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The Solution of Maximal Flow Problems Using the Method Of Fuzzy Linear Programming

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ABSTRACT

The solution of fuzzy maximal flow problems using the method of fuzzy linear programming is studied and on the basis of the study a STEP method is used to find the fuzzy optimal solution of fuzzy maximal flow problems. In the studied method, all the parameters are represented by triangular fuzzy numbers. Using the method, the fuzzy optimal solution of fuzzy maximal flow problems can be easily solved. Two examples are solved and the obtained results show that the method is good over other methods because the ranking function helps us to check our solution.

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I. INTRODUCTION

The maximal flow problem is one of the basic problems for combinatorial optimization in weighted directed graphs. It provides very useful models in a number of practical contexts including communication networks, oil pipeline systems and power systems. The maximal flow problem and its variations have wide range of applications and have been studied extensively. The maximal flow problem was proposed by Fulkerson and Dantig originally and solved it by specializing the simple method for the linear programming and Ford and Fulkerson solved it by augmenting path algorithm. There are efficient algorithms to solve the crisp maximal flow problem. In real life situations there always exist uncertainty about the parameters (e.g. costs, capacities and demands) of maximal flow problems. To deal with such type of Problems, the parameters of maximal flow problems are represented by fuzzy numbers and maximal flow problems with fuzzy parameters are known as fuzzy maximal flow problems. In the literature, the numbers of papers dealing with maximal flow problems are less. The paper by Kim and Roush is one of the first on this subject. The authors developed the fuzzy flow theory, presenting the condition to obtain an optimal flow by means of definitions on fuzzy matrices. But there were Chanas and kolodziejczyk who introduce the main works in the literature involving this subject. They approached this problem using the minimum cuts technique. In the first paper, Chanas and kolodziejczyk presented an algorithm for a graph with crisp structure and fuzzy capacities i.e. the arcs have a membership function associated in their flows. This problem was studied by Chanas and Kolodziejczyk again in this paper the flow is a real number and the capacities have upper and lower bounds with a satisfaction function. Chanas and Kolodziejczyk had also studied the inter flow and proposed an algorithm. Chanas et al studied the maximum flow when the underlying associated structure is not well defined and must be modeled as a fuzzy graph. Diamond developed interval valued versions of the max-flow min cut theorem and Karp-Edmonds algorithm and provide robustness estimates for flows in networks in an imprecise or uncertain environment. These results are extended to networks with fuzzy capacities and flows.

Liu and Kao investigated the network flow problems in that the arc lengths of the network are fuzzy numbers. Ji et al considered a generalized fuzzy version of maximal flow problem, in which arc capacities are fuzzy variables. Hernandez et al proposed an algorithm based on the classical algorithm of F1ord-Fulkerson. The algorithm uses the technique of the incremental graph and representing all the parameters as fuzzy numbers. In this research project, the fuzzy linear programming formulations of fuzzy maximal flow problems are studied and on the basis of the study, we formulate a method to find the optimal solution of fuzzy maximal flow problems. In the studied method, all the parameters are represented by triangular fuzzy numbers. By using the studied method the fuzzy optimal solution of fuzzy maximal flow problems can be easily obtained. To illustrate the studied method a numerical example is solved and the obtained results are discussed.

II. LITERATURE REVIEW

Fuzzy logic theory and applications have a vast literature. With regards to documented literature, we can classify the development in fuzzy theory and applications as having three phases; Phase I (1965-1977) can be referred to as academic phase in which the concept of fuzzy theory has been discussed in depth and accepted as a useful tool for decision making. The phase II (1978-1988) can be called as the transformation phase whereby significant advances in fuzzy set theory and a few applications were developed. The period from 1989 onwards can be the phase III, the fuzzy boom period, in which tremendous application problems in industrial and business are being tackled with remarkable success. Fuzzy sets theory was first introduced by "Prof. Lotfi Zadeh" in 1965. Successful Applications of fuzzy sets theory on controller systems in decade 80 caused to this Theory develop in other fields such as simulation, artificial intelligent, operations Research, management and many industrial applications. In the real world, many applied Problems are modeled as mathematical programming and it may be necessary to formulate these models with uncertainty. Many problems of these kinds are linear Programming with fuzzy parameters. The first formulation of fuzzy linear programming was proposed by Zimmermann (1978). After the pioneering works on fuzzy linear Programming, several kinds of fuzzy linear programming problems have appeared in the Literature and different methods have been proposed to solve such problems. One Convenient approach for solving the fuzzy linear programming problems is based on the Concept of comparison of fuzzy numbers by use of ranking function. Usually in such Methods authors define a crisp model which is equivalent to the fuzzy linear Programming problem and then use optimal solution of the model as the optimal Solution of the fuzzy linear programming problem. The duality of fuzzy parameter linear programming was first studied by Rodder and Zimmermann (1977). Maleki et al. (2000) proposed a new method based on simplex method for fuzzy number linear Programming problems by using a special ranking function. We extended their methods

for general linear ranking function and established the duality theory on fuzzy number linear programming problems. Moreover, linear programming with fuzzy variables (FVLP) problem has attracted many interests. Some methods have been developed for solving these problems by introducing and solving a certain auxiliary problems. Currently, fuzzy technique is very much applied in the field of decision making, fuzzy methods have been developed in virtually all branches of decision making including multi-objective, multi-person, and multi-stage decision making. Apart from these, other research work connected to fuzzy decision making are applications of fuzzy theory in management, business and operation research (Zimmermann, H.J 1976). In several such applications, linear membership functions such as triangular and trapezoidal have been used. The details about logistic functions that can be built on a non linear membership function have been reported recently (Watada J. 1997). A non linear membership function has been used in fuzzy linear programming (FLP) problems to encourage an interaction among decision makers to achieve a satisfactory solution.

In the applications using fuzzy linear programming, it is often difficult to determine the coefficients of the problem with precision because they are either specified subjectively by the decision maker. So, to deal with imprecision data, fuzzy intervals may be defined where coefficients of the criteria are given by intervals, (Bitran G.R 1980). Many problems in science and engineering have been considered from optimization point of view. Here , the procedure is described as how to deal with decision problems that are described by fuzzy linear programming models and formulated with elements of imprecision and uncertainty. In this respect, it is necessary to study fuzzy linear programming models in which the parameters are partially known with some degree of precision. In real-world, decision-making processes in business; decision making theory has become one of the most important fields. It uses the optimization methodology connected with single criteria, but also satisfying concepts of multiple criteria. First attempt to model decision processes with multiple criteria in business lead to the concepts of goal programming (Ignizio, 1976). In this approach, the decision maker underpins each objective with a number of goals that should be satisfied (Lootsma, 1989). Satisfying requires finding a solution to multi criteria problem which is preferred, understood and implemented with confidence. The confidence that the best solution has been found is estimated through the "ideal solution". That is the solution which optimizes all criteria simultaneously. Since this is practically unattainable, a decision maker considers feasible solution closest to the ideal solution (Zeleny, 1982).

In goal programming the preferences required from the decision maker are presented with weights, targets, trade-offs and goal levels to formulate the athe sum of these weights equal to unity. Allowing these weights to vary within the range between 0 and 1 a decision maker performs the sensitivity analysis of all these weights simultaneously. The difficulty with the steuer 's weight technique is that in many situations a decision is unwilling to specify the weights(Lootsma, 1997). Also, the technique is time consuming demanding lot of computation.

Lootsma states that apart of wasting of decision make's time in solving a particular decision problem through the goal programming models, the issue is in a significant degree of decision maker's freedom to select his/her preferences. In order to eliminate a time consuming component the improvement was suggested to applying the weighted chebychev norm in a decision process (Benayoun et al, 1971; yu and zeleny, 1995; zeleny 1974; kok and lootsma, 1985). That is to minimize the distance between the objective function values and so-called ideal values. The technique applied still suffered from the influence of powerful individual in decision making processes through determination of eihnon-dominated solution where the objective function were , the deviation from the ideal values in the respective directions of optimization were inversely proportional to the corresponding weights (Osychka, 1984). In osyczka's opinion (1984) the multi criteria optimization models, being applicable for optimization of business activities, could be satisfactorily used bin a form of linear programming. The business activities objective functions can be based on weighting coefficients. Managers determined the weighting coefficients on the basis of their intuition. Weighting coefficients can be presented by a set of weights, which is normalized to sum to 1. Known techniques for comparing this set of weights are eigenvector and weighted least square method (Hwang and Yoon, 1983). The eigenvector technique is based upon a positive pair wise comparison matrix. Since the precise value of two weighting coefficients is hard to estimate, one can use the intelligent scale of importance for activities that are broken down into important ranks. A weighted least square method involves the solution of simultaneous linear equations. Since Bellman and Zadeh's paper in 1970, the maxmin and simple additive weighting methods using membership of the fuzzy set is used in explanation of business decision making problems (Bellman and Zadeh, 1970). Lai and Hwang see the application of fuzzy set theory in decision multi criteria problems as a replacement of oversimplified (crisp) models such as goal programming and ideal nadir vector model.

Fuzzy multi criteria models are robust and flexible. Decision makers consider the existing alternatives under given constraints, but also develop new alternatives by considering all possible solutions (Lai and Hwang, 1995). The transitional step forward to fuzzy multi criteria model is models that consider some fuzzy values. Some of these models are linear mathematical formulation of multiple objective decision making presented by mainly crisp and some fuzzy values. Many authors studied such models (Cheng et al, 1986; Lai and Hwang, 1993, Lai, 1995) offered the solution for the formulation by fuzzy linear programming. Lai (1995) interactive multi objective system technique contributed to the improvement of flexibility and robustness of multiple objective decision making methodology. Lai considered several characteristics cases, which a business decision maker may encounter in his/her practice. The cases could be defined as both non-fuzzy cases and fuzzy cases. These deal with notion relevant to fuzzy set theory.

III. BASIC DEFINITIONS

Definition 1.

The characteristics function μ_A of crisp set $A \leq X$ assigns a value either 0 or 1 to each member in X

Definition 2.

A fuzzy number (a, b, c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{x-c}{b-c} & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

Where $a, b, c \in IR$

D Definition 3.

A Ranking function is a function $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\overline{\mathrm{A}} = (a, b, c)$ be a triangular fuzzy number then $\Re(\overline{\mathrm{A}}) = \dfrac{a+2b+c}{4}$

Definition 4.

A classical set (or crisp set) is defined as a collection of elements $x \in X$. Each element can either belong or not belong to a set A, $A \subseteq X$. The membership of elements x to the subset A of X can be expressed by a characteristic function in which 1 indicates membership and 0 for non-membership.

ARITHMETIC OPERATIONS.
In this subsection, arithmetic operations between two triangular fuzzy numbers defined on universal set of real numbers $\mathbb{R}$ is presented.

Let $\overline{A}_1 = (a_1, b_1, c_1)$ and $\overline{A}_2 = (a_2, b_2, c_2)$ be two triangular fuzzy number then

(i) $\overline{A}_1 \oplus \overline{A}_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2)$

$= (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

(ii) $\overline{A}_1 = \overline{A}_2$ if and only if $a_1 = a_2, b_1 = b_2$ and $c_1 = c_2$.

In this paper, all the places $\sum_i^m x_i$ and $\sum_i^m \overline{x}_i$ represents the crisp and fuzzy addition respectively i.e.

$\sum_i^m x_i = x_1 + x_2 + x_3 + \cdots + x_m$ and $\sum_i^m \overline{x}_i = \overline{x}_1 \oplus \overline{x}_2 \oplus \overline{x}_3 \oplus \cdots \oplus \overline{x}_m$, where x_i and $\overline{x}_i$ are real and fuzzy numbers respectively.

IV. CONVERSION OF INEQUALITY CONSTRAINTS INTO EQUALITY CONSTRAINTS.

In this section, the method for conversion of inequality constrains into equality constraints are reviewed.

Let $\overline{A} = (a_1, b_1, c_1)$ and $\overline{B} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers, then

$\overline{A} \preceq B$ *iff* $a_1 \le a_2,\ b_1 - a_1 \le b_2 - a_2,\ c_1 - b_1 \le c_2 - b_2$

If $\overline{A} \preceq B$, then it can be converted into $\overline{A} \oplus \overline{C} = \overline{B}$ where C is non-negative triangular fuzzy number and the symbol $\preceq$.denotes the fuzzified version of $\le$. The decision maker accepts small constrain violations but attaches different degrees of importance to violations of constrains. Thus, the fuzzy constraints are defined by membership Functions.

LINEAR PROGRAMMING FORMULATION OF MAXIMAL FLOW PROBLEM IN CRISP AND FUZZY ENVIRONMENT.
In this section, linear programming formulations of maximal flow problems in crisp and fuzzy environment are presented.

LINEAR PROGRAMMING FORMULATION OF MAXIMAL FLOW PROBLEMS IN CRISP ENVIRONMENT.

Let us consider a directed graph $G = (v, E)$ with capacities upper bound u_{ij}, and associating to each arc (i, j) a value x_{ij} corresponding to the flow in the network from source node (say s) to destination node (say t). The maximal flow problems in crisp environment may be formulated as follows:

Maximize f

Subject to

$$\sum x_{ij} = \sum x_{ki} + f \ ; \quad i = s$$

$$\sum_i x_{ij} = \sum x_{ki} + f \ ; \quad \forall\ u \ne s, t$$

$$\sum_i x_{ij} + f = \sum x_{ki} \quad ; \quad i =, t$$

$$u \le x_{ij} \le u_{ij} \qquad \forall (i,\ j) \in E$$

LINEAR PROGRAMMING FORMULATIONS OF MAXIMAL FLOW PROBLEMS IN FUZZY ENVIRONMENT.

The linear programming formulation of maximal flow problems in fuzzy environment is presented. Here, the problem can be formulated as; Let us consider a directed graph $G = (v, E)$ with fuzzy capacities upper bound $\bar{u}_{ij}$, and association to each arc (i, j) a value $\bar{x}_{ij}$ corresponding to the fuzzy flow in this arc (i, j).

Let $\bar{f}$ represent the amount of fuzzy maximal flow in the network from source node s to destination node t. The fuzzy maximal flow problems in fuzzy environment may be formulated as follows;

Maximize $\bar{f}$

Subject to $\sum \bar{x}_{ij} = \sum \bar{x}_{ki} \oplus \bar{f} \; ; \quad i = s$

$$\sum_i \bar{x}_{ij} = \sum_k^k \bar{x}_{ki} \; ; \qquad \forall \; i \ne s, t$$

$$\sum_i \bar{x}_{ij} \oplus f = \sum \bar{x}_{ki} \quad ; \quad i = t$$

$x_{ij} \le \bar{u}_{ij} \qquad \forall (i,\ j) \in E$ x_{ij} Is a non-negative fuzzy number.

STEP 1: formulate the given fuzzy maximal flow problem into the following fuzzy linear programming problem.

Maximize $\bar{f}$

Subject to

$$\sum \bar{x}_{ij} = \sum \bar{x}_{ki} \oplus \bar{f} \; ; \quad i = s$$

$$\sum \bar{x}_{ij} = \sum_k \bar{x}_{ki} \; ; \qquad \forall \; i \ne s, t$$

$$\sum_i \bar{x}_{ij} \oplus \bar{f} = \sum \bar{x}_{ki} \quad ; \quad i = t$$

$$\bar{x}_{ij} \le \bar{u}_{ij} \qquad \forall (i,\ j) \in E$$

$\bar{x}_{ij}$ is a non-negative fuzzy number

STEP 2: let all the parameters $\bar{x}_{ij}\,\bar{f}$ and $\bar{u}_{ij}$ be represented by non-negative triangular fuzzy number (a_{ij}, b_{ij}, c_{ij}), $(f_1,\ f_2,\ f_3)$ and (u_{ij}, v_{ij}, w_{ij}) respectively. Then the fuzzy linear programming formulation of fuzzy maximal flow problem obtained in step 1 may be written as:

Maximize $(f_1,\ f_2,\ f_3)$

Subject to

$$\sum (a_{ij}, b_{ij}, c_{ij}) = \sum (a_{kj}, b_{kj}, c_{kj}) \oplus \ (f_1,\ f_2,\ f_3), \ i = s$$

$$\sum_{i}^{i}\left(a_{ij},b_{ij},c_{ij}\right)=\sum_{k}\left(a_{kj},b_{kj},c_{kj}\right); \quad \forall \;\; i\neq s,t$$

$$\sum_{i}\left(a_{ij},b_{ij},c_{ij}\right)\oplus\left(f_1,\; f_2,\; f_3\right)=\sum_{k}\left(a_{ki},b_{ki},c_{ki}\right); \quad i=t$$

$$\left(a_{ij},b_{ij},c_{ij}\right)\preceq\left(u_{ij},v_{ij},w_{ij}\right) \;\forall\left(i,\; j\right)\in E$$

STEP3; converting the inequality constraints into equality constraints by introducing non-negative variable. $\bar{s}_{ij}=\left(s'_{ij},s''_{ij},s'''_{ij}\right)\;\forall\;\left(i,j\right)\in E$. The fuzzy linear programming problem, obtained in step 2, can be written as:

Maximize $\left(f_1,\; f_2,\; f_3\right)$

Subject to

$$\sum_{i}\left(a_{ij},b_{ij},c_{ij}\right)=\sum_{k}\left(a_{ki},b_{ki},c_{ki}\right); \;\; \oplus\left(f_1,\; f_2,\; f_3\right); \;\; i=s$$

$$\sum_{i}\left(a_{ij},b_{ij},c_{ij}\right)=\sum_{k}\left(a_{ki},b_{ki},c_{ki}\right) \qquad ; \;\; \forall \;\; i\neq s,t$$

$$\sum_{i}\left(a_{ij},b_{ij},c_{ij}\right)\oplus\left(f_1,\; f_2,\; f_3\right)=\sum_{k}\left(a_{ki},b_{ki},c_{ki}\right) \quad ; \;\; i=t$$

$$\left(a_{ij},b_{ij},c_{ij}\right)\oplus\left(s'_{ij},s''_{ij},s'''_{ij}\right)=\left(u_{ij},v_{ij},w_{ij}\right)\;\forall\;\left(i,j\right)\in E$$

STEP 4: Using Ranking formula and Arithmetic Operations presented in section 3. 1. 3 and 3.2 respectively. The fuzzy linear programming problem, obtained in step 3 is converted into the following crisp linear programming problem.

Maximize $\left(\dfrac{f_1+2f_2+f_3}{4}\right)$

Subject to

$$\left.\begin{aligned}
&\sum a_{ij}=\sum_{k}a_{ki}+f_1;\; i=s\\
&\sum a_{ij}=\sum_{k}a_{ki};\;\forall\;\; i\neq s,t\\
&\sum a_{ij}+f_1=\sum_{k}a_{ki};\;\; i=t
\end{aligned}\right\}\cdots(1)$$

$$\left.\begin{aligned}
&\sum_{i}b_{ij}=\sum_{k}b_{ki}+f_2;\; i=s\\
&\sum b_{ij}=\sum_{k}b_{ki};\quad i\neq s,t\\
&\sum b_{ij}+f_2=\sum_{k}b_{ki};\;\; i=t
\end{aligned}\right\}\cdots(2)$$

$$\left.\begin{array}{l}\sum_i c_{i\,j} = \sum_k c_{k\,i} + f_3\,;\; i = s \\ \sum_i c_{i\,j} = \sum_k c_{k\,i}\,;\; \forall\; i \neq s,t \\ \sum_i c_{i\,j} + f_3 = \sum_k c_{k\,i}\,;\; i = t\end{array}\right\}\cdots(3)$$

$$a_{i\,j} + s'_{i\,j} = u_{i\,j}$$
$$b_{i\,j} + s''_{i\,j} = v_{i\,j}$$
$$c_{i\,j} + s'''_{i\,j} = w_{i\,j}$$

$$b_{i\,j} - a_{i\,j} \geq 0,\; c_{i\,j} - b_{i\,j} \geq 0,\, a_{i\,j} \geq 0,\, b_{i\,j} \geq 0,\, c_{i\,j} \geq 0$$
$$s''_{i\,j} - s'_{i\,j} \geq 0, s'''_{i\,j} - s''_{i\,j} \geq 0,\, s'_{i\,j} \geq 0,\, s''_{i\,j} \geq 0,\, s''''_{i\,j} \geq 0$$
$$f_2 - f_1 \geq 0,\;\; f_3 - f_2 \geq 0,\; f_1 \geq 0,\; f_2 \geq 0,\; f_3 \geq 0, \forall (i,\, j) \in E$$

STEP 5: Find the optimal flow $f_1,\, f_2$ and f_3 by solving the crisp linear programming problem obtained in step 4 of the algorithm.

STEP 6: Find the fuzzy maximal flowby putting the values of $f_1,\, f_2$ and f_3 in $\overline{f} = (f_1, f_2, f_3)$..

METHOD OF SOLVING FUZZY LINEAR PROGRAMMING.

In this section, the fuzzy linear programming method is used to find the fuzzy optimal solution of fuzzy maximal flow problems.

ILLUSTRATIVE EXAMPLE ONE.

In this section, the fuzzy linear programming method is illustrated by the following examples.

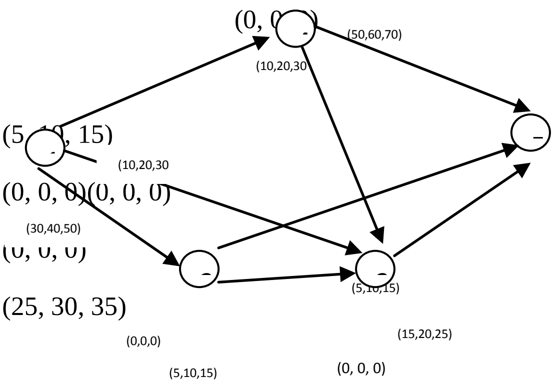

Fig 1. fuzzy value of flow a long each arc.

The problem is to find out the fuzzy maximal flow between node 1(say source node) and node 5(say destination node) on the network shown in fig. 1

__Solution:__the fuzzy maximal flow between node 1 and node 5 can be obtained by using the following steps:

STEP 1: let all the parameters $\bar{x}_{ij}$, $\bar{f}$ and $\bar{u}_{ij}$ be represented by non-negative triangular fuzzy numbers (a_{ij}, b_{ij}, c_{ij}), (f_1, f_2, f_3) and (u_{ij}, v_{ij}, w_{ij}) respectively, then the fuzzy linear programming formulation of fuzzy maximal flow problem, may be written as:

Maximize $\bar{f}$

Subject to $\bar{x}_{12} \oplus \bar{x}_{13} \oplus \bar{x}_{13} = \bar{f}$

$\bar{x}_{23} \oplus \bar{x}_{25} = \bar{x}_{12}$

$\bar{x}_{34} \oplus \bar{x}_{35} = \bar{x}_{13} \oplus \bar{x}_{23}$

$\bar{x}_{45} = \bar{x}_{14} \oplus \bar{x}_{34}$

$\bar{x}_{25} \oplus \bar{x}_{35} \oplus \bar{x}_{45} = \bar{f}$

$\bar{x}_{ij} \preceq \bar{u}_{ij} \ \forall (i, j) \in E$

$\bar{x}_{ij}$ is a non-negative fuzzy number

$$\bar{x}_{12} \preceq (20, 40, 50),\ \bar{x}_{13} \preceq (10, 20, 30),\ \bar{x}_{14} \preceq (5, 10, 15)$$
$$\bar{x}_{23} \preceq (5, 10, 15),\ \bar{x}_{25} \preceq (25, 30, 35),\ \bar{x}_{34} \preceq (5, 10, 15),$$
$$\bar{x}_{35} \preceq (15, 20, 25),\ \bar{x}_{45} \preceq (50, 60, 70).$$

STEP 2: converting the inequality constraints into equality constraint by introducing nonnegative variable $\bar{s}_{ij} = (s'_{ij}, s''_{ij}, s'''_{ij})\ \forall (i, j) \in E$. The fuzzy linear programming problem obtained in step 1may be written as:

Maximize (f_1, f_2, f_3)

Subject to:

$$(a_{12}, b_{12}, c_{12}) \oplus (s'_{12}, s''_{12}, s'''_{12}) = (30, 40, 50)$$
$$(a_{13}, b_{13}, c_{13}) \oplus (s'_{13}, s''_{13}, s'''_{13}) = (10, 20, 30)$$
$$(a_{14}, b_{14}, c_{14}) \oplus (s'_{14}, s''_{14}, s'''_{14}) = (5, 10, 15)$$
$$(a_{23}, b_{23}, c_{23}) \oplus (s'_{23}, s''_{23}, s'''_{23}) = (5, 10, 15)$$
$$(a_{25}, b_{25}, c_{25}) \oplus (s'_{25}, s''_{25}, s'''_{25}) = (25, 30, 35)$$
$$(a_{34}, b_{34}, c_{34}) \oplus (s'_{34}, s''_{34}, s'''_{34}) = (5, 10, 15)$$
$$(a_{35}, b_{35}, c_{35}) \oplus (s'_{35}, s''_{35}, s'''_{35}) = (15, 20, 25)$$
$$(a_{45}, b_{45}, c_{45}) \oplus (s'_{45}, s''_{45}, s'''_{45}) = (50, 60, 75)$$

STEP 3 the fuzzy linear programming problem is converted into the following crisp linear programming problem.

$$a_{12} + s'_{12} = 30, \quad b_{12} + s''_{12} = 40, \quad c_{12} + s'''_{12} = 50$$
$$a_{14} + s'_{14} = 5, \quad b_{14} + s''_{14} = 10, \quad c_{14} + s'''_{14} = 15$$
$$a_{23} + s'_{23} = 5, \quad b_{23} + s''_{23} = 10, \quad c_{23} + s'''_{23} = 15$$
$$a_{25} + s'_{25} = 25, \quad b_{25} + s''_{25} = 30, \quad c_{25} + s'''_{25} = 35$$
$$a_{34} + s'_{34} = 5, \quad b_{34} + s''_{34} = 10, \quad c_{34} + s'''_{34} = 15$$

$$a_{35}+s_{35}^{'}=15,\qquad b_{35}+s_{35}^{''}=20,\qquad c_{35}+s_{35}^{'''}=25$$

$$a_{45}+s_{45}^{'}=50,\qquad b_{45}+s_{45}^{''}=60,\qquad c_{45}+s_{45}^{'''}=70$$

Now, using the condition in step 4 of the algorithm.

$$a_{ij}+s_{ij}^{'}=u_{ij}$$

$$b_{ij}+s_{ij}^{''}=v_{ij}$$

$$c_{ij}+s_{ij}^{'''}=w_{ij}$$

We know that

$$a_{12}+a_{13}+a_{14}-f_1=0\Rightarrow f_1=a_{12}+a_{13}+a_{14}$$

$$b_{12}+b_{13}+b_{14}-f_2=0\Rightarrow f_2=b_{12}+b_{13}+b_{14}$$

$$c_{12}+c_{13}+c_{14}-f_3=0\Rightarrow f_3=b_{12}+b_{13}+b_{14}$$

$$u_{12}=30,\quad u_{13}=10,\quad u_{14}=5.$$

$$v_{12}=40,\quad v_{13}=20,\quad v_{14}=10.$$

$$w_{12}=50,\quad w_{13}=30,\quad w_{14}=15.$$

$$u_{12}+u_{13}+u_{14}=f_1\Rightarrow 30+10+5=f_1=45$$

$$v_{12}+v_{13}+v_{14}=f_2\Rightarrow 40+20+10=f_2=70$$

$$w_{12}+w_{13}+w_{14}=f_3\Rightarrow 50+30+15=f_3=95$$

Putting the value of $f_1=45,\quad f_2=70,$ and $f_3=95$ in $\overline{f}=\left(f_1,\ f_2,\ f_3\right)$, the fuzzy maximal flow is $\overline{f}=\left(45,\ 70,\ 95\right)$. .

4.2 Illustrative Example Two.

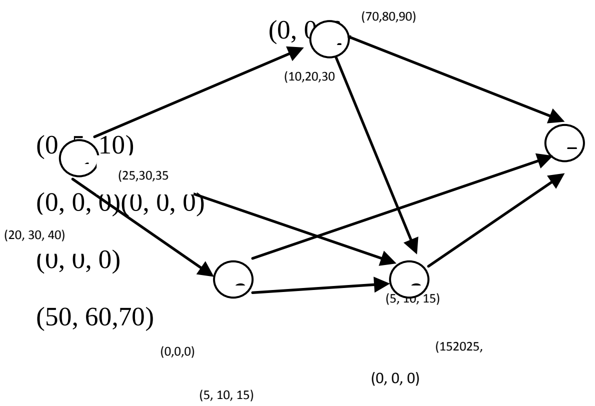

Fig 2. Fuzzy value of flow a long each arc

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$x_{12}=\left(20,30,40\right),\quad x_{13}=\left(25,\ 30,\ 35\right),\quad x_{14}=\left(0,\ 5,\ 10\right)$

$x_{23}=\left(5,\ 19,\ 15\right),\quad x_{25}=\left(50,\ 60,\ 70\right),\quad x_{34}=\left(5,\ 10,\ 15\right)\ x_{35}=\left(15,\ 20,\ 25\right),\quad x_{45}=\left(70,\ 80,\ 90\right)$

From step 3 of the algorithm, we have

$$\left(a_{12},\ b_{12},\ c_{12}\right)\oplus\left(s_{12}^{'},\ s_{12}^{''},\ s_{12}^{'''}\right)=\left(20,\ 30,\ 40\right)$$
$$\left(a_{13},\ b_{13},\ c_{13}\right)\oplus\left(s_{13}^{'},\ s_{13}^{''},\ s_{13}^{'''}\right)=\left(25,\ 30,\ 35\right)$$
$$\left(a_{14},\ b_{14},\ c_{14}\right)\oplus\left(s_{14}^{'},s_{14}^{''},s_{14}^{'''}\right)=\left(0,\ 5,\ 10\right)$$
$$\left(a_{23},\ b_{23},\ c_{23}\right)\oplus\left(s_{23}^{'},s_{23}^{''},s_{23}^{'''}\right)=\left(5,\ 10,15\right)$$
$$\left(a_{25},\ b_{25},\ c_{25}\right)\oplus\left(s_{25}^{'},s_{25}^{''},s_{25}^{'''}\right)=\left(50,\ 60,70\right)$$
$$\left(a_{34},\ b_{34},\ c_{34}\right)\oplus\left(s_{34}^{'},s_{34}^{''},s_{34}^{'''}\right)=\left(5,\ 10,\ 15\right)$$
$$\left(a_{35},\ b_{35},\ c_{35}\right)\oplus\left(s_{35}^{'},s_{35}^{''},s_{35}^{'''}\right)=\left(15,\ 20,\ 25\right)$$
$$\left(a_{45},\ b_{45},\ c_{45}\right)\oplus\left(s_{45}^{'},s_{45}^{''},s_{45}^{'''}\right)=\left(70,\ 80,\ 90\right)$$

$a_{12}+s_{12}^{'}=20.$ $\quad a_{13}+s_{13}^{'}=25$ $\quad a_{14}+s_{14}^{'}=0$
$b_{12}+s_{12}^{''}=30$ $\quad b_{13}+s_{13}^{''}=30$ $\quad b_{14}+s_{14}^{''}=5$
$c_{12}+s_{12}^{'''}=40$ $\quad c_{13}+s_{13}^{'''}=35$ $\quad c_{14}+s_{14}^{'''}=10$

$a_{23}+s_{23}^{'}=5.$ $\quad a_{25}+s_{25}^{'}=25$ $\quad a_{34}+s_{34}^{'}=5$
$b_{23}+s_{23}^{''}=10$ $\quad b_{35}+s_{35}^{''}=60$ $\quad b_{34}+s_{34}^{''}=10$
$c_{23}+s_{23}^{'''}=15$ $\quad c_{35}+s_{35}^{'''}=70$ $\quad c_{34}+s_{34}^{'''}=15$

$a_{35}+s_{35}^{'}=15.$ $\quad a_{45}+s_{45}^{'}=70$
$b_{35}+s_{35}^{''}=20$ $\quad b_{45}+s_{45}^{''}=80$
$c_{35}+s_{35}^{'''}=25$ $\quad c_{45}+s_{45}^{'''}=90$

We know that

$a_{12}+a_{13}+a_{14}=f_1,\quad b_{12}+b_{13}+b_{14}=f_2,$
$c_{12}+c_{13}+c_{14}=f_3$
$u_{12}=20,\quad u_{13}=25,\quad u_{14}=0.$
$v_{12}=30,\quad v_{13}=30,\quad v_{14}=5.$
$w_{12}=40,\quad w_{13}=35,\quad w_{14}=10.$
$f_1=u_{12}+u_{13}+u_{14}$
$f_1=20\ +\ 25+\ 0\ =45.$

$f_2=v_{12}+v_{13}+v_{14}$
$f_2=30+\ 30+5=65.$

$f_3=w_{12}+w_{13}+w_{14}$
$f_3=40\ +\ 35\ +\ 10=85.$
$\left(f_1,\ f_2,\ f_3\right)=\left(45,\ 65,\ 85\right)$

V. RESULTS AND DISCUSSION.

The obtained result in example 4.1 can be explained as follows:

(i) The amount of flow between the source and the sink is greater than 45 and less than 95 units.
(ii) The maximum numbers of persons are in favor of the amount of flow will be 70 units.
(iii) The percentage of favors for the remaining flow can be obtained as follows:

Let x represent the amount of flow, the percentage of favourness for $x = \mu_{\bar{f}}(x) \times 100$

Where $$\mu_{\bar{f}}(x) = \begin{cases} \dfrac{(x-45)}{40}, & 45 \le x \le 70 \\ \dfrac{(x-95)}{45}, & 70 \le x \le 95 \\ 0 & \text{otherwise} \end{cases}$$

The obtained result for example 4.2 can be explained as follows:

(i) The amount of flow between the source and is greater than 85.
(ii) maximum number of person are in favor of the amount of flow will be 65 unit
(iii) the percentage of favors for the remaining flow can be obtained as follows:
(iv) let x represent the amount of flow, the percentage of favourness

Where $$\mu_{\bar{f}}(x) = \begin{cases} \dfrac{(x-45)}{40}, & 45 \le x \le 65 \\ \dfrac{(x-85)}{45}, & 65 \le x \le 85 \\ 0 & \text{otherwise} \end{cases}$$

VII. CONCLUSION

The method of fuzzy linear programming based on fuzzy linear programming formulation of maximal flow problems is studied. To demonstrate the method of fuzzy linear programming, numerical examples are solved and the obtained results show that the method is good over other methods because the ranking function helps us to check our solution. Using the method of fuzzy linear programming, the fuzzy optimal solution of maximal flow problems occurring in real life situations can be easily obtained.

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