MHD Boundary Layer Flow Past a Stretching Plate With Heat Transfer

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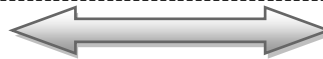[1]*Birla Institute of Technology, Extension Center-Jaipur, Jaipur-302 017 (INDIA).*
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---**Abstract**---
This paper considers the problem of a laminar electrically conducting fluid as a boundary layer flow past a stretching plate and heat transfer. The nonlinear partial differential equations were transformed and the resulting ordinary differential equations were solved by using Maple software. The effects of various parameters are presented and discussed.

Key Words: *MHD, boundary layer flow, heat transfer, stretching plate.*

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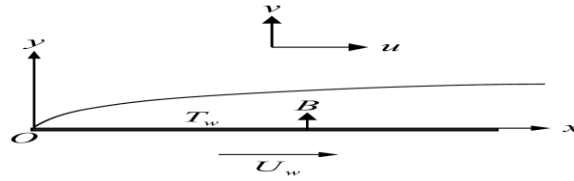## 1. Introduction:

The study of boundary layer flow over a stretching plate has generated much interest in recent years in view of its significant applications in industrial manufacturing processes such as glass-fibber and paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari & Keller [1]).In recent years, MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behaviour over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. In his pioneering work, Sakiadis [2] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [3] extended the work of Sakiadis [2] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by a number of researchers [4-7]. The radiation effects are considered by Ghaly [8] for the magnetohydrodynamics free-convection flow and Raptis et al. [9] for the steady MHD asymmetric flow of an electrically conducting fluid past a semi-infinte stationary plate. Liu [10] analyzed the hydromagnetic fluid flow past a stretching sheet in the presence of a uniform transverse magnetic field. Chen [11] investigated the fluid flow and heat transfer on a stretching vertical sheet, and his work has been extended by Ishak et al. [12] to hydromagnetic flow and they found that as the magnetic field increases, the surface skin friction as well as the surface Nusselt number decrease. Kandasamy et al. [13] analyzed effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection. Muhaimin et al.[14] studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet with suction. Rajesh [15] investigates chemical reaction and radiation effects on the transient MHD free convection flow of dissipative fluid past an infinite vertical porous plate with ramped wall temperature.In the present paper, we consider the problem of a laminar electrically conducting fluid as a boundary layer flow past a stretching plate and heat transfer.

2. Formulation of the problem:

Let us consider two dimensional laminar boundary layer flows over a stretching plate in an incompressible electrically conducting fluid, where the x-axis is along the stretching plate and y-axis perpendicular to it, the applied magnetic field B_0 is transversely to x-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equations of continuity, momentum and energy under the influence of externally imposed transverse magnetic field are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \qquad \dots (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_0^2}{\rho}u \qquad \dots (2)$$

$$v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2} \qquad \qquad \dots (3)$$

Along with the boundary conditions :
$y = 0: \quad u = mx, \; v = 0, \; T = T_p \,(m > 0)$
$y \to \infty : \quad u = 0, \qquad T = T_\infty \qquad \dots (4)$

Here since temperature filed varies with regard to y only, so $\frac{\partial T}{\partial x} = 0$. Also we introduce the following non-dimensional quantities.

$$\bar{x} = \frac{x}{h} \,, \bar{y} = \frac{y}{h} \,, \bar{u} = \frac{uh}{\nu} \,, \bar{v} = \frac{vh}{\nu}, \theta(y) = \frac{T - T_\infty}{T_p - T_\infty}$$

$$\dots (5)$$

Where T_p is the plate temperature and T_∞ is the temperature of surrounding.
Substituting equation (5) in to (1) to (3) , these equations are reduced to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \qquad \dots (6)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - Mu \qquad \dots (7)$$

Where $M = \frac{\sigma B_0^2}{\rho}\frac{h^2}{\nu}$ (non-dimensional magnetic parameter)

$$\frac{\partial^2\theta}{\partial y^2} + f(y)\, Pr\frac{\partial\theta}{\partial y} = 0 \qquad \dots (8)$$

Where bar has been dropped for our convenience

Along with the boundary conditions:
$y = 0; u = mx, v = 0, \theta = 1$
$y \to \infty : u = 0 \,, \; \theta = 0 \qquad \dots (9)$
We set the similarity solution of the form
$u = mxf'(y) \qquad \dots (10)$
Also using the continuity equation (6) with equation (10) we have
$v = -m\{f(y) - f(0)\} \qquad \dots (11)$
Using (10) and (11) equations(7) and (8) becomes
$m\{f'^2(y) - f(y)f''(y)\} = f'''(y) - Mxf'(y)$
$$\dots (12)$$
$\theta''(y) + mPrf(y)\theta'(y) = 0 \qquad \dots (13)$
Along with the boundary conditions :
$y = 0: f' = 1, \theta = 1$
$y \to \infty : f' = 0 \,, \; \theta = 0 \qquad \dots (14)$
Where we take $f(0) = 0$, without any loss of generality.

3. Numerical solutions:
The non-linear ordinary differential equations (12) and (13) subject to boundary conditions (14) are solved numerically using Runge-Kutta-Fehlberg fourth-fifth method. To solve these equations we adopted symbolic algebra software Maple. Maple uses the well know Runge-Kutta-Fehlberg fourth-fifth order (RFK45) method to generate the numerical solution of a boundary value problem. The boundary condition $y = \infty$ were replaced by those at $y = 1.5$ in accordance with standard practice in the boundary layer flow analysis also we take m=1. The effects of the parameters M and Pr are discussed and shown .
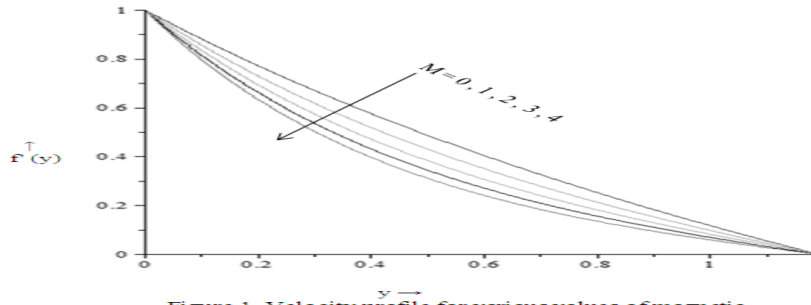
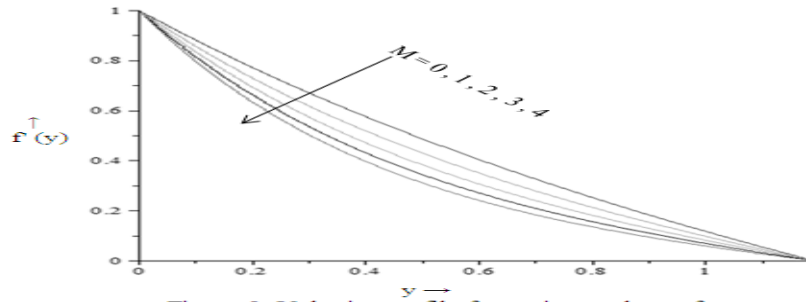Figure 1. Velocity profile for various values of magnetic parameter M, when Pr =0.7

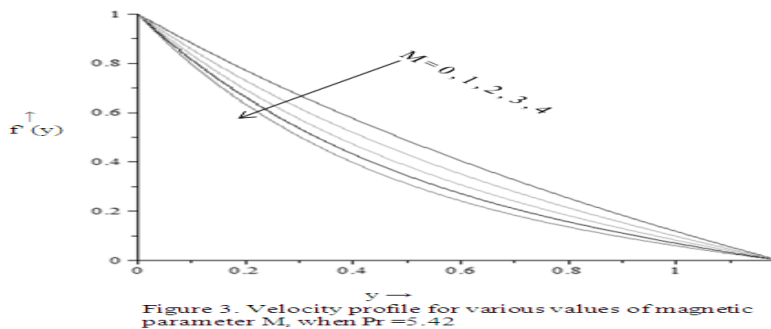Figure 2. Velocity profile for various values of magnetic parameter M, when Pr =2.5

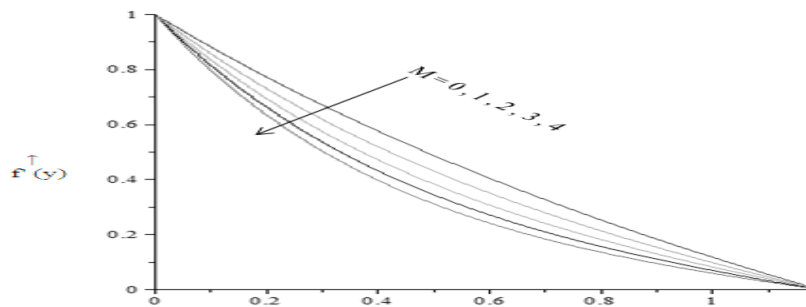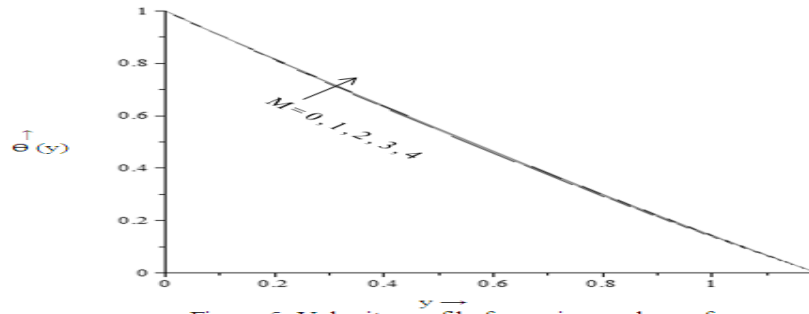Figure 3. Velocity profile for various values of magnetic parameter M, when Pr =5.42

Figure 4. Velocity profile for various values of magnetic parameter M, when Pr =7.0

Table1. Values of $f''(0)$ for various values of magnetic parameter M.

Magnetic parameter	0	1	2	3	4
$f''(0)$	-1.230843	-1.531388	-1.798980	-2.040808	-2.262076

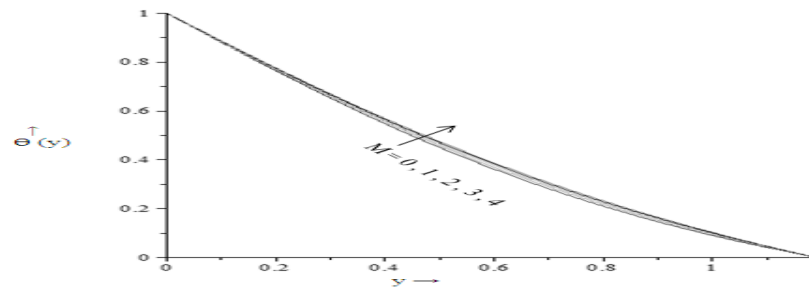Figure 5. Velocity profile for various values of magnetic parameter M, when Pr =0.7

Figure 6. Velocity profile for various values of magnetic parameter M, when Pr =2.5
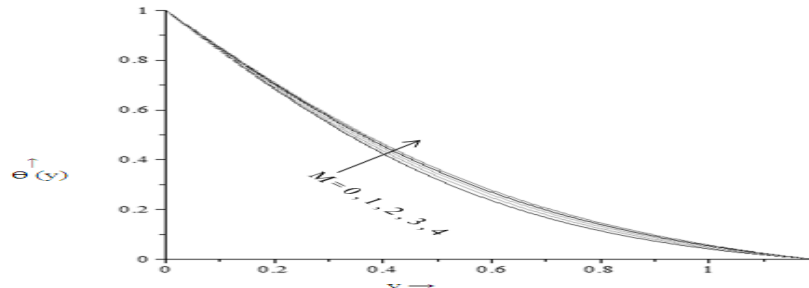

Figure 7. Velocity profile for various values of magnetic parameter M, when Pr =5.42

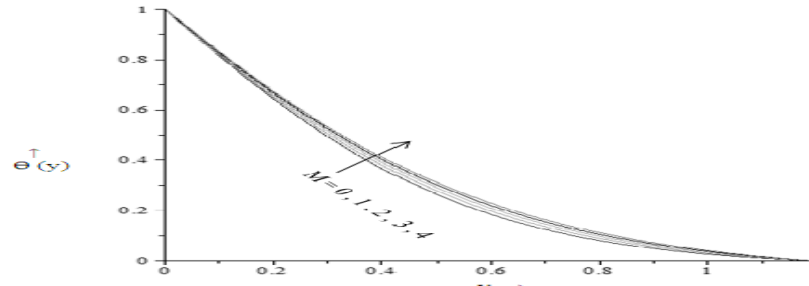
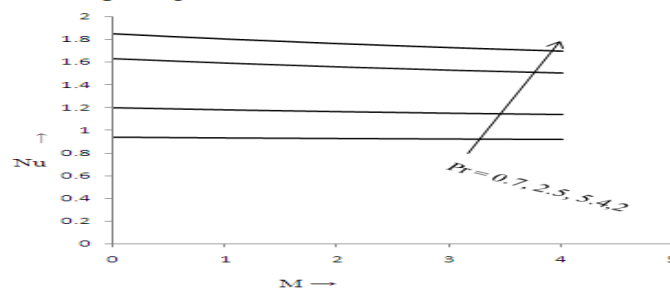Figure 8. Velocity profile for various values of magnetic parameter M, when Pr =7.0

Figure 9. Nusselt number against magnetic parameter M for various values of Pr.

4. Results and Conclusions:

A mathematical model has been presented for the laminar electrically conducting fluid as a boundary layer flow past a stretching plate and heat transfer. From the study, following conclusions can be drawn:

1. The velocity profiles for various values of the magnetic parameter M and given Pr are presented in Figures 1 to 4. It is observed that the rate of transport is considerably reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is because the variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to the transport phenomena. We note that the Prandtl number Pr has no influence on the flow field.

2. The table 1 presents the velocity gradient at the surface $f''(0)$ which represents the surface shear stress. It is observed that surface shear stress decrease as magnetic parameter M increases. Thus, the magnetic parameter M acts as a controlling parameter to control the surface shear stress.

3. Figures 5 to 8 show the temperature profile of the boundary layer for various values magnetic parameter M and given Pr. It is observed that as M increases, temperature increases and the effect of M on temperature is very small when Pr is small.

4. The local Nusselt number $-\theta'(0)$, which represents the heat transfer rate at the surface against M is shown in figure 9 for various values of Pr. It is noted that for increasing value of Pr, the Nusselt number increases but it decreases in small amount, with the increasing value of M. This is because, when Pr increases, the thermal diffusivity decreases and thus the heat is diffused away from the heated surface more slowly and in consequence increase the temperature gradient at the surface.

References:

[1] E. Magyari and B. Keller (1999): Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. J. Phys. D: Appl. Phys. 32: 577-585.

[2] B.C. Sakiadis (1961): Boundary-layer behavior on continues solid surfaces: II. The boundary-layer on continuous flat surface, AIChE J.vol. 7, pp. 221-225.

[3] L. Crane (1970): Flow past a stretching plate. Z. Angew Math Phys(ZAMP),vol.21, pp.645-647.

[4] K.B. Pavlov (1974): Magnetohydrodynamic flow of an incompressible viscous fluid caused by the deformation of a plane surface, Magn. Gidrondin, vol.4, pp.146-152.

[5] A. Chakrabarthi and A.S. Gupta (1979): A note on MHD flow over a stretching permeable surface, Q.Appl.Math.vol. 37, 1979, pp. 73-78.

[6] T. Chiam (1993): Magneto hydrodynamic boundary layer flow due to a continuous moving flate plate. Comput. Math. Appl. vol. 26, pp.1-8.

[7] N.F.M. Noor, O. Abdulaziz and I. Hashim (2010): MHD fow and heat transfer in a thin liquid film on an unsteady stretching sheet by the homotopy analysis method, International Journal for Numerical Methods in Fluids 63, pp. 357–373.

[8] A. Y. Ghaly (2002): Radiation effects on a certain MHD free convection flow. Chaos, Solutions and Fractals, 13, pp. 1843-1850.

[9] A. Raptis (2004): Effect of thermal radiation on MHD flow. Applied Mathematics and Computation, 153, pp. 645-649.

[10] I. C. Liu (2005): A note on heat and mass tranfer for a hydromagnetic flow over a stretching sheet. International Communications in Heat and Mass Transfer, 32, pp. 1075-1084.

[11] C. H. Chen (1998): Laminar mixed convection adjacent to vertical, continuously stretching sheets. Heat and Mass Transfer, 33, pp. 471-476.

[12] A. Ishak, R. Nazar and I. Pop (2008): Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet. Heat Mass Transfer, 44, pp. 921-927.

[13] R. Kandasamy, Wahid Abd.B. Md.Raj, and Azme B. Khamis (2006): Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection, Theoret. Appl. Mech., Vol (33) No (2), pp.123.

[14] Muhaimin, RamasamyKandasamy, I. Hashim and Azme B. Khamis, (2009): On the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet with suction, Theoret. Appl. Mech., Vol (36) No (2), pp.101-117.

[15] V. Rajesh (2011): Chemical Reaction and Radiation Effects on the Transient MHD Free Convection Flow of Dissipative Fluid Past an Infinite Vertical Porous Plate With Ramped Wall Temperature, Chemical Industry & Chemical Engineering Quarterly, Vol (17) No (2), pp. 189-198.