The International Journal Of Engineering And Science (IJES)
||Volume||2 ||Issue|| 9||Pages|| 46-51||2013||
ISSN(e): 2319 – 1813 ISSN(p): 2319 – 1805

Development of a Diameter Model for the Determination of the Equivalent Lengths of Composite Shafts

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ABSTRACT

The development of an equivalent shaft diameter model is presented. The model is dependent on the masses, diameters, moduli of rigidity and densities of the members of a solid series composite shaft and ensures that the equivalent shaft whose length is subsequently determined has the same mass as its composite. Calculations of the equivalent diameter, length and mass of the equivalent shaft were performed in Excel spreadsheet and its mass was compared to that of the composite. Geometric modeling and analysis of the composite shaft and its equivalent were done in Pro-Engineer software to validate the model. Also, dimensional analysis was used to check the correctness of the expression. This technique can be used in determining the torque and power requirement of shaft-machine element subassembly by converting the machine element and shaft to their equivalent shaft of equal mass and computing its diameter and length, with which the polar moment of inertia, torque and power transmitted by the subassembly are determined.

KEYWORDS: *Equivalent length, Diameter, Composite shaft, Machine elements, Torque, Power requirement,*

Date of Submission: 13, September, 2013 ⟺ Date of Acceptance: 30, September 2013

I. INTRODUCTION

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque set up within the shaft permits the power to be transferred to various machines linked up with the shaft [2]. When two shafts of different diameters are connected together to form one shaft, it is known as composite shaft. If the driving torque is at one and the resisting torque at the other end, then the shafts are said to be connected in series[2]. An example of a solid series composite shaft and its equivalent shaft is shown in figures (1) and (2) respectively.

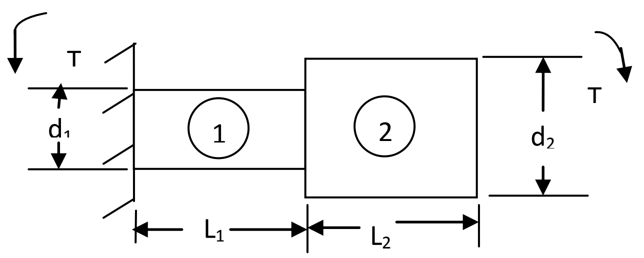

Fig. 1: Solid series composite shaft

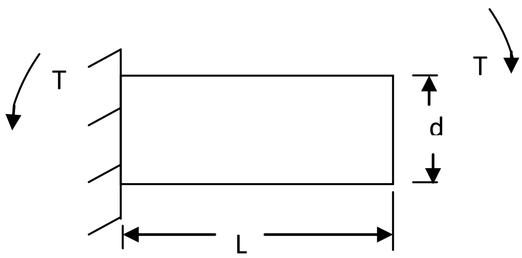

Fig. 2: Equivalent shaft

Assuming the composite shaft is made of steel of density, ℓ= 7850kg/m^3 [1], the effects of variation in the diameter of the equivalent shaft shown in fig. (2) are illustrated in table (1) below.

Table1: Effect of varying diameter on the mass and length of the equivalent shaft of a series composite shaft

d_1 (m)	d_2 (m)	d (m)	L_1 (m)	L_2 (m)	L (m)	d^2 (m^2)	V (m^3)	m (kg)
0.01	0.02	0.01	0.05	0.10	0.06	0.0001	0.000004	0.03
0.01	0.02	0.02	0.05	0.10	0.90	0.0004	0.000283	2.22
0.01	0.02	0.03	0.05	0.10	4.56	0.0009	0.003221	25.29
0.01	0.02	0.04	0.05	0.10	14.40	0.0016	0.018098	142.07

The volumes (V), (V_2), (V_3), mass (m) and the length (L) of the equivalent shaft were determined with equations (10), (11), (12), (13) and (24), respectively. The computation was done in Excel spreadsheet using the density of steel and assuming that the moduli of rigidity (G) of the composite members are the same.

The use of the torsion equation in determining the equivalent length of a composite shaft poses the challenge of having different torque values due to varying assumed diameters, which result in variation in the equivalent lengths and masses of the equivalent shaft, even when the composite shaft is made of the same material. This method is usually based on the condition that the total angle of twist equals the sum of the angles of twist of each member of a series composite shaft. Consequently, the polar moment of inertia, torque and power requirement of the resulting equivalent shaft vary as the equivalent diameter varies. This paper is geared towards exploring the case of shafts subjected to twisting moment or torque only. The series composite shaft envisaged is a shaft-machine element subassembly. The machine elements include: shafts, gears, pulleys, axles, and sprockets. More so, parts such as flywheel and impellers which are attached to shafts can be incorporated in this group. Considering the importance of these elements in machineries, the method proposed herein seeks to achieve the following objectives: (i) To develop a diameter model with which the equivalent length of a composite shaft of equal mass as its equivalent shaft can be determined; (ii) To develop a method of determining the power requirement of shaft-machine elements subassemblies; (iii) To reduce the variation in the power transmitted by composite shafts due to varying assumed equivalent shaft diameters.

II. FUNDAMENTAL CONCEPT

2.1 Determination of the equivalent length of a solid series composite shaft

Considering the composite shaft and its equivalent shown in figures (1) and (2), respectively , the calculation of equivalent length (L) is achieved on the condition that the "total angle of twist of the equivalent shaft equals the sum of angles of twist of the member shafts".

Mathematically, $\theta = \theta_1 + \theta_2 = \frac{T.L}{G.J} = \frac{T.L_1}{G_1.J_1} + \frac{T.L_2}{G_2.J_2}$ [2] ……………………………………………… (1)

Where, θ = Angle of twist of the equivalent shaft, (radian); θ_1 = Angle of twist of shaft (1), (radian); θ_2 = Angle of twist of shaft (2), (radian); J = Polar moment of inertia of the equivalent shaft, (m^4); J_1 = Polar moment of inertia of shaft (1), (m^4); J_2 = Polar moment of inertia of shaft (2), (m^4); G = Modulus of rigidity of the equivalent shaft, (N/m^2); G_1 = Modulus of rigidity shaft (1), (N/m^2); G_2 = Modulus of rigidity shaft (2), (N/m^2); L, L_1, L_2 are the lengths of the equivalent shaft, shaft (1) and shaft (2), (m), respectively; T= Driving Torque = Resisting Torque, (Nm).

Notice that, $J = \frac{\pi d^4}{32}$ --- (2), $J_1 = \frac{\pi d_1^4}{32}$ ------ (3), $J_2 = \frac{\pi d_2^4}{32}$ ……… (4), where d, d_1 and d_2 are the diameters of the equivalent shaft, shaft (1) and (2), respectively. Dividing equation (1) by T , gives

$$\frac{L}{G.J} = \frac{L_1}{G_1.J_1} + \frac{L_2}{G_2.J_2} \quad \text{……………………………………………} \quad (5)$$

Substituting the relations for J, J_1 , J_2 into equation (5), gives

$$\frac{L}{G.d^4} = \frac{L_1}{G_1.d_1^4} + \frac{L_2}{G_2.d_2^4} \quad \text{……………………………………………} \quad (6)$$

Multiplying equation (6) by Gd^4 , yields

$$L = \frac{G.L_1.d^4}{G_1.d_1^4} + \frac{G.L_2.d^4}{G_2.d_2^4} \quad \text{……………………………………………} \quad (7)$$

Hence, equation (7) can be used to determine the equivalent length of the composite shaft, knowing the modulus of rigidity, diameter and length of each member shaft and assuming an equivalent diameter.

2.1 Application of the equivalent diameter and length of a composite shaft

When the equivalent diameter and length of a composite are known, the moment of inertia, J, twisting moment of the shaft and power transmitted by the shaft can be determined as shown in equations (2), (9) and (10), respectively.

$$J = \frac{\pi d^4}{32} \ldots\ldots\ldots (2)$$

Having known G of the shaft material and assumed value for θ, which in most cases does not exceed 1^0 , as long as L< 20d, T is given by the relation

$$T = \frac{G.\theta.J}{L}, (Nm)\ [2] \ldots\ldots\ldots (8)$$

Knowing the speed, of the shaft and the torque,T, the power transmitted by the shaft is given by the formula

$$P = \frac{2\pi.N.T}{60}, (W)\ [3] \ldots\ldots\ldots (9)$$

Where, N = Number of revolutions per minute of the equivalent shaft

Determining the power transmitted by the shaft, paves way for the selection of a driver or prime mover of higher power output.

III. METHODOLOGY

3.1 Derivation of the equivalent diameter model of a solid series composite shaft

Consider the solid series composite shaft shown in fig. (1), from which the equivalent length was determined as

$$L = \frac{G.L_1.d^4}{G_1.d_1^4} + \frac{G.L_2.d^4}{G_2.d_2^4} \ldots\ldots\ldots (7)$$

Let the volume of the equivalent shaft, $V = \frac{\pi d^2 L}{4} \ldots\ldots (10)$, volume of shaft-1, $V_1 = \frac{\pi d_1^2 L_1}{4}, \ldots\ldots (11)$, volume of shaft-2, $V_2 = \frac{\pi d_2^2 L_2}{4}, \ldots\ldots (12)$, where, L, L_1, L_2, d, d_1,and d_2 have been previously defined.

For the mass of the composite shaft to equate that of the equivalent shaft, there is need to introduce the density of the shaft material or densities of the member materials. The density, ℓ, of any substance is defined as its mass per unit volume.

Mathematically, $\ell = \frac{m}{V} \ldots\ldots\ldots (13)$

Let ℓ, V, and m , be the density, volume, and mass of the equivalent shaft. Similarly, ℓ_1, ℓ_2, m_1,and m_2, are the densities and masses of the composite shaft members.

Then, $V = \frac{m}{\ell} \ldots\ldots\ldots (14)$, $V_1 = \frac{m_1}{\ell_1}, \ldots\ldots (15)$, $V_2 = \frac{m_2}{\ell_2}, \ldots\ldots (16)$,

Substituting equations (14), (15) and (16) into equations (10), (11), and (12) accordingly, and making L, L_1, L_2 the subjects, yield

$$L = \frac{4m}{\pi \ell d^2} \ldots\ldots\ldots (17),\quad L_1 = \frac{4m_1}{\pi \ell_1 d_1^2}, \ldots\ldots (18),\quad L_2 = \frac{4m_2}{\pi \ell_2 d_2^2}, \ldots\ldots (19)$$

Substituting equations (17), (18) and (19) into equation (7), gives

$$\frac{4m}{\pi \ell d^2} = \frac{4m_1}{\pi \ell_1 d_1^2}\left(\frac{G}{G_1}\right)\left(\frac{d}{d_1}\right)^4 + \frac{4m_2}{\pi \ell_2 d_2^2}\left(\frac{G}{G_2}\right)\left(\frac{d}{d_2}\right)^4 \quad \ldots (20)$$

Multiplying equation (20) by $\pi\ell/4m\,d^4$, yields

$$\frac{1}{d^6} = \left(\frac{m_1 G\ell}{m G_1 \ell_1}\right)\left(\frac{1}{d_1}\right)^6 + \left(\frac{m_2 G\ell}{m G_2 \ell_2}\right)\left(\frac{1}{d_2}\right)^6 \quad \ldots (21)$$

Taking the reciprocal of the 6th root of equation (21), yields

$$d = \left[\left(\frac{m_1 G\ell}{m G_1 \ell_1}\right)\left(\frac{1}{d_1}\right)^6 + \left(\frac{m_2 G\ell}{m G_2 \ell_2}\right)\left(\frac{1}{d_2}\right)^6\right]^{\frac{-1}{6}} \quad \ldots (22)$$

Equation (22) is the equivalent diameter of the shaft that will give the same mass as the mass of the solid series composite shaft shown in fig. (1).Notwithstanding the materials that the composite members are made of, and the number of members, provided the shaft is in solid series composite form, the equivalent diameter can be determined from equation (23), below:

$$d = \left[\sum_{i=1}^{n}\left(\frac{m_i G\ell}{m G_i \ell_i}\right)\left(\frac{1}{d_i}\right)^6\right]^{\frac{-1}{6}} \quad \ldots (23)$$

Where n is the number of member shafts that make up the composite shaft and i ranges from 1 to n.

Equation (23) is the general model for determining the equivalent diameter of the shaft that has equal mass as its solid series composite. The equivalent length of the shaft can be determined by substituting the computed equivalent diameter into equation (7).

If the shaft is made of the same material, then, $\ell = \ell_1 = \ell_2 = \ell_i,$ and $G = G_1 = G_2 = G_i$ hence, equations (7), (22) and (23) reduce to equations (24), (25) and (26), respectively.

$$L = L_1\left(\frac{d}{d_1}\right)^4 + L_2\left(\frac{d}{d_2}\right)^4 \quad \ldots (24)$$

$$d = \left[\left(\frac{m_1}{m}\right)\left(\frac{1}{d_1}\right)^6 + \left(\frac{m_2}{m}\right)\left(\frac{1}{d_2}\right)^6\right]^{\frac{-1}{6}} \quad \ldots (25)$$

$$d = \left[\sum_{i=1}^{n}\left(\frac{m_i}{m}\right)\left(\frac{1}{d_i}\right)^6\right]^{\frac{-1}{6}} \quad \ldots (26)$$

3.2 Comparison of the mass of the solid series composite shaft with its equivalent

Table2: Computation of the mass of the solid series composite

d_1 (m)	d_2 (m)	L_1 (m)	L_2 (m)	V_1 (m^3)	V_2 (m^3)	m_1 (kg)	m_2 (kg)	m_c (kg)
0.01	0.02	0.05	0.10	0.00000393	0.00003142	0.03083088	0.24664700	0.2775

Table2 shows the computation of the mass (m_c), of the composite shaft shown in fig. (1), in Excel spreadsheet. Steel of density 7850kg/m^3 was assumed to be the shaft material. Equations (11) and (12) were used in calculating V_1 and V_2. The masses m_1 and m_2 were determined from equations (15) and (16), while m_c is the sum of m_1 and m_2.

Table3: Computation of the mass (m_e), of the equivalent shaft of the solid series composite

d_1 (m)	d_2 (m)	m_1 (kg)	m_2 (kg)	m_c (kg)	d (m)	L (m)	V (m^3)	m_e (kg)
0.01	0.02	0.03083100	0.24664700	0.2775	0.014142128	0.225000338	0.00003535	0.2775

Prior to the calculation of the mass (m_e), of the equivalent mass, equations (25), (24) and (10), were used to determine the values of d, L and V, respectively. The total mass of the composite (m_c) was used as m in equation (25). Then, the mass (m_e) was computed with equation (13) using the density of steel, 7850kg/m^3. The computation was done in Excel spreadsheet.

Notice that,” the mass (m_c) of the composite shaft equals the mass (m_e) of the equivalent shaft”.

3.3 Validation of the diameter model using dimensional analysis

Recall that, $$d = \left[\sum_{i=1}^{n}\left(\frac{m_i G \ell}{m G_i \ell_i}\right)\left(\frac{1}{d_i}\right)^6\right]^{\frac{-1}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (23)$$

Notice that, the ratio $m_i G\ell/mG_i\ell_i$ is dimensionless, hence, the dimension of d and d_i is L.

Note: Dimensional analysis makes use of the fact that dimensions can be treated as algebraic quantities. Consequently, quantities can be added or subtracted only if they have the same dimensions [4]. For equation (23) to be correct, the dimension on the left hand side (LHS) must be equal to that on the right hand side (RHS).

Dimension on the LHS = [L]

Dimension on the RHS = $\left[\frac{1}{L^6}\right]^{\frac{-1}{6}} = [L^{-6}]^{\frac{-1}{6}} = [L]^{\frac{6}{6}} = [L]^1 = [L]$

Therefore, equation (23) is dimensionally correct.

3.4 Validation of the diameter model in pro-engineer software

Validation of the diameter model in Pro-Engineer software was achieved via geometric modeling of the composite shaft with diameters and lengths of the members and determining its mass using the analysis tool. Similarly, the equivalent shaft was drawn using the calculated diameter and length and its mass was generated using the analysis tool. The analysis steps include: Draw the composite and equivalent shaft models as shown in figures (3) and (4) in different windows - click analysis icon – scroll to model – click mass properties – change density value to 0.00000000785 – click compute current analysis for preview – highlight and copy volume to mass – paste in MS word. Note: Density of steel = 7850kg/m^3 = 0.00000000785tonne/mm^3.
The results of the analysis of the composite shaft and its equivalent are as shown in table 4.
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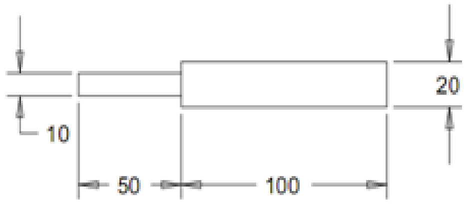

Fig. 3: 2D-geometric model of the composite shaft

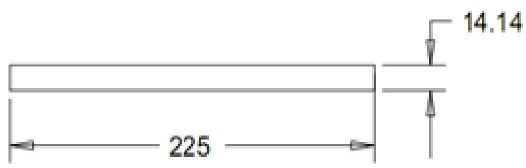

Fig. 4: 2D-geometric model of the equivalent shaft

Table4: Selected mass properties of the composite shaft and its equivalent from Pro-Engineer software

Mass Properties	Composite Shaft	Equivalent Shaft
VOLUME	3.5342917e+04 MM^3	3.5342932e+04 MM^3
SURFACE AREA	8.4823002e+03 MM^2	1.0310655e+04 MM^2

DENSITY	7.8500000e-09 TONNE / MM^3	7.8500000e-09 TONNE / MM^3
MASS	2.7744190e-04 TONNE	2.7744202e-04 TONNE

Notice that, "the mass of the composite shaft equals that of the equivalent".

IV. RESULT AND DISCUSSION

4.1 Result

Table 5: Comparison of the calculated and generated mass properties

Mass Properties	Composite Shaft		Equivalent Shaft	
	Calculated Values	Generated Values	Calculated Values	Generated Values
Volume (m3)	0.00003535	0.00003543	0.00003535	0.00003543
Mass (kg)	0.2775	0.2774	0.2775	0.2774

The negligible deviations of the generated values of the mass and volume values from the calculate values are due to approximations in the calculated equivalent diameter, length and mass. However, the calculated mass and volume of the composite shaft are equal to that of the equivalent shaft. Likewise, their generated masses and volumes are equal.

4.2 Discussion

Though the generated and calculated values of the masses and volumes of the composite shaft and its equivalent are the same, the surface areas generated from the Pro-E software are not equal. Table 4 shows that the surface area of the equivalent shaft is higher than that of the composite. This deviation in surface area may not affect the torque value and power transmitted by the shaft, as the torque is directly proportional to the fourth power of the shaft diameter and inversely proportional to the length.

V. CONCLUSION AND RECOMMENDATION

5.1 Conclusion

Having shown that the developed diameter model is dimensionally correct and the volume and mass of the solid series composite shaft equals that of its equivalent shaft, it can be inferred that the use of the model in determining the equivalent diameters and lengths of composite shafts will result in less deviation from the mass properties, torque and power transmitted by such shafts and related shaft-machine element assemblies. The model showed that the diameter of the equivalent shaft that will give the same mass as its composite shaft is dependent on the moduli of rigidity, masses, densities and diameters of the member shafts. If the composite shaft members are made of the same material, then, diameter of the equivalent shaft will depend on the masses and diameters of the member shafts only.

5.2 Recommendation

Validation of the calculated equivalent diameter and length values in Pro-E software when the shaft members are drawn in different windows and assembled in one window can be achieved through the following steps: Draw each member in different windows – assemble the shaft members in the assembly window – highlight each part and open in different windows - click analysis icon – scroll to model – click mass properties – change density value to the density of the shaft member material (Hint: $1000kg/m^3 = 0.000000001tonne/mm^3$) – click compute current analysis for preview – repeat the same analysis step for other shaft members –[do not close the members window and the analysis results] – activate the assembly window – click analysis icon – scroll to model – click mass properties - click compute current analysis for preview - highlight and copy volume to mass – paste in MS word.

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