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Optimal control of two-phase M/M/1 queueing system with server Start-up, N-Policy, unreliable server and Balking

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ABSTRACT
In this paper, an M/M/1 queueing system with server Start-up, N-Policy, breakdowns at two phases of service and Balking is analyzed. Customers arrive at the system according to a Poisson process. They receive the first phase service as a batch followed by second essential phase of individual service. Arriving customers may balk with a certain probability and may depart without getting service due to impatience. Lack of service occurs when the server is on vacation, busy during the service mode or due to the sudden breakdowns of the server. For this model the probability generating functions for the number of customers present in the system at various states of the server are derived and obtained the closed-form expressions for various performance measures of interest. Further a total expected cost model is formulated to determine the optimal threshold of N at a minimum cost. Finally, numerical examples are given.

KEYWORDS: *Vacation, Start-up, Server Breakdowns, Balking, Cost model.*

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I. INTRODUCTION

We consider the optimal policy of a removable and un-reliable server for an M/M/1 two-phase queueing system, where the removable server operates an *N* policy.

Two-phase queuing systems have been discussed in the past for their applications in various areas, such as computer, communication, manufacturing, and other stochastic systems. In many computer and communication service systems, the situation in which arriving packets receive batch mode service in the first phase followed by individual services in the second phase is common. As related literature we should mention some papers [1,7,12] arising from distributed system control where all customers receives batch mode service in the first phase followed by individual service in the second phase.

The concept of the *N* policy, was first introduced by Yadin and Naor [13]. Past work regarding queueing systems under the *N* policy may be divided into two categories: (i) cases with server vacations, and (ii) cases with server breakdowns.

Queueing systems with server vacations have attracted much attention from numerous researchers since the paper was presented by Levy and Yechiali[9]. Server vacations are useful for the system where the server wants to utilize his idle time for different purposes. An excellent survey of queueing systems with server vacations can be found in papers by Doshi and Takagi[3,4,5,11]. Queueing models with server vacations accommodate the real-world situations more closely. Such model frequently occurs in areas of computer and communications, or manufacturing systems. For example, consider an assembly line where a worker may have some idle time between subsequent jobs. To utilize the time effectively, managers can assign secondary jobs to the worker. However, it is important that the worker must return to do his primary jobs when he completes the secondary jobs.

Queueing systems with impatient customers have been studied by a number of authors [2,10].There is an extensive amount of literature based on this kind of model .The source of impatience has always been taken to be either a long wait already experienced upon arrival at a queue, or a long wait anticipated by a customer upon arrival.Haight[6] first considered an M/M/1 queue with balking.

However, to the best of our knowledge, for two –phase queueing systems with N-Policy, server breakdowns, there is no literature which takes customers' impatience into consideration. This motivates us to study a two-phase queueing system with N-policy, server start-up, breakdowns and balking. Thus, in this present paper, we consider two-phase M/M/1 queueing system with server Start-up, N-Policy, unreliable server and Balking where customers become impatient when the server is unavailable.

The article is organized as follows. A full description of the model is given in Section. 2. The steady-state analysis of the system state probabilities is performed through the generating in Section. 3 while some, very useful for the analysis, results on the expected number of customers in different states and some special cases are given in Section. 4. In Section. 5 the characteristic features of the system are investigated. Optimal control policy is explained in section.6, while, in Section. 7, numerical results are obtained and used to compare system performance under various changes of the parameters through sensitivity analysis.

The main objectives of the analysis carried out in this paper for the optimal control policy are:

i. To establish the steady state equations and obtain the steady state probability distribution of the number of customers in the system in each state.
ii. To derive expressions for the expected number of customers in the system when the server is in vacation, in startup, in batch service (working and broken conditions) and in individual service (working and broken conditions) respectively.
iii. To formulate the total expected cost functions for the system, and determine the optimal value of the control parameter N.
iv. To carry out sensitivity analysis on the optimal value of N and the minimum expected cost for various system parameters through numerical experiments.

II. THE SYSTEM AND ASSUMPTIONS

We consider the M/M/1 queueing system with N-policy, two phases of service, server Breakdowns and balking with the following assumptions:

1. Customers are assumed to arrive according to Poisson process with mean arrival rate λ and join the batch queue. Customers will get the service in the order in which they arrive.
2. The service is in two phases. The first phase of service is batch service to all customers waiting in the queue. On completion of batch service, the server immediately proceeds to the second phase to serve all customers in the batch individually. Batch service time is assumed to follow exponential distribution with mean $1/\beta$ which is independent of batch size. Individual service times are assumed to be exponentially distributed with mean $1/\mu$. On completion of individual service, the server returns to the batch queue to serve the customers who have arrived. If the customers are waiting, the server starts the batch service followed by individual service to each customer in the batch. If no customer is waiting the server takes a vacation.
3. Whenever the system becomes empty, the server is turned off. As soon as the total number of arrivals in the queue reaches or exceeds the pre-determined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. The server needs a startup time which follows an exponential distribution with mean $1/\theta$. As soon as the server finishes startup, it starts serving the first phase of waiting customers.
4. The customers who arrive during the batch service are also allowed to join the batch queue which is in service.
5. The breakdowns are generated by an exogenous Poisson process with rates ξ_1 for the first phase of service and α_1 for the second phase of service. When the server fails it is immediately repaired at a repair rate ξ_2 in first phase and α_2 in second phase, where the repair times are exponentially distributed. After repair the server immediately resumes the concerned service.
6. A customer may balk from the queue station with probability b_0 when the server is in vacation or may balk with a probability b_1 when the server is in service made due to impatience.

III. STEADY-STATE ANALYSES

In steady – state the following notations are used.

$p_{0,i,0}$ = The probability that there are i customers in the batch queue when the server is on vacation, where $i = 0,1,2,3,\ldots,N-1$

$p_{1,i,0}$ = The probability that there are i customers in the batch queue when the server is doing pre-service (startup work), where i = N,N+1,N+2,…

$p_{2,i,0}$= The probability that there are i customers in the batch queue when the server is in batch service where i = 1,2,3,…

$p_{3,i,0}$= The probability that there are i customers in batch queue when the server is working but found to be broken down, where i = 1,2,3,…

$p_{4,i,j}$= The probability that there are i customers in the batch queue and j customers in individual queue when the server is in individual service, where i=0,1,2… and j=1,2,3,…

$p_{5,i,j}$=The probability that there are i customers in the batch queue and j customers in individual queue when the server is working but found to be broken down, where i = 0,1,2,… and j = 1,2,3, …

The steady-state equations governing the system size probabilities are as follows:

$\lambda b_0 p_{0,0,0} = \mu p_{4,0,1}$. (1)

$\lambda b_0 p_{0,i,0} = \lambda b_0 p_{0,i-1,0}\,; 1 \le i \le N-1.$ (2)

$(\lambda b_1 + \theta) p_{1,N,0} = \lambda b_0 p_{0,N-1,0}$. (3)

$(\lambda b_1 + \theta) p_{1,i,0} = \lambda b_1 p_{1,i-1,0}; i > N.$ (4)

$(\lambda b_1 + \beta + \xi_1) p_{2,i,0} = \lambda b_1 p_{2,i-1,0} + \mu\, p_{4,i,1} + \xi_2 p_{3,i,0}\,; 1 \le i \le N-1.$ (5)

$(\lambda b_1 + \beta + \xi_1) p_{2,i,0} = \lambda b_1 p_{2,i-1,0} + \mu\, p_{4,i,1} + \xi_2 p_{3,i,0} + \theta p_{1,i,0}; i \ge N$. (6)

$(\lambda b_1 + \xi_2) p_{3,i,0} = \lambda b_1 p_{3,i-1,0} + \xi_1 p_{2,i,0}\,; i \ge 1$.(7)

$(\lambda b_1 + \alpha_1 + \mu) p_{4,0,j} = \mu p_{4,0,j+1} + \beta p_{2,j,0} + \alpha_2 p_{5,0,j}; j \ge 1$.(8)

$(\lambda b_1 + \alpha_1 + \mu) p_{4,i,j} = \mu p_{4,i,j+1} + \lambda b_1 p_{4,i-1,j} + \alpha_2 p_{5,i,j}; i,j \ge 1$(9)

$(\lambda b_1 + \alpha_2) p_{5,0,j} = \alpha_1 p_{4,0,j}; j \ge 1$. (10)

$(\lambda b_1 + \alpha_2) p_{5,i,j} = \alpha_1 p_{4,i,j} + \lambda b_1 p_{5,i-1,j}; i,j \ge 1$. (11)

To obtain the analytical closed expression of$p_{0,0,0}$, the technique of probability generating function can be successfully applied as detailed below. Define probability generating functions associated with marginal queue size distributions as follows:

$G_0(z) = \sum_{i=0}^{N-1} p_{0,i,0} z^i$, $G_1(z) = \sum_{i=N}^{\infty} p_{1,i,0} z^i$,

$G_2(z) = \sum_{i=1}^{\infty} p_{2,i,0} z^i$, $G_3(z) = \sum_{i=1}^{\infty} p_{3,i,0} z^i$,

$G_4(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} p_{4,i,j} z^i y^j$, $G_5(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} p_{5,i,j} z^i y^j$

and$R_j(z) = \sum_{i=0}^{\infty} p_{4,i,j} z^i$.

Multiplication of equation (2) by z^i and adding over i (1 ≤ i ≤ N-1) gives

$G_0(Z) = \frac{(1-z^N)}{(1-z)} p_{0,0,0}$. (12)

Multiplication of equations (3) and (1.4) by z^i and adding over i ($i \geq N$) gives

$$(\lambda b_1(1-z)+\theta)\ \ G_1(Z) = (\lambda b_0 z^N)p_{0,0,0}. \tag{13}$$

Multiplication of equations (1.5) and (1.6) by z^i and adding over i ($i \geq 1$) gives

$$(\lambda b_1(1-z)+\ \beta\ +\xi_1)\, G_2(Z) = \xi_2 G_3(z) +\ \mu\, R_1(z) +\ \theta\ G_1(z) -\ \lambda b_0 p_{0,0,0}\ . \tag{14}$$

Multiplication of equation (1.7) by z^i and adding over i ($i \geq 1$) gives

$$(\lambda b_1(1-z)+\xi_2)\, G_3(Z) = \xi_1 G_2(z). \tag{15}$$

Multiplication of equations (1.8) and (1.9) by $z^i y^j$ and adding over Corresponding values of i and j gives

$$\Big(\lambda b_1 y(1-z) + \alpha_1 y -\ \mu(1-y)\Big)\, G_4(z,y) =$$

$$\Big(\alpha_2 G_5(z,y) +\ \beta\ G_3(y) -\ \mu R_1(z)\Big)\, y. \tag{16}$$

Multiplication of equations (10) and (11) by $z^i y^j$ and adding over Corresponding values of i and j gives

$$(\lambda b_1(1-z)+\alpha_2) G_5(z,y) = \alpha_1 G_4(z,y). \tag{17}$$

The total probability generating function G (z, y) is given by

$$G(z,y) = G_0(z) + G_1(z) + G_2(z) + G_3(z) + G_4(z,y) + G_5(z,y). \tag{18}$$

The normalizing condition is

$$G(1,1) = G_0(1) + G_1(1) + G_2(1) + G_3(1) + G_4(1,1) + G_5(1,1) = 1 \tag{19}$$

From equations (12) to (19)

$$G_0(1) = N p_{0,0,0}\ \ , \tag{20}$$

$$G_1(1) = \left(\frac{\lambda b_0}{\theta}\right) p_{0,0,0}, \tag{21}$$

$$G_2(1) = \left(\frac{\mu}{\beta}\right) R_1(1), \tag{22}$$

$$G_3(1) = \left(\frac{\xi_1}{\xi_2}\right) G_2(1) \tag{23}$$

$$G_4(1,1) = \left(\frac{\alpha_2\big(\beta G_2'(1) - \mu R_1'(1)\big)}{\mu\alpha_2 - \lambda b_1(\alpha_1+\alpha_2)}\right)$$

let $t_1 = \big(\mu\alpha_2 - \lambda b_1(\alpha_1+\alpha_2)\big)$, then

$$G_4(1,1) = \left[\frac{\frac{\left(\left(t_1\left(1 - p_{0,0,0}\left(\frac{\lambda b_0}{\theta}+N\right)\right)\right) + (\alpha_1+\alpha_2)\frac{\lambda b_0(\lambda b_1+N\theta)}{\theta}\right)\lambda b_{1\mu}\ (\xi_1+\xi_2)\Big)}{\mu^2\alpha_2(\xi_1+\xi_2)} + \left(\frac{\lambda b_0(\lambda b_1+N\theta)}{\theta}\right)p_{0,0,0}}{t_1}\right]\alpha_2. \tag{24}$$

$$G_5(1,1) = \left(\frac{\alpha_1}{\alpha_2}\right) G_4(1,1). \tag{25}$$

The normalizing condition (19) gives,

$$R_1(1) = \frac{\left(\left(t_1\left(1-p_{0,0,0}\left(\frac{\lambda b_0}{\theta}+N\right)\right)+(\alpha_1+\alpha_2)\frac{\lambda b_0(\lambda b_1+N\theta)}{\theta}\right)\beta\xi_2\right)}{\mu^2\alpha_2(\xi_1+\xi_2)} .$$

Substituting the value of $R_1(1)$ from (22) to (25) gives $G_2(1)$, $G_3(1)$, $G_4(1,1)$ and $G_5(1,1)$.

Probability that the server is neither in batch service nor in individual service is given by $G_0(1) + G_1(1) = 1 - \left(\frac{\lambda b_1}{\beta}\left(1+\frac{\xi_1}{\xi_2}\right) + \frac{\lambda b_1}{\mu}\left(1+\frac{\alpha_1}{\alpha_2}\right)\right)$.

This gives $p_{0,0,0} = (1-\rho)\frac{\theta}{(\lambda b_0+N\theta)}$. (26)

Where $\rho = \left(\frac{\lambda b_1}{\beta}\left(1+\frac{\xi_1}{\xi_2}\right) + \frac{\lambda b_1}{\mu}\left(1+\frac{\alpha_1}{\alpha_2}\right)\right)$ is the utilizing factor of the system.

From Equation (26) we have $\rho < 1$, which is the necessary and sufficient condition under which steady state solution exits.

Under steady state conditions, let $p_0, p_1, p_2, p_3, p_4,$ *and* p_5 be the probabilities that the server is in vacation, startup, in batch service, in batch service with break down, in individual service and in individual service with break-down states respectively. Then,

$p_0 = G_0(1),$ (27)

$p_1 = G_1(1),$ (28)

$p_2 = G_2(1),$ (29)

$p_3 = G_3(1),$ (30)

$p_4 = G_4(1,1).$ (31)

$p_5 = G_5(1,1).$ (32)

4.1 Expected number of customers at different states of the server

Using the probability generating functions expected number of customers in the system at different states are presented below.

Let L_0, L_1, L_2, L_3, L_4 and L_5 be the expected number of customers in the system when the server is in idle, startup, batch service, break down in batch service, individual service and break down in individual states respectively.

Then

$L_0 = \sum_{i=0}^{N-1} i\, p_{0,i,0} = G_0'(1) = \frac{N(N-1)}{2} p_{0,0,0}.$ (33)

$L_1 = \sum_{i=N}^{\infty} i\, p_{1,i,0} = G_1'(1) = \frac{\lambda b_0(\lambda b_1+N\theta)}{\theta^2} p_{0,0,0}.$ (34)

$L_2 = \sum_{i=1}^{\infty} i\, p_{2,i,0} = G_2'(1) = \left(\frac{\lambda b_1(\xi_1+\xi_2)G_2(1)+\theta\xi_2 G_1'(1)}{t_1\beta\xi_2}\right)\mu\alpha_2.$ (35)

$L_3 = \sum_{i=1}^{\infty} i\, p_{3,i,0} = G_3'(1) = \frac{\xi_1(G_2'(1)\xi_2+\lambda b_1 G_2(1))}{\xi_2^2}.$ (36)

$L_4 = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} (i+j)\, p_{4,i,j} = G_4'(1,1)$

$$= \frac{\left[\alpha_2\left(\beta G_2''(1) - \mu R_1''(1)\right) + 2(\alpha_2 - \lambda b_1)\left(\beta G_2'(1) - \mu R_1'(1)\right) - 2\lambda b_1 G_4(1,1)(\lambda b_1 - (\alpha_1 + \alpha_2 + \mu))\right]}{2t_1}. \quad (37)$$

$L_5 = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} (i+j)\, p_{5,i,j} = G_5'(1,1)$

$$= \frac{\alpha_1}{\alpha_2} L_4 + \frac{\lambda b_1}{\alpha_2} G_5(1,1). \quad (38)$$

The expected number of customers in the system is given by

$$L(N) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5. \quad (39)$$

4.2 Special cases

In this section we present some existing results in the literature which are special cases of our system.

(i) When there is no startup, breakdowns in both the phases of service, balking and N=1,the equation (39) becomes

(ii) $L(N) = \frac{\lambda}{(\mu-\lambda)} + \frac{\lambda\mu}{\beta(\mu-\lambda)}$. (39a)

This is the expected system length for the two-phase M/M/1 queueing system without gating. The obtained expression coincides with that of Krishna and Lee (1990).

(iii) When there is no batch service, the equation (39 a) becomes

$$L(N) = \frac{\lambda}{(\mu-\lambda)}. \quad (39b)$$

This is the expected system length for the M/M/1 queueing system.

IV. CHARACTERISTIC FEATURES OF THE SYSTEM

In this section, we obtain the expected system length when the server is in different states. Let E_0, E_1, E_2, E_3, E_4 *and* E_5 denote the expected length of vacation period, startup period, batch service period , batch service breakdown period, individual service period, and breakdown period during individual service respectively. Then the expected length of a busy cycle is given by

$$E_c = E_0 + E_1 + E_2 + E_3 + E_4 + E_5.$$

The long run fractions of time the server is in different states are as follows:

$$\frac{E_0}{E_c} = p_0, \quad (40)$$

$$\frac{E_1}{E_c} = p_1, \quad (41)$$

$$\frac{E_2}{E_c} = p_2, \quad (42)$$

$$\frac{E_3}{E_c} = p_3, \quad (43)$$

$$\frac{E_4}{E_c} = p_4, \quad (44)$$

$$\frac{E_5}{E_c} = p_5. \quad (45)$$

Expected length of vacation period is given by

$$E_0 = \frac{N}{\lambda b_0}. \quad (46)$$

Hence,

$$E_c = \frac{1}{(\lambda b_0 p_{0,0,0})}. \qquad (47)$$

V. OPTIMAL CONTROL POLICY

In this section, we determine the optimal value of N that minimizes the long run average cost of two- phase M/M/1, N-policy queue with server break downs and balking. To determine the optimal value of N, we consider the following cost structure:

Let T (N) be the average cost per unit of time, then

$$T(N) = C_h L(N) + C_o\left(\frac{E_2}{E_c}+\frac{E_4}{E_c}\right) + C_m\left(\frac{E_5}{E_c}\right) + C_{b1}\left(\frac{E_3}{E_c}\right) + C_{b2}\left(\frac{E_5}{E_c}\right) + C_s\left(\frac{1}{E_c}\right)$$

$$+C_b\big(\lambda(1-b_0)p_0 + \lambda(1-b_1)(p_1+p_2+p_3+p_4+p_5)\big) - C_r\left(\frac{E_0}{E_c}\right).$$

(48)

Where

C_h= Holding cost per unit time for each customer present in the system,

C_o = Cost per unit time for keeping the server on and in operation,

C_m = Startup cost per unit time,

C_s = Setup cost per cycle,
C_{b1}= Break down cost per unit time for the unavailable server in batch service mode,
C_{b2}= Break down cost per unit time for the unavailable server in individual service mode,
C_b = Cost per unit time when a customer balks,
C_r = Reward per unit time as the server is doing secondary work in vacation.

For the determination of the optimal operating N-policy, minimize T (N) in equation 48.

An approximate value of the optimal threshold N* can be found by solving the equation

$$\left.\frac{dT_1(N)}{dN}\right|_{N=N^*} = 0. \qquad (49)$$

A computational algorithm translated in MATLAB is used to obtain the optimum values.

VI. SENSITIVITY ANALYSIS

In order to verify the efficiency of our analytical results, we perform numerical experiment by using MATLAB. The variations of different parameters (both monetary and non-monetary) on the optimal threshold N*, mean number of jobs in the system and minimum expected cost are shown.

Parameters for which the model is relatively sensitive would require more attention of researchers, as compared to the parameters for which the model is relatively insensitive or less sensitive.

We perform the sensitivity analysis by fixing

Non –monetary parameters as

$\lambda=0.5, \mu=8, \alpha_1=0.2, \alpha_2=3.0, \xi_1=0.2, \xi_2=0.3, \theta=6, \beta=12$, $b_0=0.4$, $b_1=0.2$ and monetary parameters as

$C_r=15, C_{b1}=50, C_{b2}=75, C_b=15, C_m=200, C_h=5$ and $C_s=1000$;

7.1. Effect of variation in the non-monetary parameters

Table 1: Effect of **λ** on N*, expected system length and minimum expected cost

λ	0.9	3	1.7	2.1	2.5	2.9
N*	12	14	16	18	19	20
L(N*)	6	7	7	8	9	9
T(N*)	51.58	69.7	81.97	96.12	107.39	117.67

It is observed from the above table that with increase in values of λ,

a) N* is increasing.
b) Mean number of customers in the system is increasing.
c) Minimum expected cost is increasing.

Table 2: Effect of on **μ andβ** N*, expected system length and minimum expected cost

μ,β μ,β	9,12.05	10,12.10	11,12.15	12.12.20	13,12.25	14,12.30
N*	9,9	9,9	9,9	9,9	9,9	9,9
L(N*)	4,4	4,4	4,4	4,4	4,4	4,4
T(N*)	31.71, 31.92	31.58, 31.91	31.48, 31.91	31.39, 31.91	31.31, 31.91	31.25, 31.91

It is observed from the above Table that, with increase in values of **μ and β,**
a) N* is almost insensitive.
b) Mean number of customers in the system is almost insensitive.
c) Minimum expected cost is decreasing.

Table 3: Effect of **α_1 and α_2**on N*, expected system length and minimum expected cost

α_1, α_2	0.205, 1.1	0.21, 1.2	0.215, 1.3	0.220, 1.4	0.225, 1.5	0.230, 1.6
N*	9,9	9,9	9,9	9,9	9,9	9,9
L(N*)	4,4	4,4	4,4	4,4	4,4	4,4
T(N*)	31.92, 31.87	31.92, 31.87	31.92, 31.86	31.93, 31.86	31.93, 31.86	31.93, 31.85

It is observed from the above table that with increase in values of **α_1 and α_2,**

a) N* is almost insensitive.
b) Mean number of customers in the system is insensitive.
c) Minimum expected cost is slightly increasing with**α_1**andslightly decreasing with **α_2.**

Table 4: Effect of **θ** on N*, expected system length and minimum expected cost

θ	7	8	9	10	11	12
N*	9	9	9	9	9	9
L(N*)	4	4	4	4	4	4
T(N*)	31.72	31.64	31.58	31.54	31.51	30.89

It is observed from the above table that with the increase in values of θ,
a) N* is decreasing.
b) Mean number of customers in the system is decreasing.
c) Minimum expected cost is decreasing.

Table 5: Effect of ξ_1 **and** ξ_2 on N*, expected system length and minimum expected cost

ξ_1, ξ_2	0.3, 0.305	0.4, 0.31	0.5, 0.315	0.6, 0.320	0.7, 0.325	0.8, 0.330
N*	9,9	9,9	9,9	9,9	9,9	9.9
L(N*)	4,4	4,4	4,4	4,4	4,4	4,4
T(N*)	31.87, 31.92	31.91, 31.91	34.00, 31.91	34.16, 31.91	34.39, 31.91	34.7, 31.91

It is observed from the above Table that, with increase in values of ξ_1 **and** ξ_2 **,**

a) N* is almost insensitive.
b) Mean number of customers in the system is almost insensitive.
c) Minimum expected cost is increasing with the increase of ξ_1 and slightly decreasing with the increase of ξ_2 **.**

Table 6: Effect of b_0 **and** b_1 on N*, expected system length and minimum expected cost

b_0, b_1	0.4015, 0.25	0.4030, 0.30	0.4045, 0.35	0.4060, 0.40	0.4075, 0.45	0.4090, 0.50
N*	9,9	9,9	9,9	9,9	9,9	9,9
L(N*)	4,4	4,4	4,4	4,4	4,4	4,4
T(N*)	31.99, 34.05	34.07, 34.17	34.14, 34.29	34.22, 34.41	34.3, 34.53	34.37, 34.64

It is observed from the above Table that with the increase in values of b_0 **and** b_1**,**

a) N* is slightly increasing.
b) Mean number of customers in the system is slightly increasing.
c) Minimum expected cost is increasing with both b_0 **and** b_1

7.2. Effect of variation in the monetary parameters

Table 7: Effect of C_r on N*, expected system length and minimum expected cost

Cr	17	19	21	23	25	27
N*	9	9	9	9	9	9
L(N*)	4	4	4	4	4	4
T(N*)	31.98	30.04	28.10	26.16	24.23	22.29

It is observed from the above table that with the increase in values of C_r,

a) N* is slightly increasing.
b) Mean number of customers in the system is slightly increasing.
c) Minimum expected cost is decreasing.

Table 8: Effect of C_{b1} **and** C_{b2} on N*, expected system length and minimum expected cost

C_{b1}, C_{b2}	52,80	54,85	56,90	58,95	60.100	62,105
N*	9,9	9,9	9,9	9,9	9,9	9,9
L(N*)	4,4	4,4	4,4	4,4	4,4	4,4
T(N*)	31.92, 31.9	31.92, 31.94	31.92, 31.94	31.92, 31.95	31.92, 31.96	31.92, 31.97

It is observed from the above table that with the increase in values of C_{b1} **and** C_{b2},

a) N* is almost insensitive.
b) Mean number of customers in the system is almost insensitive.
c) Minimum expected cost is insensitive with C_{b1} and slightly increasing with C_{b2}.

Table 9: Effect of $\mathbf{C_b}$ on N*, expected system length and minimum expected cost

C_b	20	25	30	35	40	45
N*	9	9	9	9	9	9
L(N*)	4	4	4	4	4	4
T(N*)	35.43	36.94	38.463	39.97	41.49	43.01

It is observed from the above table that with increase in values of $\mathbf{C_b}$,

a) N* is increasing.
b) Mean number of customers in the system is increasing.
c) Minimum expected cost is increasing.

Table 10: Effect of $\mathbf{C_m}$ **and** $\mathbf{C_o}$on N*, expected system length and minimum expected cost

C_m, C_o	215,55	230,60	245,65	260,65	275,70	290,75
N*	9,9	9,9	9,9	9,9	9,9	9,9
L(N*)	4,4	4,4	4,4	4,4	4,4	4,4
T(N*)	31.97,34.04	34.03,34.17	34.08,34.3	34.13,34.42	34.19,34.55	34.24,34.67

It is observed from the above table that with increase in values of $\mathbf{C_m}$ **and** $\mathbf{C_o}$**,**

a) N* is insensitive.
b) Mean number of customers in the system is insensitive.
c) Minimum expected cost is increasing.

Table 11: Effect of $\mathbf{C_h}$ **and** $\mathbf{C_s}$ on N*, expected system length and minimum expected cost

C_h, Cs	6,1100	7,1200	8,1300	9,1400	10,1500	11,1600
N*	8,9	8,10	7,10	7,11	6,11	6,11
L(N*)	4,4	3,4	3,5	3,5	3,5	3,5
T(N*)	37.74, 36.03	42.2, 38.06	44.42, 40.00	47.41, 41.87	50.20, 41.68	52.84, 45.43

It is observed from the above table that with the increase in values of $\mathbf{C_h}$ **and** $\mathbf{C_s}$
a) N* is decreasing.
b) Mean number of customers in the system is decreasing with C_h and increasing with Cs.
c) Minimum expected cost is increasing with both C_h and C_s.

VII. CONCLUSIONS

- Two–phase N-policy M/M/1 queueing system with server breakdowns and balking is studied. The closed expressions for the steady state distribution of the number of customers in the system when the server is at different states are obtained and hence the expected system length is derived.
- Total expected cost function for the system is formulated and determined the optimal value of the control parameter N that minimizes the expected cost.
- Sensitivity analysis is performed to discuss how the system performance measures can be affected by the changes of the both non-monetary and monetary input parameters.

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