

## Autocorrelation effects of seemingly unrelated regression (sur) model on some estimators

Olanrewaju S.O<sup>1</sup> & Ipinyomi R.A<sup>2</sup>

<sup>1</sup>Department of Statistics University of Abuja, Abuja, Nigeria.

<sup>2</sup>Department of Statistics, University of Ilorin, Ilorin, Nigeria

### -----ABSTRACT-----

The seemingly unrelated regression (SUR) proposed by Zellner consists of  $L$  regression equations each of which satisfies the assumptions of the standard regression model. These assumptions are not always satisfied mostly in Economics, Social Sciences and Agricultural Economics which may lead to adverse consequences on the estimator parameters properties. Literature has revealed that multicollinearity often affects the efficiency of SUR estimators and the efficiency in the SUR formulation increases, the more the correlations between error vector differ from zero and the closer the explanatory variables for each response being uncorrelated. This study therefore examined the effect of correlation between the error terms and autocorrelation on seven methods of parameter estimation in SUR model using Monte Carlo approach.

A two equation model was considered in which the first equation has the presence of autocorrelation and correlation between the error terms exists between the two equations. The levels of correlation between the error terms were specified as  $CR = -0.99, -0.9, -0.8, -0.6, -0.4, -0.2, 0, +0.2, +0.4, +0.6, +0.8, +0.9$  and  $+0.99$  and autocorrelation levels  $RE = -0.99, -0.9, -0.8, -0.6, -0.4, -0.2, 0, +0.2, +0.4, +0.6, +0.8, +0.9$  and  $+0.99$ . A Monte Carlo experiment of 1000 trials was carried out at five sample sizes 20, 30, 50, 100 and 250. The seven estimation methods; Ordinary Least Squares (OLS), Cochran – Orcut (GLS2), Maximum Likelihood Estimator (MLE), Multivariate Regression, Full Information Maximum Likelihood (FIML), Seemingly Unrelated Regression (SUR) Model and Three Stage Least Squares (3SLS) were used and their performances were critically examined. Finite properties of estimators' criteria such as bias, absolute bias, variance and mean squared error were used for methods comparison.

The results show that the performances of the estimators cannot be solely determined by the evaluation given by bias criterion because it always behaves differently from other criteria. For the eight different cases considered in this study, it was observed that when the sample size is small (i.e 20 or 30) there is high variability among the estimators but as the sample size increases the variance of the estimator decreases and the performances of the estimators become asymptotically the same.

In the presence of correlation between the error terms and autocorrelation, the estimator of MLE is preferred to estimate all the parameters of the model at all the level of sample sizes.

**KEYWORDS:** SUR, Autocorrelation, Error terms, mean square error, Bias, Absolute Bias, Variance.

Date of Submission: 17 May 2013



Date of Publication: 10 May 2014

### I. INTRODUCTION

The seemingly unrelated regression (SUR) model is well known in the Econometric literature (Zellner, 1962, Srivastava and Giles, (1987), Greene (1993) but is less known elsewhere, its benefits have been explored by several authors and more recently the SUR model is being applied in Agricultural Economics (O' Dorell et al 1999), Wilde et al (1999). Its application in the natural and medical sciences is likely to increase once scientists in the disciplines are exposed to its potential.

The SUR estimation procedures which enable an efficient joint estimation of all the regression parameters was first reported by Zellner (1962) which involves the application of Aitken's Generalised Least Squares (AGLS), (Powell 1965) to the whole system of equations. Zellner (1962 & 1963), Zellner & Theil (1962) submitted that the joint estimation procedure of SUR is more efficient than the equation-by-equation estimation procedure of the Ordinary Least Square (OLS) and the gain in efficiency would be magnified if the contemporaneous correlation between each pair of the disturbances in the SUR system of equations is very high and explanatory variables (covariates) in different equations are uncorrelated. In other words, the efficiency in the SUR formulation increases the more the correlation between error vector differs from zero and the closer the explanatory variables for each response are to being uncorrelated.

After the much celebrated Zellner's joint generalized least squares estimator, several other estimators for different SUR systems were developed by many scholars to address different situations being investigated. For instance, Jackson (2002) developed an estimator for SUR system that could be used to model election returns in a multiparty election. Sparks (2004) developed a SUR procedure that is applicable to environmental situations especially when missing and censored data are inevitable. In share equation systems with random coefficients, Mandy & Martins-Filho (1993) proposed a consistent and asymptotically efficient estimator for SUR systems that have additive heteroscedastic contemporaneous correlation. They followed Amemiya (1977) by using Generalized Least Squares (GLS) to estimate the parameters of the covariance matrix. Furthermore, Lang, Adebayo & Fahrmeir (2002), Adebayo (2003), and Lang *et al* (2003) in their works also extended the usual parametric SUR model to Semiparametric SUR (SSUR) and Geometric SUR models within a Bayesian context. Also O'Donnell *et al* (1999) and Wilde *et al* (1999) developed SUR estimators that are applicable in Agricultural Economics. More recently, Foschi (2004) provided some new numerical procedures that could successively and efficiently solve a large scale of SUR model. In all the estimation procedures developed for different SUR situations as reported above, Zellner's basic recommendation for high contemporaneous correlation between the error vectors with uncorrelated explanatory variables within each response equation was also maintained. However, in most practical situations, the explanatory variables across the different equations in SUR systems are often correlated. Also, it may be necessary to jointly regress the demand for two or more complementary products like automobiles and gasoline on people's income and expenditures on other products within the SUR framework. While the two demands (responses) would obviously correlate through their error, satisfying the first basic requirement of SUR estimation, people's income and their expenditure on other products should not be expected to be uncorrelated thereby, violating the second important condition. Therefore, the existence of this kind of relationship needed to be recognized and accorded proper management within the SUR context such that the efficiency of SUR estimator would not be compromised. It is now obvious, due to several instances of SUR highlighted above, that the independent variables are often correlated (collinear).

The seemingly unrelated regression proposed by Zellner (Zellner; 1962) consists of L regression equations each of which satisfies the assumptions of the standard regression model:

$$\begin{aligned}
 y_1 &= X_1\beta_1 + u_1 \\
 y_2 &= X_2\beta_2 + u_2 \\
 &\cdot \quad \cdot \quad \cdot \\
 &\cdot \quad \cdot \quad \cdot \\
 y_L &= X_L\beta_L + u_L
 \end{aligned}
 \tag{1.1}$$

Where  $y_i$  and  $u_i$  are  $N \times 1$  vectors and  $X_i$  is a  $N \times K$  matrix. Notice that by stacking all equations together we can write this system as

$$\begin{pmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_L \end{pmatrix} = \begin{pmatrix} X_1 & \dots & \dots & 0 \\ \cdot & \cdot & & \\ \cdot & & \cdot & \\ \cdot & & & 0 \\ 0 & \dots & \dots & X_L \end{pmatrix} \begin{pmatrix} \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_L \end{pmatrix} = \begin{pmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_L \end{pmatrix}
 \tag{1.2}$$

$N \times 1$                    $N \times \sum_i k_i$                    $\sum_i k_i \times 1$      $N \times 1$

$$Y = X\beta + U
 \tag{1.3}$$

So we stack all the equations together into a system of the OLS form in (1.3). This suggests as an estimation procedure to run OLS on this system, i.e to consider

$$\hat{\beta} = (X'X)^{-1} X'Y$$

## 1.2 Efficiency of SUR model for estimating regression coefficients.

Note that if either  $\sigma_{ij} = 0$  for all  $j \neq i$  or  $X_i = X_j$  for all  $j \neq i$ , then, the two model formulations produce estimator identical to

$$\beta_j = (X_j' X_j)^{-1} X_j' Y_j \text{ for } j=1,2,\dots,p$$

The efficiency in the SUR formulation increases, the more the correlations between error vector differ from zero and the closer the explanatory variables for each response being uncorrelated (e.g. see Sparks (1987) ) discussed how to select variables and parameter estimators for the SUR model. The standard errors for the set of the regression parameter estimates in the SUR formulation are given by the diagonal elements of  $(X'\Sigma^{-1}X)^{-1}$  while for the unrelated formulation they are the appropriate diagonal elements of  $(X_j'X_j)^{-1}$  for  $j=1,2,\dots,p$ . These can be used to gain an idea of the relative merits of the SUR model formulation for estimating the regression parameters of the model.

Generally when  $\sigma_{ij} = 0$  and the covariances are known, it can be shown that the diagonal elements of  $(X_j'X_j)^{-1}$  are larger than the corresponding diagonal elements of  $(X'\Sigma^{-1}X)^{-1}$  for each  $j$ . The mean square error of prediction using the generalized least squares estimate is smaller. This is not generally true when the covariances are unknown but depend on the sample size  $n$  (Zellner, 1963; Kmenta and Gilbert, 1968; Revankar, 1974 ; Mehta and Swamy ,1976; and Maeshiro ,1980). When fitting regression models with small sample sizes, is unlikely that the seemingly unrelated regression formulation and related generalized least squares estimates are going to add much value.

However, the larger the sample size, the more reliable the estimate of  $\sigma_{ij}$  and hence the more likely an advantage is gained from the seemingly unrelated regression formulation of the model.

Consequently, this study examines the performances of some estimation methods in Seemingly Unrelated Regression model in the presence of autocorrelation with the intention of studying their effects on the estimators and identifying the preferred estimator(s) of the model parameters.

Very specifically, the study aims at the following:

- (i) Examine the effect of sample size on the performance of the estimators
- (ii) Examine the effect of autocorrelation (RE) and correlation between the error terms (CR) on the performance of seven estimators.
- (iii) Identify the estimator that yields the most preferred estimates under separate or joint influence of the three correlation effects under consideration.

## II. THE MODEL FORMULATION

The Seemingly Unrelated regression (SUR) Model used in this research work is given as

$$y_{1t} = \beta_{01} + \beta_{11}x_{1t} + \beta_{12}x_{2t} + u_{1t} \tag{3.1}$$

where  $u_{1t} = \rho u_{1(t-1)} + e_{1t}$  ,  $e_{1t} \approx (0, \sigma^2)$  .

$$y_{2t} = \beta_{02} + \beta_{21}x_{1t} + \beta_{22}x_{3t} + u_{2t} \text{ , } u_{2t} \approx N(0, \sigma^2) \tag{3.2}$$

**NOTE:** (1) Autocorrelation exists in equations (3.1)

(2) There is correlation between  $U_1$  and  $U_2$  of the two equations

(3) There is no correlation between  $X_1$  and  $X_3$  in equation (3.2), thus, equation (3.2) appears as control equation.

### III. EQUATION USED FOR GENERATING VALUES IN SIMULATION

The equation used for generating values of the variables in the simulation study as proposed by Ayinde K.(2007) is given below

Suppose  $W_i \sim N(\mu, \sigma_i^2)$   $i = 1, 2$ . If these variables are correlated, then,  $W_1$  and  $W_2$  can be generated with the equations

$$\begin{aligned} W_1 &= \mu_1 + \sigma_1 z_1 \\ W_2 &= \mu_2 + \rho \sigma_2 z_1 + \sigma_2 z_2 \sqrt{1 - \rho^2} \end{aligned} \tag{3.3}$$

where  $Z_i \sim N(0,1)$   $i = 1, 2$  and  $|\rho| < 1$  is the value of the correlation between the two variables.

#### 3.1 Other Specifications

1. Sample Size(n) of 20, 30, 50, 100 and 250 were used in the simulation
2. The following levels were used for the correlations studied:
  - a. Autocorrelation(RE) : -0.99, -0.9, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99
  - b. Correlation between error term (CR) : -0.99, -0.9, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99
  - c. Replication (RR) : we make use of 1000 replications
  - d. Two RUNS were done for the simulations which were averaged at analysis stage.

#### 3.2 Criteria for comparison

Evaluation and comparison of the seven (7) estimators were examined using the finite sampling properties of estimators which include Bias (BB), Absolute Bias (AB), Variance (VB) and the Mean Square Error (MB) criteria.

Mathematically, for any estimator  $\hat{\beta}_i$  of  $\beta_i$  of model (3.1) & (3.2)

$$(i) \bar{\hat{\beta}}_i = \frac{1}{R} \sum_{j=1}^R \hat{\beta}_{ij} \qquad (ii) Bias(\hat{\beta}_i) = \frac{1}{R} \sum_{j=1}^R (\hat{\beta}_{ij} - \beta_j) = \bar{\hat{\beta}}_i - \beta_i$$

$$(iii) AB(\hat{\beta}_i) = \frac{1}{R} \sum_{j=1}^R |\hat{\beta}_{ij} - \beta_j| \qquad (iv) Var(\hat{\beta}_i) = \frac{1}{R} \sum_{j=1}^R (\hat{\beta}_{ij} - \bar{\hat{\beta}}_i)^2$$

$$(v) MSE(\hat{\beta}_i) = \frac{1}{R} \sum_{j=1}^R (\hat{\beta}_{ij} - \beta_i)^2, \text{ for } i = 0, 1, 2 \text{ and } j = 1, 2, \dots, R.$$

Using a computer program which was written with TSP software package to estimate all the model parameters and the criteria, the performances of seven estimation methods; Ordinary Least Squares (OLS), Cochran – Orcut (COCR), Maximum Likelihood Estimator (MLE), Multivariate Regression, Full Information Maximum Likelihood (FIML), Seemingly Unrelated Regression (SUR) and Three Stage Least Squares (3SLS) were examined by subjecting the results obtained from each finite properties of the estimators into a multi factor analysis of variance model. Consequently, the highest order significant interaction effect which has “method” as a factor is further examined using the Least Significance Difference (LSD) test. The estimated marginal mean of the factor was investigated out at a particular combination of levels of the correlations in which estimators were preferred. An estimator is most preferred at a particular combination of levels of the correlation if the marginal means is the smallest. All estimators whose estimated marginal means are not significantly different from the most preferred are also preferred.

### IV. RESULTS

The summary of results from the Analysis of variance tables of the criteria showing the effect of the estimators, correlation between the error term sand autocorrelation on  $\beta_i$  are presented in Table 4.5.1 below.

TABLE 4.5.1: ANOVA for sample size of 20

n	SOV	EQN	$\beta_i$	df	TYPE III SUM OF SQUARES			
					Bias	Absolute Bias	Variance	Mean Square
20	RE	1	$\beta_{01}$	12	892.446***	115926.509***	2445822.237***	3951716.298***
			$\beta_{11}$	12	.029***	32.515***	95.927***	96.084***
			$\beta_{21}$	12	.012	24.373***	23.509***	87.017***
		2	$\beta_{02}$	12	.112	103.206***	122116.658***	128548.527***
			$\beta_{12}$	12	.063	.628***	.093***	.091***
			$\beta_{22}$	12	.132***	.605***	.113***	.125***
	CR	1	$\beta_{01}$	12	.670	.003	.003	.005
			$\beta_{11}$	12	.001	6.016E-5	8.532E-5	8.897E-5
			$\beta_{21}$	12	7.468***	3.807***	3.176***	5.004***
		2	$\beta_{02}$	12	3.519	45.130***	113879.706***	119769.347***
			$\beta_{12}$	12	.513***	.032	.224***	.032***
			$\beta_{22}$	12	3.006***	.404***	.139***	.011***
	M	1	$\beta_{01}$	6	315.786***	83483.317***	4080093.223***	6466311.896***
			$\beta_{11}$	6	.000	4.612***	5.977***	5.990***
			$\beta_{21}$	6	.007	5.564***	2.320***	9.779***
		2	$\beta_{02}$	6	.042	45.091***	232859.705***	243905.100***
			$\beta_{12}$	6	.476***	.141***	.002***	.006***
			$\beta_{22}$	6	.086***	2.096***	.361***	.391***
	RE*CR		$\beta_{01}$	144	.458	.026	.021	.037
			$\beta_{11}$	144	.001	.000	.001	.001
			$\beta_{21}$	144	5.046	1.759	7.126***	19.761**
			$\beta_{02}$	144	5.506	195.745***	360069.375***	378405.077***
			$\beta_{12}$	144	.048	.423***	.054***	.052***
			$\beta_{22}$	144	.019	.256***	.053***	.052***
RE*M	1	$\beta_{01}$	72	5540.631***	454326.369***	1.038E7***	1.557E7***	
		$\beta_{11}$	72	.011	15.816***	40.207***	40.276***	
		$\beta_{21}$	72	.014	15.506***	13.080***	56.497***	
	2	$\beta_{02}$	72	.675	199.404***	716966.573***	755012.778***	
		$\beta_{12}$	72	.201	.078	.007**	.007	
		$\beta_{22}$	72	.116***	.529***	.131***	.134***	
CR*M	1	$\beta_{01}$	72	.515	.002	.002	.004	
		$\beta_{11}$	72	.001	4.520E-5	6.406E-5	6.680E-5	
		$\beta_{21}$	72	5.943	2.889***	2.473**	3.902	
	2	$\beta_{02}$	72	3.940	196.384***	683436.471***	721549.650***	
		$\beta_{12}$	72	.243	.407***	.004	.022***	
		$\beta_{22}$	72	.148***	1.340***	.289***	.246***	
RE*CR*M	1	$\beta_{01}$	864	.348	.020	.017	.030	
		$\beta_{11}$	864	.001	.000	.000	.000	
		$\beta_{21}$	864	3.917	1.358	5.731	15.895	
	2	$\beta_{02}$	864	33.150	884.547***	2141981.317***	2251208.104***	
		$\beta_{12}$	864	.072	.059	.006	.005	
		$\beta_{22}$	864	.082	.433	.102	.103	
ERROR	1	$\beta_{01}$	1183	3595.810	8759.488	8834975.252	8871627.167	
		$\beta_{11}$	1183	.245	11.841	50.548	50.564	
		$\beta_{21}$	1183	84.299	23.465	28.185	128.394	
	2	$\beta_{02}$	1183	214.134	135.089	1384793.426	1438377.807	
		$\beta_{12}$	1183	16.754	2.534	.089	.150	
		$\beta_{22}$	1183	.659	1.140	.197	.256	
TOTAL	1	$\beta_{01}$	2365	10346.814	662542.071	2.575E7	3.487E7	
		$\beta_{11}$	2365	.288	64.783	192.657	192.913	
		$\beta_{21}$	2365	106.707	78.742	85.610	326.275	
	2	$\beta_{02}$	2365	261.126	1804.890	5756781.519	6037516.906	
		$\beta_{12}$	2365	18.374	4.302	.480	.365	
		$\beta_{22}$	2365	4.259	6.818	1.387	1.320	

\*\*\* Significant at 0.05 level of significance

\*\* Significant at 0.1 level of significance

#### 4.5.1 EFFECT ON $\beta_0$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimatorexceptGLS2 are preferred to estimate  $\beta_0$  at all the levels of autocorrelation. In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria except in bias criterion. The results of the LSD further test visa- vice their estimated marginal means as shown in revealed that all estimatorexceptGLS2 are preferred to estimate  $\beta_0$  at all levels of autocorrelation and correlation between the error terms.

#### EFFECT ON $\beta_1$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation under all criteria except for bias. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all the levels of autocorrelation.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria except in bias criterion. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_1$  at all levels of autocorrelation and correlation between the error terms.

#### EFFECT ON $\beta_2$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all the levels of autocorrelation.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that SUR and 3SLS estimators are preferred to estimate  $\beta_2$  at all levels of autocorrelation and correlation between the error terms EXCEPT for -0.9 and -0.8 levels of correlation between the error terms under bias that is significantly different.

**Summarily, GLS2, MLE, SUR and 3SLS are preferred to estimate the model at sample size of 20**

**TABLE4.5.2: ANOVA for the sample size of 30**

n	SOV	EQN	$\beta_i$	df	TYPE III SUM OF SQUARES			
					Bias	Absolute Bias	Variance	Mean Square
30	RE	1	$\beta_{01}$	12	1368.073***	165272.612***	1.008E12***	1.009E12***
			$\beta_{11}$	12	.029***	37.228***	125.437***	127.031***
			$\beta_{21}$	12	.075	29.385***	69.897***	83.604***
		2	$\beta_{02}$	12	.095	51.854***	6251392.175***	6258276.131***
			$\beta_{12}$	12	.005	.334***	.025***	.025***
			$\beta_{22}$	12	.011	.175***	.008***	.043***
	CR	1	$\beta_{01}$	12	.147	10.954	1.102E8	1.102E8
			$\beta_{11}$	12	.271***	.043	.144	.137
			$\beta_{21}$	12	1.980***	5.721***	5.706***	5.296***
		2	$\beta_{02}$	12	.200	13.613***	5187309.671***	5192414.391***
			$\beta_{12}$	12	2.338***	.012	.096***	.001
			$\beta_{22}$	12	.695	15.467***	.018***	2.358***
	M	1	$\beta_{01}$	6	187.891***	82932.248***	6.302E11	6.312E11***
			$\beta_{11}$	6	.013**	5.296***	7.289***	7.403***
			$\beta_{21}$	6	.009	4.316***	8.386***	8.620***
		2	$\beta_{02}$	6	.007	9.955***	5467535.629***	5474905.799***
			$\beta_{12}$	6	.029	.034	.004***	.004
			$\beta_{22}$	6	.001	.114***	.052***	.040***
RE*CR		$\beta_{01}$	144	1.846	131.775	1.362E9	1.363E9	
		$\beta_{11}$	144	.143**	.163	1.140	1.113	
		$\beta_{21}$	144	1.132	2.945***	20.161***	18.791***	
		$\beta_{02}$	144	1.196	70.924***	3.657E7***	3.661E7***	
		$\beta_{12}$	144	.024	.222	.016***	.016	
		$\beta_{22}$	144	.034	.108	.005***	.016	

	RE*M	1	$\beta_{01}$	72	7396.149***	696530.165***	6.041E12***	6.050E12***
			$\beta_{11}$	72	.012	17.083***	51.754***	52.348***
			$\beta_{21}$	72	.045	15.567***	46.940***	48.988***
		2	$\beta_{02}$	72	583	43.075***	3.735E7***	3.739E7***
			$\beta_{12}$	72	.002	.034	.003***	.004
			$\beta_{22}$	72	.028	.105	.016***	.026
	CR*M	1	$\beta_{01}$	72	.988	45.595	6.622E8	6.623E8
			$\beta_{11}$	72	.203***	.032	.108	.103
			$\beta_{21}$	72	1.491***	4.279***	4.281	3.968
		2	$\beta_{02}$	72	.863	40.424***	3.189E7***	3.193E7***
			$\beta_{12}$	72	.012	.085	.004***	.005
			$\beta_{22}$	72	.009	.414***	.035***	.065
RE*CR*M	1	$\beta_{01}$	864	10.321	546.890	8.173E9	8.174E9	
		$\beta_{11}$	864	.107	.121	.855	.835	
		$\beta_{21}$	864	.853	2.202	15.134	14.102	
	2	$\beta_{02}$	864	7.135	249.877***	2.190E8***	2.192E8***	
		$\beta_{12}$	864	.001	.041	.003	.003	
		$\beta_{22}$	864	.014	.174	.013***	.030	
ERROR	1	$\beta_{01}$	1183	3150.131	4943.259	4.933E10	5.003E10	
		$\beta_{11}$	1183	.916	19.633	110.203	112.032	
		$\beta_{21}$	1183	13.579	13.356	69.548	66.923	
	2	$\beta_{02}$	1183	44.274	32.901	2.213E7	2.212E7	
		$\beta_{12}$	1183	7.545	6.245	.041	.357	
		$\beta_{22}$	1183	123.484	4.942	.011	1.476	
TOTAL	1	$\beta_{01}$	2365	12115.647	950459.247	7.739E12	7.751E12	
		$\beta_{11}$	2365	1.693	79.598	296.927	301.002	
		$\beta_{21}$	2365	19.164	77.788	240.059	250.302	
	2	$\beta_{02}$	2365	54.357	512.744	3.639E8	3.642E8	
		$\beta_{12}$	2365	9.965	7.003	.192	.413	
		$\beta_{22}$	2365	124.277	21.562	.158	4.059	

#### 4.5.2 EFFECT ON $\beta_0$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all the levels of autocorrelation exceptfor GLS2 which differed significantly at 0.8, 0.9 and 0.99 autocorrelation levels.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria except in bias criterion. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all levels of autocorrelation and correlation between the error terms exceptfor GLS2 which differed significantly at autocorrelation level of 0.9 and correlation between the error terms of 0.99 under bias criterion.

#### EFFECT ON $\beta_1$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all the levels of autocorrelation and correlation between the error terms.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under variance criterion. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all levels of autocorrelation and correlation between the error terms.

#### EFFECT ON $\beta_2$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all the levels of autocorrelation and correlation between the error terms except that we have to be cautious when using them at some levels of autocorrelation.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under variance criterion. The results of the LSD further test *visa- vice* their estimated marginal means revealed that all estimators except OLS, GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all levels of autocorrelation and correlation between the error terms.

**Summarily, GLS2 and MLE estimators are preferred to estimate the model at sample size of 30**

**TABLE4.5.3: ANOVA for sample size of 50**

n	SOV	$\beta_i$	df	TYPE III SUM OF SQUARES			
				Bias	Abs.Bias	Var	MSE
50	RE	$\beta_{01}$	12	452.571***	74575.669***	1.764E11***	1.770E11***
		$\beta_{11}$	12	.050***	18.709***	24.791***	24.976***
		$\beta_{21}$	12	1.014***	6.985***	1.255***	2.662***
		$\beta_{02}$	12	.515***	35.964***	251158.322***	252591.912***
		$\beta_{12}$	12	.417***	.167***	.004***	.007***
		$\beta_{22}$	12	.129**	.174***	.001***	.003***
	CR	$\beta_{01}$	12	1.992	234.178	1.404E9	1.406E9
		$\beta_{11}$	12	.022***	.972***	1.780***	1.786**
		$\beta_{21}$	12	5.131***	.177**	.052	.030
		$\beta_{02}$	12	1.353***	6.205***	161713.711***	162539.579***
		$\beta_{12}$	12	3.505***	1.093***	.026***	.024***
		$\beta_{22}$	12	.221***	.373***	.003***	.003***
	M	$\beta_{01}$	6	227.569***	24107.884***	8.971E10***	9.003E10***
		$\beta_{11}$	6	.001	3.527***	2.193***	2.209***
		$\beta_{21}$	6	.085	2.311***	.619***	.709***
		$\beta_{02}$	6	.105***	1.285***	178487.825***	179459.300***
		$\beta_{12}$	6	.021***	.003	.002***	8.218E-5
		$\beta_{22}$	6	.023	.307***	.010***	.012***
	RE*CR	$\beta_{01}$	144	23.036	2764.733	1.713E10	1.714E10
		$\beta_{11}$	144	.019	4.251***	12.667***	12.699***
		$\beta_{21}$	144	1.698	1.158	.538***	1.049***
		$\beta_{02}$	144	2.520***	28.684***	1365792.064***	1373566.001***
		$\beta_{12}$	144	.136***	.165***	.005***	.009
		$\beta_{22}$	144	.058	.055	.001	.001
	RE*M	$\beta_{01}$	72	3285.331***	280727.544***	1.056E12***	1.060E12***
		$\beta_{11}$	72	.021	8.107***	10.064***	10.139***
		$\beta_{21}$	72	.847	3.449***	1.724***	1.696***
		$\beta_{02}$	72	2.363***	10.635***	1480512.907***	1488990.865***
		$\beta_{12}$	72	.132***	.218***	.003***	.008***
		$\beta_{22}$	72	.009	.064	.002***	.003
	CR*M	$\beta_{01}$	72	11.459	1223.011	8.561E9	8.571E9
		$\beta_{11}$	72	.016	.241	.671	.673
		$\beta_{21}$	72	3.879***	.469	.307***	.207
		$\beta_{02}$	72	1.161***	6.135***	982226.051***	987452.884***
		$\beta_{12}$	72	.012	.092***	.002**	.007***
		$\beta_{22}$	72	.010	.276***	.006***	.007***
	RE*CR*M	$\beta_{01}$	864	137.160	14657.361	1.026E11	1.027E11
		$\beta_{11}$	864	.013	1.423	4.996	5.009
		$\beta_{21}$	864	1.263	.490	1.123	.968
		$\beta_{02}$	864	13.881***	59.010***	8150369.132***	8196863.794***
		$\beta_{12}$	864	.048	.067	.002	.003
		$\beta_{22}$	864	.008	.050	.002	.002
ERROR	$\beta_{01}$	1183	6296.390	82375.378	8.427E11	8.460E11	
	$\beta_{11}$	1183	.278	8.667	23.218	23.233	
	$\beta_{21}$	1183	12.234	8.004	2.939	4.219	
	$\beta_{02}$	1183	4.175	9.073	1571334.088	1580748.512	
	$\beta_{12}$	1183	.661	.922	.026	.070	
	$\beta_{22}$	1183	6.136	1.510	.012	.043	
TOTAL	$\beta_{01}$	2365	10435.662	480676.871	2.295E12	2.303E12	
	$\beta_{11}$	2365	.420	45.888	80.372	80.717	
	$\beta_{21}$	2365	26.153	23.042	8.557	11.540	
	$\beta_{02}$	2365	26.079	157.021	1.414E7	1.422E7	
	$\beta_{12}$	2365	4.946	2.729	.070	.128	
	$\beta_{22}$	2365	6.593	2.811	.038	.073	



### 4.5.3.1 EFFECT ON $\beta_0$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all the levels of autocorrelation exceptfor GLS2 which differed significantly at 0.99 autocorrelation levels.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all levels of autocorrelation and correlation between the error terms exceptfor GLS2 which differed significantly at autocorrelation level of 0.9 & 0.99 and correlation between the error terms of 0.99 under all criteria.

### 4.5.3.2 EFFECT ON $\beta_1$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all the levels of autocorrelation and correlation between the error terms.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under variance criterion. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all levels of autocorrelation and correlation between the error terms.

### 4.5.3.3 EFFECT ON $\beta_2$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all the levels of autocorrelation and correlation between the error terms.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators except OLS, GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all levels of autocorrelation and correlation between the error terms.

**Summarily, GLS2 and MLE estimator are preferred to estimate the model at sample size of 50**

**TABLE 4.5.4: ANOVA for sample size of 100**

n	SOV	$\beta_i$	df	TYPE III SUM OF SQUARES			
				Bias	Abs.Bias	Var	MSE
100	RE	$\beta_{01}$	12	47.699	48743.394***	7.898E9***	7.945E9***
		$\beta_{11}$	12	.022***	21.108***	33.739***	33.776***
		$\beta_{21}$	12	.014	7.287***	3.435***	4.171***
		$\beta_{02}$	12	.031	23.608***	27762.719***	27802.782***
		$\beta_{12}$	12	.004	.122***	.002***	.002***
		$\beta_{22}$	12	.007	.019***	.001***	.001***
	CR	$\beta_{01}$	12	.001	.001	.000	.000
		$\beta_{11}$	12	.011***	.002	.006	.006
		$\beta_{21}$	12	1.366***	1.539***	.352***	.320***
		$\beta_{02}$	12	.486	.139***	12228.857***	12185.714***
		$\beta_{12}$	12	.047***	.692***	.018***	.016***
		$\beta_{22}$	12	.057	1.315***	.002***	.019***
	M	$\beta_{01}$	6	12.616	13036.510***	3.909E9***	3.932E9***
		$\beta_{11}$	6	.005***	3.739***	2.145***	2.151***
		$\beta_{21}$	6	.000	1.601***	.568***	.585***
		$\beta_{02}$	6	.044	.218***	13231.759***	13252.284***
		$\beta_{12}$	6	.003	.058***	.002***	.002***
		$\beta_{22}$	6	.002	.095***	.004***	.005***
	RE*CR	$\beta_{01}$	144	.001	.008	.002	.002
		$\beta_{11}$	144	.022***	.026	.077	.077
		$\beta_{21}$	144	.810***	.784***	1.083***	.978***
		$\beta_{02}$	144	.204	15.466***	147798.362***	148004.003***
		$\beta_{12}$	144	.011	.053***	.002***	.002***
		$\beta_{22}$	144	.002	.021	.000***	.000
RE*M	$\beta_{01}$	72	151.558	156345.485***	4.691E10***	4.719E10***	
	$\beta_{11}$	72	.015***	9.522***	13.766***	13.781***	

		$\beta_{21}$	72	.005	4.081***	2.357***	2.457***
		$\beta_{02}$	72	.167	2.498***	158513.192***	158738.340***
		$\beta_{12}$	72	.011	.044***	.001***	.001***
		$\beta_{22}$	72	.009	.029***	.001***	.001***
	CR*M	$\beta_{01}$	72	.001	7.294E-6	5.759E-5	6.569E-5
		$\beta_{11}$	72	.009***	.001	.004	.004
		$\beta_{21}$	72	1.024***	1.158***	.264***	.240***
		$\beta_{02}$	72	.063	1.170***	73197.801***	73298.063***
		$\beta_{12}$	72	.001	.052***	.001***	.001***
		$\beta_{22}$	72	.002	.123***	.002***	.002***
	RE*CR*M	$\beta_{01}$	864	.001	.000	.001	.001
		$\beta_{11}$	864	.018	.016	.058	.057
		$\beta_{21}$	864	.610	.587	.813	.734
		$\beta_{02}$	864	.899	13.854***	877548.891***	878767.713***
		$\beta_{12}$	864	.011	.029	.001	.001
		$\beta_{22}$	864	.005	.038	.001***	.001
	ERROR	$\beta_{01}$	1183	3458.358	1982.236	5.755E10	5.739E10
		$\beta_{11}$	1183	.034	8.157	27.089	27.121
		$\beta_{21}$	1183	1.548	1.645	1.959	2.328
		$\beta_{02}$	1183	28.879	4.729	142202.156	142616.134
		$\beta_{12}$	1183	.422	.148	.005	.005
		$\beta_{22}$	1183	13.178	.257	.000	.009
	TOTAL	$\beta_{01}$	2365	3670.242	220115.863	1.163E11	1.165E11
		$\beta_{11}$	2365	.137	42.568	76.883	76.972
		$\beta_{21}$	2365	5.378	18.685	10.833	11.814
		$\beta_{02}$	2365	30.774	61.678	1452558.902	1454739.728
		$\beta_{12}$	2365	.510	1.199	.031	.030
		$\beta_{22}$	2365	13.260	1.905	.012	.039

#### 4.5.4.1 EFFECT ON $\beta_0$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation under absolute bias, variance and mean square error criteria. The results of the LSD further test *visa- vice* their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all the levels of autocorrelation except for GLS2 which differed significantly at 0.99 autocorrelation levels.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria. The results of the LSD further test *visa- vice* their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all levels of autocorrelation and correlation between the error terms except for GLS2 which differed significantly at autocorrelation level of 0.99 and correlation between the error terms of - 0.99 and +0.99 under all criteria considered.

#### 4.5.4.2 EFFECT ON $\beta_1$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms under all criteria. The results of the LSD further test *visa- vice* their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all the levels of autocorrelation and correlation between the error terms.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under all the criteria. The results of the LSD further test *visa- vice* their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all levels of autocorrelation and correlation between the error terms.

#### 4.5.4.3 EFFECT ON $\beta_2$

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms under all criteria. The results of the LSD further test *visa- vice* their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all the levels of autocorrelation and correlation between the error terms, although they too are significantly different at some limited levels of autocorrelation.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under variance criterion. The results of the LSD further test *visa- vice* their estimated marginal means as shown in Table EB4.8 of appendix 5 revealed that all estimators except OLS, GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all levels of autocorrelation and correlation between the error terms.

Summarily, GLS2, SUR and MLE estimator are preferred to estimate the model at sample size of 100

TABLE4.5.5: ANOVA for sample size of 250

n	SOV	$\beta_i$	df	TYPE III SUM OF SQUARES			
				Bias	Abs.Bias	Var	MSE
250	RE	$\beta_{01}$	12	1.478***	4632.931***	1.059E8***	1.059E8***
		$\beta_{11}$	12	.030***	6.158***	2.703***	2.730***
		$\beta_{21}$	12	.001	2.812***	.540***	.658***
		$\beta_{02}$	12	.003	6.709***	83.297***	83.466***
		$\beta_{12}$	12	.319	.035***	17.412	.001**
		$\beta_{22}$	12	.002	.021***	8.986E-5***	.000***
	CR	$\beta_{01}$	12	.008	6.104E-5	8.877E-5	8.761E-5
		$\beta_{11}$	12	.001	5.698E-5	3.105E-5	3.798E-5
		$\beta_{21}$	12	.205***	.627***	.062***	.058***
		$\beta_{02}$	12	.338***	.295***	30.040	29.626
		$\beta_{12}$	12	.778**	.356***	17.229	.006***
		$\beta_{22}$	12	.036	.303***	.000***	.001***
	M	$\beta_{01}$	6	.190***	873.346***	5.133E7***	5.133E7***
		$\beta_{11}$	6	7.732E-5	1.168***	.229***	.230***
		$\beta_{21}$	6	.001	.583***	.102***	.105***
		$\beta_{02}$	6	.001	.021	28.674	28.775
		$\beta_{12}$	6	.182	.007	8.763	.000
		$\beta_{22}$	6	.001	.073***	.001***	.001***
	RE*CR	$\beta_{01}$	144	.005	.000	.001	.001
		$\beta_{11}$	144	.012	9.808E-5	.000	.000
		$\beta_{21}$	144	.176	.311**	.155***	.148***
		$\beta_{02}$	144	.027	2.815***	352.222	353.150
		$\beta_{12}$	144	3.609	.015	209.071	.000
		$\beta_{22}$	144	.005	.012	6.516E-5***	.000
	RE*M	$\beta_{01}$	72	2.434	10508.921***	6.160E8***	6.161E8***
		$\beta_{11}$	72	.011***	2.858***	1.109***	1.120***
		$\beta_{21}$	72	.001***	1.528***	.355***	.376***
		$\beta_{02}$	72	.003	.195	344.710***	345.580***
		$\beta_{12}$	72	1.811	.071	104.517	.001
		$\beta_{22}$	72	.002	.042***	.000***	.000***
	CR*M	$\beta_{01}$	72	.006	4.581E-5	6.662E-5	6.574E-5
		$\beta_{11}$	72	.001	3.947E-5	2.313E-5	2.817E-5
		$\beta_{21}$	72	.152**	.469***	.046	.044
		$\beta_{02}$	72	.005	.090	162.122	162.533
		$\beta_{12}$	72	1.763	.013	104.616	.000
		$\beta_{22}$	72	.006	.075***	.001***	.001***
	RE*CR*M	$\beta_{01}$	864	.004	.000	.001	.001
		$\beta_{11}$	864	.009	8.591E-5	.000	.000
		$\beta_{21}$	864	.131	.233	.116	.111
		$\beta_{02}$	864	.055	1.121	1945.383	1950.633
		$\beta_{12}$	864	21.396	.030	1254.431	.001
		$\beta_{22}$	864	.004	.021	.000***	.000
ERROR	$\beta_{01}$	1183	3.854	16061.289	7.821E8	7.822E8	
	$\beta_{11}$	1183	.082	3.122	2.240	2.268	
	$\beta_{21}$	1183	1.781	2.054	.749	.792	
	$\beta_{02}$	1183	3.320	7.305	2945.654	2953.427	
	$\beta_{12}$	1183	37.560	1.475	1717.611	.034	
	$\beta_{22}$	1183	3.873	.314	6.234E-5	.003	
TOTAL	$\beta_{01}$	2365	7.981	32077.122	1.555E9	1.556E9	
	$\beta_{11}$	2365	.146	13.305	6.281	6.348	
	$\beta_{21}$	2365	2.449	8.618	2.125	2.292	
	$\beta_{02}$	2365	3.756	18.552	5892.407	5907.490	
	$\beta_{12}$	2365	67.419	2.003	3433.660	.042	
	$\beta_{22}$	2365	3.929	.863	.003	.007	

**4.5.5.1 EFFECT ON  $\beta_0$**

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all the levels of autocorrelation except for GLS2 which differed significantly at 0.99 autocorrelation levels.

In equation 2, the estimators are affected by autocorrelation under variance and mean square error criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that all estimators are preferred to estimate  $\beta_0$  at all levels of autocorrelation except for GLS2 which differed significantly at autocorrelation level of 0.99 in both criteria considered.

Summarily, we can infer that all the estimators are preferred to estimate  $\beta_0$  except GLS2 at all five sample sizes under consideration.

**4.5.5.2 EFFECT ON  $\beta_1$**

Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all the levels of autocorrelation.

In equation 2, the estimators are neither affected by autocorrelation nor correlation between the error terms under all criteria.

Summarily, we can infer that GLS2 and MLE estimators are preferred to estimate  $\beta_1$  at all five sample sizes under consideration and at all levels of autocorrelation and correlation between the error terms.

**4.5.5.3 EFFECT ON  $\beta_2$**

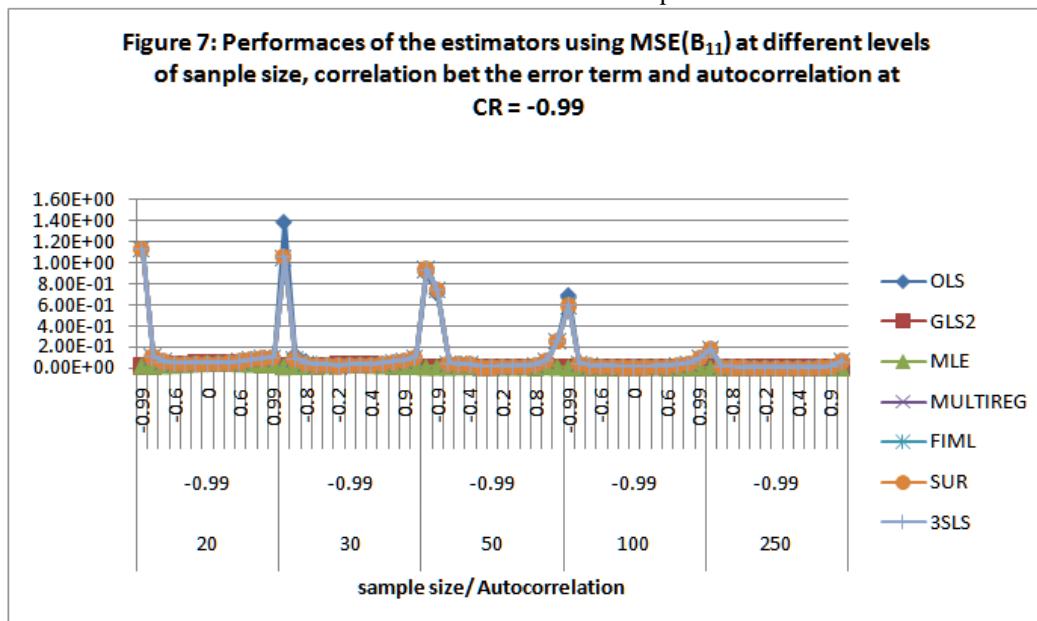
Consequently in equation 1, it can be inferred that the performances of the estimators are affected by autocorrelation and correlation between the error terms under all criteria. The results of the LSD further test visa- vice their estimated marginal means revealed that GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all the levels except at -0.99 and +0.99 correlation between the error terms under absolute bias.

In equation 2, the estimators are affected by autocorrelation and correlation between the error terms under variance criterion. The results of the LSD further test visa- vice their estimated marginal means as shown in revealed that all estimators except OLS, GLS2 and MLE estimators are preferred to estimate  $\beta_2$  at all levels of autocorrelation and correlation between the error terms.

Summarily, we can infer that GLS2 and MLE estimators are preferred to estimate  $\beta_2$ .

**Summarily, MLE estimator is preferred to estimate the model at sample size of 250**

Conclusively, the estimator of MLE is preferred to estimate all the parameters of the model in the presence of correlation between the error terms and autocorrelation at all the sample sizes.



In fig.7, the plot of MSE values against different sample sizes for all the estimators revealed appreciable increase in efficiency (lower MSE) of the estimators as sample size increases with MLE estimator showing a better performance over GLS2.

### V. SUMMARY OF THE FINDINGS

#### 5.1.5 When there is correlation between the error terms and Autocorrelation

The summary of results from the Analysis of variance tables of the criteria showing the performances of the estimators and sample sizes on parameters of the two equation model when there is presence of correlation between the error terms and autocorrelation are presented in Table 5.1.5

**Table 5.1.5:** Summary of results when there is presence of correlation between the error terms and autocorrelation

n	EQ N	PARAM ETERS	PREFERRED	OVERALL ASSESSMENT	MOST PREFERRED
20	1	$\beta_{01}$	All except CORC	CORC, MLE	MLE
		$\beta_{11}$	CORC, MLE		
		$\beta_{21}$	CORC, MLE		
	2	$\beta_{02}$	All except CORC	All except CORC	
		$\beta_{12}$	All		
		$\beta_{22}$	SUR, 3SLS		
30	1	$\beta_{01}$	All except CORC	CORC, MLE	MLE
		$\beta_{11}$	CORC, MLE		
		$\beta_{21}$	CORC, MLE		
	2	$\beta_{02}$	All except CORC	MLE.SUR,3SLS	
		$\beta_{12}$	CORC, MLE		
		$\beta_{22}$	MulReg,FIML,SUR,3SLS		
50	1	$\beta_{01}$	All	CORC, MLE	CORC, MLE
		$\beta_{11}$	CORC, MLE		
		$\beta_{21}$	CORC, MLE		
	2	$\beta_{02}$	All	CORC,MLE.SUR,3SLS	
		$\beta_{12}$	CORC, MLE		
		$\beta_{22}$	MulReg,FIML,SUR,3SLS		
100	1	$\beta_{01}$	All except CORC	CORC, MLE	MLE
		$\beta_{11}$	CORC, MLE		
		$\beta_{21}$	CORC, MLE		
	2	$\beta_{02}$	All except CORC	MLE,MulReg,FIML,SUR,3SLS	
		$\beta_{12}$	CORC, MLE		
		$\beta_{22}$	MulReg,FIML,SUR,3SLS		
250	1	$\beta_{01}$	All except CORC	CORC, MLE	MLE
		$\beta_{11}$	CORC, MLE		
		$\beta_{21}$	CORC, MLE		
	2	$\beta_{02}$	All except CORC	All except CORC	
		$\beta_{12}$	All		
		$\beta_{22}$	MulReg,FIML,SUR,3SLS		

From table 5.1.5 when there is presence of correlation between the error terms and autocorrelation in the model under the equation 1 in all the five sample sizes, all the estimating methods except CORC are equally good in estimating the parameters  $\beta_{01}$ , meanwhile, for parameters  $\beta_{11}$  and  $\beta_{21}$  CORC and MLE estimators are good for their estimation thus it can be concluded that MLE estimating method is preferred in estimating all the model parameters in equation 1.

Under equation 2, it was observed that all estimation methods except CORC are good in estimating all the parameters of the model at all level of the sample sizes.

However, observing the two equations together, we can conclude that MLE is the most preferred in estimating all the parameters of the two equations among all the estimation methods used.

## 5.2. RECOMMENDATION

The research work has revealed that MLE method of estimation is the most preferred estimator in estimating all the parameters of the model based on the four criteria used namely; Bias, Absolute Bias, Variance and Mean Square Error under the five level of sample sizes considered. It can therefore be recommended that when the validity of other correlation assumptions cannot be authenticated in seemingly unrelated regression model, the most preferred estimator to use is MLE. Meanwhile, for any SUR model without any form of correlation, SUR estimation method is most preferred.

## 5.3. SUGESTION FOR FURTHER STUDY

This study considered two- equation model with two depended variables in each equation, a future research may consider situation in which more than two equations and as many depended variables as possible.

One may still consider a Bayesian estimation approach as one of the estimation methods in order to test its own potential.

## REFERENCES

- [1]. **Adebayo, S. B., (2003)**, Semiparametric Bayesian Regression for Multivariate Response. *Hieronymus, Munich*.
- [2]. **Amemiya, T., (1977)**, A note on a Heteroscedastic Model. *Journal of Econometrics*, 6: 365 – 370
- [3]. **Ayinde K.&Oyejola B.A. (2007)**: A comparative Study of the Performances of the OLS and Some GLS Estimators when Stochastic Regressors are correlated with the error terms. *Research Journal of Applied Sciences*. 2 (3):215 -220. Published by Medwell Journals, Faisalabad, Pakistan.
- [4]. **Foschi, P., (2004)**, Numerical Methods for estimating Linear Econometric Models. Ph. D. Thesis. Institute d'informatique, Université de Neuchâtel, Switzerland.
- [5]. **Green,W.(2003)**, Econometric Analysis (5<sup>th</sup> Edition). New york Macmillan. Pearson Education Inc. USA.
- [6]. **Jackson, J.E., (2002)**, A Seemingly Unrelated Regression Model for Analyzing Multiparty Elections. *Political Analysis*. 10: (1), 49-65.
- [7]. **Kmenta,J.&Gilbert,R.F (1968)**,Small sample properties of alternative estimators of seemingly unrelated regressions.*J. Amer. Statist. Asso*.63:1180-1200.
- [8]. **Lang, S., Adebayo, S. B., and Fahrmeir, L., (2002)**, Bayesian Semiparametric Seemingly Unrelated Regression. In: *Härdle, W. and Roenz, B. (eds):195 – 200. Physika-Verlag, Heidelberg*.
- [9]. **Lang, S., Adebayo, S. B., Fahrmeir, L. and Steiner, W. J., (2003)**, Bayesian Geoadditive Seemingly Unrelated Regression. *Computational Statistics*, 18(2):263 – 292.
- [10]. **Locke, C. and Spurrier, J. D. (1977)**. The Use of U – Statistics for Testing Normality against Alternatives with both tail Light. *Biometrika*, 64, 638-640.
- [11]. **Maeshiro, A., (1980)**, New evidence on Small sample Properties of Estimators of SUR models with Autocorrelated Disturbances. *Journal of Econometrics*. 12: 177-187.
- [12]. **Mandy, D.M. and Martins-Filho ,C. (1993)**, "Seeming Unrelated Regressions Under additive heteroscedasticity: theory and share equation," *Journal of Econometrics*,58, 315-346.
- [13]. **Markov, A.A. (1900)**. *Wahrscheinlichkeitsrechnung*. Leipzig: Tuebner.
- [14]. **O'Donnell,C.J.;Shumway,C.R.&Ball,V.E.(1999)**. Input demands and inefficiency in US agriculture. *American Journal of Agricultural Economics*.81:865-880.
- [15]. **Powell, A. A., (1965)**, Aitken Estimators as a tool in Allocating Predetermined Aggregates. *Journal of the American Statistical Association*.64: 913 – 922.
- [16]. **Revankar,N.S.(1974)**. Some finite sample results in the context of seemingly unrelated regression equations. *J. Amer. Statist. Asso*.69:187-190.
- [17]. **Sparks, R., (2004)**, SUR Models Applied To an Environmental Situation with Missing Data and Censored Values.*Journal of Applied Mathematics and Decision Sciences*. 8: (1), 15-32.
- [18]. **Sparks,R.S.(1987)**.Selecting estimators and variables in the seemingly unrelated regression model.*Commun.Statist.-simula*, 16:99-127.
- [19]. **Srivasta,V.K.&Giles,D.E.A.(1987)**. SURE equations model: Estimation and inference, NewYork: Marcel Dekker.
- [20]. **TSP (2005)**. Users Guide and Reference Manual. Time Series Processor. New York.
- [21]. **Wilde,P.E.; McNamara,P.E.&Raney,C.K.(1999)**. The effect of income and food programs on dietary quality: A seemingly unrelated regression analysis with error components. *Amer. J. of Agric. Economics*81(4):959-971.
- [22]. **W. B. Yahya et al(2008)** Effects of non-orthogonality on the efficiency of seemingly unrelated regression (SUR) models. *InterStat Journal May 2008*
- [23]. **Zellner,A.(1962)**.An efficient method of estimating seemingly unrelated regression equations and test for aggregation bias.*J. Amer. Statist. Asso*.57:348-368.
- [24]. **Zellner,A.(1963)**. Estimators for seemingly unrelated regression equations: Some exact finite sample results *J. Amer. Statist. Asso*.58:977-992.