The International Journal Of Engineering And Science (IJES)
|| Volume || 4 || Issue || 11 || Pages || PP -55-61|| 2015 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

Buckling and Postbuckling Loads Characteristics of All Edges Clamped Thin Rectangular Plate

Oguaghamba O. A.[1], Ezeh J. C.[2], Ibearugbulem M. O.[3], and Ettu L. O.[4]
[1]*Civil Engineering Department, Imo State University, Owerri, Imo State, Nigeria*
[2,3,4]*Civil Engineering Department, Federal University of Technology, Owerri, Imo State, Nigeria*

ABSTRACT
Previous studies on the buckling and postbuckling loads characteristics of thin rectangular plates that are subjected to uniaxial uniformly distributed in-plane loads were limited to all edges simply supported (SSSS) plate. Those studies were carried out using assumed displacement and stress profiles in the form of double trigonometric functions, never minding their inadequacies. Hence, major associated parameters: displacement parameter, Wuv, stress coefficient, Wuv^2 and load factor, K_{cx} for such plate could not be determined. No study has considered the buckling and postbuckling loads characteristics of thin rectangular plate having all the four edges clamped (CCCC). This paper obtained the exact displacement and stress profiles of the buckling and postbuckling characteristics of thin rectangular CCCC plates by applying the direct integration theory to the Kirchhoff's linear governing differential equation and von Karman's non–linear governing differential compatibility equation respectively. With these exact profiles, the buckling and postbuckling load expression of the CCCC plate was obtained by applying work principle to the Von Karman's non–linear governing differential equilibrium equation. Yield/maximum stress of the plate and those major related parameters were determined. Results of this present study show that for a CCCC plate material having yield stress of 250MPa, failure would occur at 0.0478h postbuckling out of plane deflection, contrary to the presumed critical buckling load. Hence, CCCC accommodates additional load beyond critical buckling load.

Keywords: *Buckling, Coupled Equations, Direct Integration, Postbuckling, Work Principle, Yield stress*

Date of Submission: 09 November 2015 Date of Accepted: 25 November 2015

I. Introduction

Postbuckling of plates may readily be understood through an analogy to a simple grillage model, as shown in Fig. 1. In the grillage model, the continuous plate is replaced by vertical columns and horizontal ties. Under loading on the x – edges, the vertical columns will buckle. If they were not connected to the ties, they would buckle at the same load and no postbuckling reserve would exist. However, the ties are stretched as the columns buckle outward, thus restraining the motion and providing postbuckling reserve. The columns nearer to the supported edge are restrained more by the ties than those in the middle. This occurs too in a real plate, as more of the longitudinal in-plane compression is carried nearer the edges of the plate than in the center. Thus, the grillage model provides a working analogy for both the source of the postbuckling reserve and its most important result; i.e., re-distribution of longitudinal stresses.

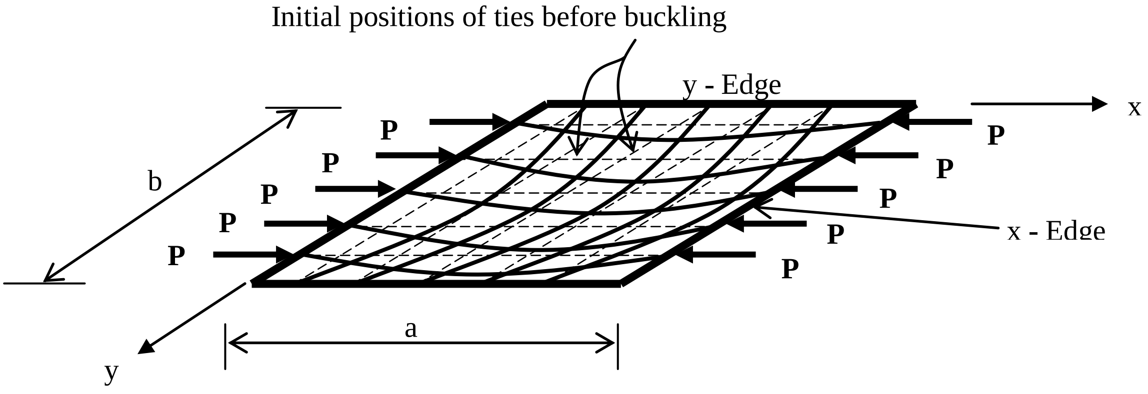

Figure 1. Post-buckling model of a thin plate under in-plane loads

In his context, Chaje [1] defined postbuckling load as the increase in stiffness with increase in deflection characteristic of the plate. This represents possible resistance of axial load by plate at excess of the critical load subsequent to buckling. Hence, the postbuckling response of thin elastic plates is very important in engineering analysis. Therefore, concerted effort to thoroughly studying thin plates postbuckling behaviour becomes imminent.

Postbuckling load analysis of thin plates accounts for the membrane stretching and their corresponding strains and stresses, while buckling analysis accounts also for the membrane stretching but do not consider the corresponding strains and stresses developed by the stretching. Postbuckling load analysis of plate involves nonlinear large-deflection plate bending theory, contrary to buckling load study which is based on classical or Kirchhoff's linear theory of plates. Researchers have not done much on postbuckling behaviour of thin plates as its analysis involves nonlinear large-deflection plate theory, which usually reduces to two indeterminate nonlinear governing differential equations originally derived by Von Karman in 1910 [2, 3]. These equations are written as follows:

$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \quad (1)$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{h}{D}\left[\frac{\partial^2 \phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 \phi}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right] \quad (2)$$

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] = N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{dxdy} + N_y\frac{\partial^2 w}{\partial y^2} \quad (3)$$

where, ϕ is the stress function, w is deflection function, h is the plate's thickness and D is flexural rigidity.

Equation 1 is the "Compatibility Equation". It ensures that in an elastic plate the in-plane and out-of-plane displacements are compatible. Equations 2 and 3 are based on equilibrium principles of stress and in-plane loads respectively. They are termed "Equilibrium equations" [2, 3]. Equations 1 and 2 are usually called Von Karman's coupled equations.

The exact solutions of these equations have been a rigor from the conceptual time to the recent time, in which the coupled solutions would give the buckling/postbuckling load of plates from which the true failure load is determined. This exact solutions of these equation is imminent, as the critical load predicted by buckling analysis is adjudged unsatisfactory [1, 4].

Despite these revelations, very few researchers have made effort to solving these coupled equations to obtain the expressions for the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under compression. Researchers such as: Von Karman*et. al* [5], Marguerre [6], Levy [7], Timoshenko and Woinowsky – Krieger [8], Volmir [9], Iyengar [10], Ventsel and Krauthammer [11], Chai [12]; and Yoo and Lee [1] have tried to solve these equations to obtain the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under uniaxial compression. They tried to solve the problem by assuming double trigonometric solutions for deflection, w and stress,ϕ functions to solve the governing differential equations of thin rectangular plates. In which case, the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under compression they obtained would also be said to be assumed, as the solutions of the governing differential equations of the plate (deflection and stress functions) were assumed abinitio. No researcher has bothered to solve for these parameters by the direct solution of these coupled governing differential equations.

In addition, these researchers restricted themselves to the use of either direct variational or indirect variational energy methods to finally evaluate the buckling/postbuckling load of this simply supported edges thin rectangular plate. None of the researchers considered applying direct work principle to finally evaluate the buckling/postbuckling loads of the CCCC plate or any other plate.

Von Karman evaluated the final buckling/postbuckling loads characteristics of SSSS plate by solving the equilibrium equation 3, after assuming trigonometric functions for deflection and stress. Marguerre [6], Timoshenko and Woinowsky – Krieger [8] and Volmir [9] also assumed doubled trigonometric functions of deflection and stress; and employed the principle of minimum potential energy, rather than the equilibrium equation to furnish the final solution for the same SSSS plate. Iyengar [10], Ventsel and Krauthammer [11] and Yoo and Lee [13] also assumed doubled trigonometric functions of deflection and stress used Galerkin's energy methods to obtain the final buckling/postbuckling load of SSSS plate.

Researchers in later years very often assumed doubled trigonometric functions of deflection and stress and used a similar type of approach, i.e., combining an exact solution of the compatibility equation with either evaluation and minimization of the potential energy, or an approximate solution (for example, using Galerkin's method, Ritz method or Rayleigh-Ritz method) of the equilibrium equation.

In all these, none of these researchers obtained the displacement parameter, W_{uv}, stress coefficient, W_{uv}^2 and load factor, K_{cx} associated with CCCC plate buckling and postbuckling characteristics, or any other plate. This situation has been the bane of comprehensive solution of the buckling/postbuckling characteristics of plates, as the actual yield/maximum stress of the plate could not be obtained, which this paper addressed.

II. The Direct Integration Approach for Exact General Deflection and Stress Profile for Buckling and Postbuckling of CCCC Plate

Oguaghamba [14] used direct integral calculus approach and evaluated equation 3 to obtain the exact general displacement function of a buckled plate. The deflection function, W in its non – dimensional coordinates: R and Q is given as:

$$W(R,Q) = \Lambda \sum_{m=0}^{4} \sum_{n=0}^{4} U_m R^m V_n Q^n \tag{4}$$

wherenon – dimensional coordinates:R and Q in equation 4 relates to the usual independent coordinates x and y by the relation:

$$x = aR: \;\; 0 \le R \le 1 \; and \; y = bQ: \;\; 0 \le Q \le 1 \tag{5}$$

$\mathbf{U_m}$ **and** $\mathbf{V_n}$ arecoefficients to be determined.

(6)

Solving equation 1 by direct integral calculus approach, the stress distribution of the plate prior to buckling to obtain the exact general stress function, $\phi(x,y)$ of buckling and postbuckling load of plate is obtained [14]. The expression in non-dimensional coordinates, R and Q is given as:

$$\phi(R,Q) = \frac{Ep^2\Lambda^2}{(1+2p^2+p^4)}\Bigg[\Bigg[\left(\frac{U_1^{\,2}}{24}R^4 + \frac{U_1U_2}{30}R^5 + \frac{1}{180}[2U_2^{\,2} + 3U_1U_3]R^6 + \frac{1}{210}[2U_1U_4 + 3U_2U_3]R^7 + \right.$$

$$\left.\frac{1}{1680}[16U_2U_4 + 9U_3^{\,2}]R^8 + \frac{U_3U_4}{126}R^9 + \frac{U_4^{\,2}}{315}R^{10}\right) \times \left(\frac{V_1^{\,2}}{24}Q^4 + \frac{V_1V_2}{30}Q^5 + \frac{1}{180}[2V_2^{\,2} + 3V_1V_3]Q^6 + \right.$$

$$\left.\frac{1}{210}[2V_1V_4 + 3V_2V_3Q^3]Q^7 + \frac{1}{1680}[16V_2V_4 + 9V_3^{\,2}]Q^8 + \frac{V_3V_4}{126}Q^9 + \frac{V_4^{\,2}}{315}Q^{10}\right)$$

$$-\left(\frac{U_0U_2}{12}R^4 + \frac{1}{60}[3U_0U_3 + U_1U_2]R^5 + \right.$$

$$\frac{1}{180}[6U_0U_4 + 3U_1U_3 + U_2^{\,2}]R^6 + \frac{1}{210}[3U_1U_4 + 2U_2U_3]R^7 + \frac{1}{840}[3U_3^{\,2} + 7U_2U_4]R^8$$

$$\left. + \frac{U_3U_4}{168}R^9 + \frac{U_4^{\,2}}{420}R^{10}\right)\left(\frac{V_0V_2}{12}Q^4 + \right.$$

$$\frac{1}{60}[3V_0V_3 + V_1V_2]Q^5 + \frac{1}{180}[6V_0V_4 + 3V_1V_3 + V_2^{\,2}]Q^6 + \frac{1}{210}[3V_1V_4 + 2V_2V_3]Q^7 + \frac{1}{840}[3V_3^{\,2} + 7V_2V_4]Q^8$$

$$+ \frac{V_3V_4}{168}Q^9$$

$$+\frac{V_4^{\,2}}{420}Q^{10}\Big)\Big]$$
$$-\frac{N_{cx}b^2}{2h}Q^2 \qquad (7)$$

U_m and V_n coefficients in equation 4 and were determined by Oguaghamba [14] using the Benthem's boundary conditions of CCCC plate as follows:

$U_o = 0;\ U_1 = 0;\ U_2 = U_4; U_3 = -2U_4;\ U_4 = U_4;\ V_o = 0;\ V_1 = 0;\ V_2 = V_4; V_3 = -2V_4;\ V_4 = V_4$

Hence, the CCCC plate displacement and stress profiles in buckling and postbuckling regimes are obtained by substituting these coefficients into equations 4 and 7 as:

$$W(R,Q) = W_{uv}(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

$$W(R,Q) = W_{uv}h_1(R,Q) \quad (8)$$

where, $W_{uv} = \Lambda U_4 V_4$

$$h_1(R,Q) = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \qquad (9)$$

$$\phi(R,Q) = \frac{\varphi W_{uv}^2}{6350400}[4(14R^6 - 36R^7 + 39R^8 - 20R^9 + 4R^{10})(14Q^6 - 36Q^7 + 39Q^8 - 20Q^9 + 4Q^{10}) - (14R^6 - 48R^7 + 57R^8 - 30R^9 + 6R^{10})(14Q^6 - 48Q^7 + 57Q^8 - 30Q^9 + 6Q^{10})] - \frac{N_{cx}b^2}{2h}Q^2 \qquad (10)$$

$$\phi(R,Q) = \varphi W_{uv}^2 h_2(R,Q) - \frac{N_{cx}b^2}{2h}Q^2 \qquad (11)$$

where,

$$\varphi = \frac{Ep^2}{(1 + 2p^2 + p^4)} \qquad (12)$$

$W_{uv}^2 =$ Stress function coefficient for a plate in postbuckling regime

$\Lambda^2 =$ Consolidated coefficient factor of stress in postbuckling regime

$h_2(R,Q) =$ Non-dimensional stress shape (profile) function of the slightly bent plate, given as:

$$h_2(R,Q) = \frac{1}{6350400}[4(14R^6 - 36R^7 + 39R^8 - 20R^9 + 4R^{10})(14Q^6 - 36Q^7 + 39Q^8 - 20Q^9 + 4Q^{10}) - (14R^6 - 48R^7 + 57R^8 - 30R^9 + 6R^{10})(14Q^6 - 48Q^7 57Q^8 - 30Q^9 + 6Q^{10})] \qquad (13)$$

Expressions for the deflection and stress functions factors, W_{uv} *and* W_{uv}^2 of the plate behaviour under pre – buckling, buckling and post buckling regimes deduced by Oguaghamba [14] is given as:

$$W_{uv} = 256\alpha h;\ W_{uv}^2 = W_{uv}^2 = 65536\,\alpha^2 h^2 \qquad (14)$$

III. Work Principle Application for Buckling and Postbuckling Load and Stress of CCCC Plate

Oguaghamba [14] applied the work principle according to Ibearugbulem*et al.* [15, 16] to equation 2 in non – dimensional coefficient and obtained the exact general buckling and postbuckling load, N_{cx} (R, Q) of thin rectangular plates in non – dimensional coordinates as in equation 15.

$$N_{cx} = -\frac{\frac{49}{484}\int\int_{0,0}^{1,1}\left(\frac{\partial^4 h_1}{p^2\partial R^4}\cdot h_1 + 2\frac{\partial^4 h_1}{\partial R^2\partial Q^2}\cdot h_1 + p^2\frac{\partial^4 h_1}{\partial Q^4}\cdot h_1\right)dRdQ}{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial R^2}\cdot h_1\right)dRdQ}\frac{\pi^2 D}{b^2} + \frac{\frac{294}{121}(1-\mu^2)p^2 W_{uv}^2}{(1+2p^2+p^4)h^2}\frac{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial Q^2}\frac{\partial^2 h_2}{\partial R^2} + \frac{\partial^2 h_1}{\partial R^2}\cdot\frac{\partial^2 h_2}{\partial Q^2} - 2\frac{\partial^2 h_1}{\partial R\partial Q}\frac{\partial^2 h_2}{\partial R\partial Q}\right)\cdot h_1 dRdQ}{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial R^2}\cdot h_1\right)dRdQ}\frac{\pi^2 D}{b^2} \qquad (15)$$

where the first and the second terms account for critical buckling load of the plate and the gain in load of the plate at postbuckling regime respectively.
Substituting the expressions of $h_1(R,Q)$ and $h_2(R,Q)$ into equation 15; solving out the resulting integrand expressions gave the buckling and postbuckling load expression for a CCCC thin rectangular plate as:

$$N_{cx} = \left[\left(\frac{4.25206612}{P^2} + 2.42975207 + 4.25206612P^2\right) + 1.18730631 \times 10^{-6}\frac{(1-\mu^2)p^2 W_{uv}^2}{(1+2p^2+p^4)h^2}\right]\frac{D\pi^2}{b^2} \quad (16)$$

Introducing the expression of W_{uv}^2 given in equation 14 into equation 16; the buckling and postbuckling load expression for a CCCC thin rectangular plate reduced to:

$$N_{cx} = \left[\left(\frac{4.25206612}{P^2} + 2.42975207 + 4.25206612P^2\right) + 7.08082886 \times 10^{-2}\frac{p^2\alpha^2}{(1+2p^2+p^4)}\right]\frac{D\pi^2}{b^2} \quad (17)$$

$$N_{cx} = K_{cx}\frac{D\pi^2}{b^2} \quad (18)$$

$$K_{cx} = \left(\frac{4.25206612}{P^2} + 2.42975207 + 4.25206612P^2\right) + 7.08082886 \times 10^{-2}\frac{p^2\alpha^2}{(1+2p^2+p^4)} \quad (19)$$

where, K_{cx} is the buckling and postbuckling load coefficient.
Oguaghamba [14] also evaluated the inplane and bending buckling and postbuckling yield stress developed by the CCCC as:

$$\sigma_{cri} = \left[\left(\frac{4.25206612}{P^2} + 2.42975207 + 4.25206612P^2\right) + 7.08082886 \times 10^{-2}\frac{p^2\alpha^2}{(1+2p^2+p^4)}\right]\frac{D}{hb^2}\pi^2 + 96\left[\frac{1}{p^2} + 0.3\right]\frac{D}{hb^2}\alpha \quad (20)$$

IV. Results and Discussions

Fig. 2 shows a CCCC thin rectangular plate subjected to uniaxial compression loads on the R - edges. The interest is to evaluate the buckling and postbuckling load of the plate.

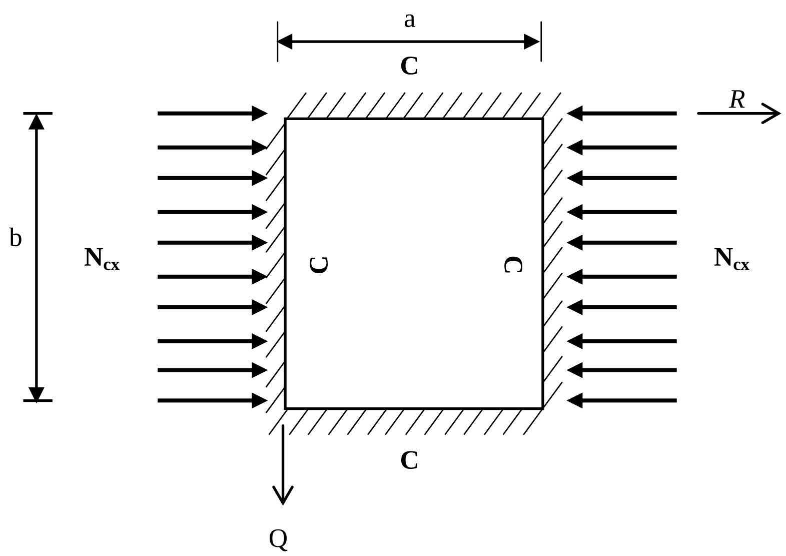

Figure 2: CCCC – Thin Rectangular Plate under Uniaxial Load

Iyengar [10]; Ventsel and Krauthammer [11]; Szilard [4]; and Yoo and Lee [13] in their separate works obtained the buckling and postbuckling load of SSSS – thin rectangular plate only as:

$$\mathbf{N_{cx}} = \left[\left(\frac{1}{P^2} + 2 + P^2\right) + 3\left(\frac{1}{p^2} + p^2\right)\frac{W_{11}^2}{h^2}\frac{(1-\mu^2)}{4}\right]\frac{D\pi^2}{b^2} \qquad (21)$$

The stress function coefficient, W_{uv}^2 in the present study is well defined in equation 14 for CCCC plate. This is not the case for the stress function coefficient, W_{11}^2 in the literature formulation for SSSS plate, as given in equation 21; while that for CCCC plate has not been studied. Hence, the stress function coefficient, W_{11}^2 in the literature formulation has no empirical interpretation. This leaves the literature formulation as a mere theoretical exercise rather than real life adventure. The present study clearly defined these parameters: the displacement parameter, W_{uv}, stress coefficient, $W_{uv}{}^2$ and load factor, K_{cx}. With this parameters, the present study obtained critical yield stress of the CCCC plate under buckling and postbuckling loads as given in equation 20. Therefore, equations 19 and 20 can be used to obtain the actual value of the buckling and postbuckling load and critical yield stress of an SSSS plate, knowing other parameters: deflection coefficient, α; Poisson ration, μ; breadth, b; aspect ratio, p and thickness, h of the plate.

For instance, an ASTM grade A36 thin rectangular steel plate possessing CCCC edge conditions; subjected to uniformly distributed in-plane load on its R – edge, b and having the following physical and geometric properties as: breadth, b = 4000mm; thickness of plate, h = 20mm; yield load, σ_{ys} = 250 MPa; Ultimate Stress, σ_u = 400 – 550 MPa; Poisson's ratio, μ = 0.30; Modulus of elasticity, E = 200 GPa; density of plate, ρ = 7,800 kg/m^3. The buckling and postbuckling load coefficient and critical yield stress of the plate through unit aspect ratio and deflection coefficients range: $0 \leq \alpha \leq 5.0$ are shown in Fig. 3 and Fig. 4 respectively.

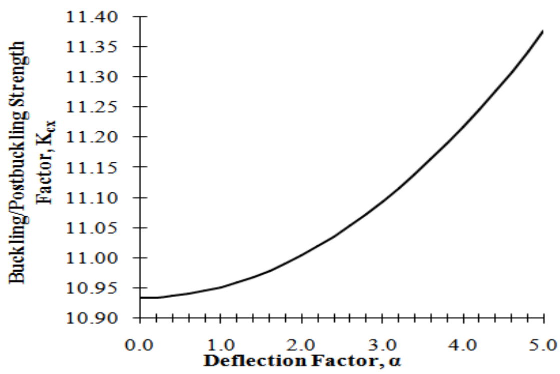

Figure 3. Buckling and Postbuckling Load Coefficient, K_{cx} and Deflection Factor, α at aspect ratio of unity for CCCC – Plate

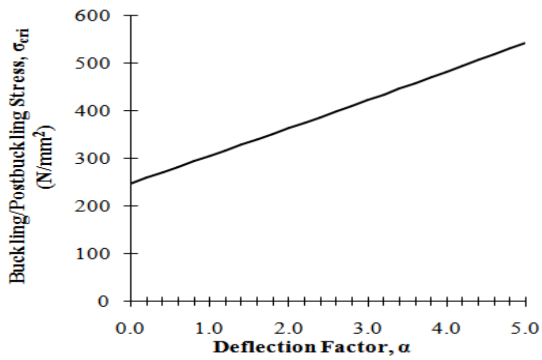

Figure 4. Buckling and postbuckling load critical yield/maximum stress, σ_{cri} and deflection Factor, α at aspect ratio of unity for CCCC – Plate

In Fig. 3, the graph shows that the buckling and postbuckling load parameter, K_{cx} increases quadratically as the out of plane deflection factor, α increases. The buckling and postbuckling load parameters, K_{cx} are higher at other aspect ratios lower than 1.0. Thus, the behaviour of buckling and postbuckling load parameter which is a function of the buckling and postbuckling load means that the buckling and postbuckling load would continue to increase as the out of plane deflection increases. This is contrary to the literature's hypothesis that the axial stiffness reduces, as the plate as a whole sustains increase in load after buckling or deflection [14].

However, this hypothesis is clarified in Fig. 4. The linear relationship in the yield stress behaviour against out of plane deflection explained that the plate would resist extra in-plane load after buckling, while reduces in material stiffness. That is, the plate resists further in-plane load due to postbuckling reserve but loses stiffness due to in-plane bending stress developed. Where the in-plane load bending stress is not considered, the plate would behave as if it had higher yield stress, which it does not. Fig. 4 also show that for a CCCC plate material having yield stress of 250MPa, failure of such plate under in-plane loading would not occur at critical buckling stress. For instance, at zero deformation, the yield stress of the plate reaches 247MPa, which is close to the yield stress of the plate's material. Extra 3MPa stress of applied load on the plate would thereafter lead to collapse of the plate.

V. Conclusion

Whereas the previous study did not analysed the buckling and postbuckling load characteristics of CCCC plate, this paper analysed the buckling and postbuckling load characteristics of CCCC plate. Whereas the double trigonometric functions have been adjudged inadequate for the analysis of thin plates' postbuckling load characteristics, this study obtained exact displacement and stress profiles of buckling and postbuckling load characteristics of CCCC plate by direct integration of the governing differential equations of the plate and implored the work principle technique to finally evaluating the buckling and postbuckling load of CCCC plate. In addition to the buckling and postbuckling load and yield stress obtained for CCCC plate, the study obtained other parameters of the CCCC plate under buckling and postbuckling regimes such as: displacement parameter, W_{uv}, stress coefficient, W_{uv}^2 and load factor, K_{cx}. With all these, the study explained stiffness loss behaviour of plate in postbuckling regime. Thus, the study found out that CCCC plate would accommodate more loads beyond the critical buckling load, prior to actual material failure in its postbuckling regime. For CCCC plate's of higher yield stress, failure would be due to geometric orpermissible deflection criteria. The study also revealed that plate deforms along the transverse direction, leading to the stretching of the longitudinal fibers of the plate, when uniaxially loaded. In this way, the longitudinal fibers of the plate would undergo stress redistribution, as well as develop transverse tensile stresses. These tensile stresses provide the postbuckling reserve load.

References

[1] A. Chaje, Principles of Structural Stability Theory, London: Prentice - Hall Inc., 1974.

[2] J. Rhodes, "Buckling of Thin Plates and Members — and Early Work on Rectangular Tubes," Thin-Walled Structures vol. 40, pp. 87–108, 2002.

[3] J. Rhodes, "Some observations on the Postbuckling Behaviour of Thin-Walled Members," Thin-Walled Structures, vol. 41, pp. 207 – 26, 2003.

[4] R. Szilard, Theories and Application of Plates Analysis: Classical, Numerical and Engineering Methods, New Jersey: John Wiley and Sons Inc., 2004.

[5] T. Von Karman, E. E. Sechler, and L. H. Donnel, "Strength of Thin Plates in Compression," Trans ASME, vol. 54, pp. 53 – 4, 1932.

[6] K. Marguerre, "Effective Width of Plates in Compression," National Advisory Council for Aeronautics, NACA Technical Note, Washington D. C., No. 833, 1937.

[7] S. Levy, "Bending of Rectangular Plates with Large Deflections," National Advisory Council for Aeronautics, NACA Technical Report, Washington D.C., No. 737, 1942.

[8] S. P. Timoshenko and S. Woinowsky - Krieger, Theory of Plates and Shells, 2nd ed., Auckland: McGraw-Hill Inc., 1959.

[9] A. S. Volmir,"A Translation of Flexible Plates and Shells," Air Force Flight Dynamics Laboratory, Ohio: Wright-Patterson Air Force Base, Technical Report No. 66-216, 1967.

[10] N. G. Iyengar, Structural Stability of Columns and Plates, New York: Ellis Horwood Limited, 1988.

[11] E. Ventsel, and T. Krauthammer, Thin Plates and Shells: Theory, Analysis and Applications, New York: Maxwell Publishers Inc., 2001.

[12] H. Chai, "Contact Buckling and Postbuckling of Thin Rectangular Plates," Journal of the Mechanics and Physics of Solids, vol. 49, pp. 209 – 30, 2001.

[13] C. H. Yoo and S. C. Lee, Stability of Structures: Principles and Applications, Amsterdam: Butterworth-Heinemann, 2011.

[14] O. A. Oguaghamba, "Analysis of Buckling and Postbuckling Loads of Isotropic Thin Rectangular Plates,"Ph.D. Thesis Submitted to Postgraduate School, Federal University of Technology, Owerri, Nigeria, 2015.

[15] O. M. Ibearugbulem, L. O. Ettu, and J. C. Ezeh, "Direct Integration and Work Principle as New Approach in Bending Analyses of Isotropic Rectangular Plates," The International Journal of Engineering and Science (IJES), vol. 2 (3), pp. 28 – 36, March 2013.

[16] O. M. Ibearugbulem, O. A. Oguaghamba, K. O. Njoku, and M. Nwaokeorie, "Using Line Continuum to Explain Work Principle Method for Structural Continuum Analysis," International Journal of Innovative Research and Development, vol. 3 (9), pp. 365 – 70, September