Extended Kalman Filter, Proceedings of the 2005 IEEE Engineering in Medicine and Biology 27th Annual Conference, 2005.
- [4] P. E. McSharry, G. Clifford, L. Tarrasenko, and L. A. Smith, "A dynamical model for generating synthetic electrocardiogram signals", *IEEE Trans. Biomed. Eng.* vol 50, no. 3 pp. 289-294, 2003
- [5] M. S. Grewal and A. P. Andrews, *Kalman Filtering: Theory and Practice Using MATLAB*, 2nd ed., John Wiley and Sons Inc., 2001. A.Barros, A.Mansour, and N.Ohnishi, "Removing Artifacts from ECG signals using independent components analysis", *Neuro-computing*, vol.22, pp-173-186, 1998.
- [6] Sreeraman Rajan, R. Doraiswami, M. Stevenson and R.Waltrous, "Wavelet based bank of correlators approach for phonocardiogram signal classification", *IEEE*, pp.
- [7] O. Sayadi, M.B. Shamsollahi and G.D. Clifford, "Synthetic ECG generation and Bayesian filtering using a Gaussian wave-based dynamical model", *Physiological Measurement*, 31(2010) 1309-1329.
- [8] D. Dubin, *Rapid Interpretation of EKG's*, Third Edition. Cover Publishing Company, Tampa, FL, 1981.
- [9] R. Sameni, M. B. Shamsollahi, and C. Jutten, Filtering Electrocardiogram Signals Using the Extended Kalman Filter, Proceedings of the 2005 IEEE Engineering in Medicine and Biology 27th Annual Conference, 2005.