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Matrix Representation of Soft Sets and Its Application To Decision Making Problem

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Abstract

In this paper, we present soft matrices which are representations of soft sets. We also present soft matrix operations and their basic properties. Finally, we demonstrate application of soft matrices to a decision making problem.

Keywords and Phrases :*Soft sets, soft matrices, soft matrix operations, soft max-min decision making method.*

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I. Introduction

Real life problems more often than not inherently involve uncertainties and vagueness. In 1999,Molodtsov [1] initiated the concept of soft set theory for solving problems especially in economics, engineering, social sciences, medical sciences and in other fields that deal with uncertainties.In the past ten years or so, works on soft set theory and its applications have been progressing rapidly.Maji *et al*.[2,3] developed many new operations on soft sets, established their properties and presented an application to a decision making problem.Cagman and Enginoglu [4,5] developed a theory of soft matrices and successfully applied it to a decision-making problem. In this paper, we describe soft sets and soft matrices. We also present operations on soft matrices and their basic properties .Finally, we apply soft matrices to a decision making problem.

II. Soft Sets And Soft Matrices

We describe representations of soft sets by matrices, called *soft matrices*, which are found useful as they can be efficiently stored in a computer.

Definition 2.1 [1] (Soft set)

Let U be an initial universe set, $P(U)$ be the power set of U, E be the set of all possible parameters under consideration with respect to U, and $A \subseteq E$. Usually parameters are attributes, characteristics or properties of objects in U. A pair $\left(f_A, E\right)$, also denoted (F,A), is called a *soft set* over the universe U is defined by the set of ordered pairs:

$\left(f_A, E\right) = \left\{\left(e, f_A(e)\right) : e \in E,\, f_A(e) \in P(U)\right\}$ where , $f_A : E \to P(U)$ such that $f_A(e) = \varnothing$ (the empty set) if $e \notin A$.

Thus a soft set over U is a parameterized family of subsets of U. Here f_A is called an approximate function of the soft set $\left(f_A, E\right)$,and the value $f_A(e)$ is a set called

e-approximate element of the soft set for all $e \in E$

Example2.1

Suppose that there are six candidates under consideration given by the universe set -
U = { u_1 , u_2 , u_3 , u_4 , u_5 , u_6 } and
E = { e_1 , e_2 , e_3 , e_4 , e_5 } is the set of all parameters, where e_i,(i= 1,2,3,4,5) stand for the parameters experience, computer knowledge, young age, higher education and state of origin respectively.

Let a soft set (f_A,E) describe the capability of the candidates who are to fill a position for a company.

In this case, to define a soft set means to point out experienced candidates , computer knowledge candidates and so on.

Let A = { e_1 , e_3 , e_4 } ⊂ E and $f_A(e_1)$ = { u_2 , u_4 }, $f_A(e_3)$ = U and $f_A(e_4)$ = { u_1 , u_3 , u_5 }. Then the soft set (f_A,E) consists of the following set of ordered pairs:

(f_A,E) = { (e_1 ,{ u_2 , u_4 }), (e_3, U), (e_4, {u_1 , u_3 , u_5 })}.

Definition 2.2 [4] (Soft Matrix)

Let (f_A, E) be a soft set over *U*. Then a subset R_A of $U \times E$,uniquely defined as $R_A = \{(u,e) : e \in A, u \in f_A(e)\}$, is called a *relation form* of the soft set (f_A, E) .

The *characteristic function* of R_A is defined as

$$\chi_{R_A} : U \times E \to \{0,1\},$$

where

$$\chi_{R_A}(u,e) = \begin{cases} 1, & (u,e) \in R_A; \\ 0, & (u,e) \notin R_A. \end{cases}$$

Let $U = \{u_1, u_2, \ldots, u_m\}, E = \{e_1, e_2, \ldots, e_n\}$ and $A \subseteq E$. Then R_A can be represented by a matrix as follows:

Let $a_{ij} = \chi_{R_A}(u_i, e_j)$. We can represent a matrix

$$\left[a_{ij}\right]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

and call it an $m \times n$ *soft matrix* of the soft set (f_A, E) over *U*. In other words, a soft set is uniquely represented by its corresponding soft matrix.

Example 2.2

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universe set, and

$E = \{e_1, e_2, e_3, e_4\}$ be a set of all parameters with respect to *U*.

Let $A = \{e_1, e_3, e_4\}$, $f_A(e_1) = \{u_3, u_4\}$, $f_A(e_3) = \varnothing$,and $f_A(e_4) = \{u_1, u_3, u_5\}$.Then the soft set (f_A, E) is given by

$(f_A, E) = \{(e_1, \{u_3, u_4\}), (e_4, \{u_1, u_3, u_5\})\}$.

The relation form R_A of (f_A, E) is given by $R_A = \{(u_3, e_1), (u_4, e_1), (u_1, e_4), (u_3, e_4), (u_5, e_4)\}$. Hence the soft matrix $\left[a_{ij}\right]$ of the soft set (f_A, E) is given by

$$\left[a_{ij}\right]=\begin{bmatrix}0&0&0&1\\0&0&0&0\\1&0&0&1\\1&0&0&0\\0&0&0&1\end{bmatrix},\ i=1,2,\ldots,5;\ j=1,2,\ldots,4.$$

As noted earlier, $f_A(e_3)=\varnothing$, since there is no element in *U* related to the parameter $e_3\in A$, it does not appear in the aforesaid description of the soft set (f_A,E).

Definition 2.3 [4](Types of Soft Matrices)
Let the set of all mxn soft matrices over *U* be denoted *SM(U)*$_{mxn}$ or just *SM(U)*
Let $\left[a_{ij}\right]\in SM(U)$. Then $\left[a_{ij}\right]$ is called

(a) A *zero soft matrix,* denoted $\left[\tilde{0}\right]$, if $a_{ij}=0\ \ \forall\ \ i$ and *j*;

(b) An *A-universal soft matrix,* denoted $\left[a_{ij}\right]$, if $a_{ij}=1\ \ \forall\ \ j\in I_A=\{j:e_j\in A\}$ and *i*.

(Note that it is so called, since $a_{ij}=1$ only for the parameters appearing in the set $A\subset E$); and

(c) A *universal soft matrix* denoted $\left[\tilde{I}\right]$, if $a_{ij}=1\ \ \forall\ \ i$ and j

Example 2.3
Let $U=\{u_1,u_2,u_3,u_4\}$, $E=\{e_1,e_2,e_3,e_4\}$ and $\left[a_{ij}\right],\left[b_{ij}\right],\left[c_{ij}\right]\in SM(U)_{4\times4}$. If $A=\{e_1,e_2,e_3\}$, $f_A(e_1)=f_A(e_2)=f_A(e_3)=\varnothing$ then $\left[a_{ij}\right]=\left[\tilde{0}\right]$ is a zero soft matrix given by

$$\left[\tilde{0}\right]=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}.$$

If $B=\{e_2,e_4\}$, $f_B(e_2)=U=f_B(e_4)$, then $\left[b_{ij}\right]$ is a *B*-universal soft matrix given by

$$\left[\tilde{b}_{ij}\right]=\begin{bmatrix}0&1&0&1\\0&1&0&1\\0&1&0&1\\0&1&0&1\end{bmatrix}.$$

If $C=E$, $f_e(e_i)=U$ for each *i*, then $\left[\tilde{c}_{ij}\right]=\left[\tilde{I}\right]$ is a universal soft matrix given by

$$\left[\tilde{I}\right]=\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}.$$

Definition 2.4 [4](Soft Submatrices)
Let $M=\left[a_{ij}\right], N=\left[b_{ij}\right]\in SM(U).$ Then we define the following:

(i) M is a *soft submatrix* of *N*, denoted $M\,\tilde{\subseteq}\,N$ if $a_{ij}\le b_{ij}$ for each *i* and *j*.

In this case, we also say that *M* is *dominated* by *N* or *N dominates M*. Note that similar to $R^k(k>1),$ the *k*-dimensional real space, $\le$ holds without the holding of either < or =.
We define *M* and *N comparable*, denoted $M\|N,$ iff $M\,\tilde{\subseteq}\,N\;or\;N\,\tilde{\subseteq}\,M;$

(ii) *M* is a *proper soft submatrix* of *N*, denoted $M\,\tilde{\subset}\,N,$ if $\left[a_{ij}\right]\tilde{\subset}\left[b_{ij}\right]$ and for at least one term $a_{ij}<b_{ij}$ for all *i* and *j*. In this case, we say that *M* is *properly dominated* by *N*.

(iii) *M* is a *strictly proper soft submatrix* of *N*, denoted if $M\,\tilde{\subset}\,N$ and $a_{ij}<b_{ij}$ $M\,\underset{\neq}{\tilde{\subseteq}}\,N,$ for each *i* and *j*. In this case we say that *M* is *strictly dominated* by *N*.

(iv) *M* and *N* are *soft equal matrices* denoted $M\cong N$ if $a_{ij}=b_{ij}$ for each *i* and *j*. Equivalently, if $M\,\tilde{\subseteq}\,N$ and $N\,\tilde{\subseteq}\,M,$ then
$M\cong N.$ It is immediate to see that $\tilde{\subseteq}$ is a partial ordering (reflexive, antisymmetric and transitive) on the class of soft matrices.

III. Operations On Soft Matrices

We discuss the operations of union, intersection complement, difference and products of soft matrices and their basic properties.

Definition 3.1 [4] (Soft Matrix Operations)
Let $M=\left[a_{ij}\right], N=\left[b_{ij}\right]\in SM(U)$. Then a soft matrix $P=\left[c_{ij}\right]\in SM(U)$ is called the

(i) *union* of *M* and *N*, denoted $M\,\tilde{\cup}\,N$, if

$$c_{ij}=\max\left\{a_{ij},b_{ij}\right\} \text{ for all } i \text{ and } j;$$

(ii) *intersection* of *M* and *N*, denoted $M\,\tilde{\cap}\,N$

$$c_{ij}=\min\left\{a_{ij},b_{ij}\right\} \text{ for all } i \text{ and } j;$$

(iii) *complement* of *M*, denoted M^0, if

$$c_{ij}=1-a_{ij} \text{ for all } i \text{ and } j;$$

(iv) *difference* of *N* from *M*, also called the *relative complement* of *N* in *M*, denoted *M – N* or *M\N* if $P=M\,\tilde{\cap}\,N^0$.

In view of the (ii) above, *M* and *N* are said to be *disjoint* if $M\,\tilde{\cap}\,N=\left[\tilde{0}\right]$.

Example 3.1
Let

$$M=\begin{bmatrix}0&0&1&1\\1&0&1&0\\0&1&0&1\\0&0&0&0\\0&0&0&1\end{bmatrix} \text{ and } N=\begin{bmatrix}1&0&0&0\\0&1&0&1\\1&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix}.$$

Then,

(i) $M \tilde{\cup} N = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$;

(ii) $M \tilde{\cap} N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \left[\tilde{0}\right]$ which implies that M and N are disjoint;

(iii) $M^0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ and $M \tilde{\cup} M^0 = \left[\tilde{I}\right]$, $M \tilde{\cap} M^0 = \left[\tilde{0}\right]$;

(iv) $N^0 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$;

(v) $M - N = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M \tilde{\cap} N^0$; and

(vi) $N - M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = N \tilde{\cap} M^0$.

Proposition 3.1: Properties of Soft Matrix Operations

Let $M = \left[a_{ij}\right],\ N = \left[b_{ij}\right],\ P = \left[c_{ij}\right] \in SM(U)$.

(i) $M \tilde{\cup} M = M;\quad M \tilde{\cap} M = M$ (Idempotent laws)

(ii) $M \tilde{\cup} \left[\tilde{0}\right] = M;\quad M \tilde{\cap} \left[\tilde{I}\right] = M$ (Identity laws)

(iii) $M \tilde{\cup} \left[\tilde{I}\right] = \left[\tilde{I}\right];\quad M \tilde{\cap} \left[\tilde{0}\right] = \left[\tilde{0}\right]$ (Domination laws)

(iv) $\left[\tilde{0}\right]^0 = \left[\tilde{I}\right]$; $\left[\tilde{I}\right]^0 = \left[\tilde{0}\right]$ (De Morgan's laws)

(v) $M \tilde{\cup} M^0 = [\tilde{I}]$; $M \tilde{\cap} M^0 = [\tilde{0}]$(De Morgan's laws)

(vi) $(M \tilde{\cup} N)^0 = M^0 \cap N^0$; $(M \tilde{\cap} N)^0 = M^0 \tilde{\cup} N^0$

(De Morgan's laws)

(vii) $(M^0)^0 = M$ for all M (Involution law)

(viii) $M \tilde{\cup} N = N \tilde{\cup} M$; $M \tilde{\cap} N = N \tilde{\cap} M$(Commutative laws)

(ix) $M \tilde{\cup} (N \tilde{\cup} P) = (M \tilde{\cup} N) \tilde{\cup} P$;

$M \tilde{\cap} (N \tilde{\cap} P) = (M \tilde{\cap} N) \tilde{\cap} P$(Associative laws)

(x) $M \tilde{\cup} (N \tilde{\cap} P) = (M \tilde{\cup} N) \tilde{\cap} (M \tilde{\cup} P)$;

$M \tilde{\cap} (N \tilde{\cup} P) = (M \tilde{\cap} N) \tilde{\cup} (M \tilde{\cap} P)$. (Distributive Laws)

Proof: Most of the proofs follow from definitions.
Let us, for example, prove the first parts of (vi), (ix) and (x).

(vi) For each *i* and *j*,

$$\begin{aligned}
(M \tilde{\cup} N)^0 &= \left([a_{ij}] \tilde{\cup} [b_{ij}]\right)^0 \\
&= \left[\max\{a_{ij}, b_{ij}\}\right]^0 \\
&= \left[1 - \max\{a_{ij}, b_{ij}\}\right] \\
&= \left[\min\{1 - a_{ij},\ 1 - b_{ij}\}\right] \\
&= [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0 \\
&= M^0 \tilde{\cap} N^0.
\end{aligned}$$

(ix)

$$\begin{aligned}
M \tilde{\cup} (N \tilde{\cup} P) &= [a_{ij}] \tilde{\cup} \left([b_{ij}] \tilde{\cup} [c_{ij}]\right) \\
&= \left[\max\{a_{ij}, \max(b_{ij}, c_{ij})\}\right] \\
&= \left[\max\{\max(a_{ij}, b_{ij}), c_{ij}\}\right] \\
&= \left([a_{ij}] \tilde{\cup} [b_{ij}]\right) \tilde{\cup} [c_{ij}] \\
&= (M \tilde{\cup} N) \tilde{\cup} P.
\end{aligned}$$

(x)

$$\begin{aligned}
M \tilde{\cup} (N \tilde{\cap} P) &= [a_{ij}] \tilde{\cup} \left([b_{ij}] \tilde{\cap} [c_{ij}]\right) \\
&= \left[\max\{a_{ij}, \min(b_{ij}, c_{ij})\}\right] \\
&= \left[\min\{\max\{a_{ij}, b_{ij}\}, \max\{a_{ij, c_{ij}}\}\}\right] \\
&= \left([a_{ij}] \tilde{\cup} [b_{ij}]\right) \tilde{\cap} \left([a_{ij}] \tilde{\cup} [c_{ij}]\right) \\
&= (M \tilde{\cup} N) \tilde{\cap} (M \tilde{\cup} P).
\end{aligned}$$

Definition 3.2 [4] (Product of Soft Matrices)
Let $M=\left[a_{ij}\right], N=\left[b_{ik}\right]\in SM(U)_{m\times n}$. Then

(i) *AND-product* of *M* and *N*, denoted $M\wedge N$ is defined
$\wedge: SM(U)_{m\times n}\times SM(U)_{m\times n}\to SM(U)_{m\times n^2}$ such that $\left[a_{ij}\right]\wedge\left[b_{ik}\right]=\left[c_{ip}\right]$, where
$c_{ip}=\min\left\{a_{ij},b_{ik}\right\}$ and $P=n(j-1)+k$.

(ii) *OR-product* of *M* and *N*, denoted $M\vee N$ is defined $\vee: SM(U)_{m\times n}\times SM(U)_{m\times n}\to SM(U)_{m\times n^2}$
such that $\left[a_{ij}\right]\vee\left[b_{ik}\right]=\left[c_{ip}\right]$, where $c_{ip}=\max\left\{a_{ij},b_{ik}\right\}$ and $P=n(j-1)+k$.

(iii) *AND-NOT-product* of *M* and *N*, denoted $M\barwedge N$ is defined
$\barwedge: SM(U)_{m\times n}\times SM(U)_{m\times n}\to SM(U)_{m\times n^2}$ such that $\left[a_{ij}\right]\barwedge\left[b_{ik}\right]=\left[c_{ip}\right]$, where
$c_{ip}=\min\left\{a_{ij},1-b_{ik}\right\}$ and $P=n(j-1)+k$.

(iv) *OR-NOT-product* of *M* and *N*, denoted $M\veebar N$ is defined
$\veebar: SM(U)_{m\times n}\times SM(U)_{m\times n}\to SM(U)_{m\times n^2}$ such that $\left[a_{ij}\right]\veebar\left[b_{ik}\right]=\left[c_{ip}\right]$, where
$c_{ip}=\max\left\{a_{ij},1-b_{ik}\right\}$ and $P=n(j-1)+k$.

Note: Products of soft matrices hold if the two matrices are of the same order or have the same number of rows.

Proposition 3.2 [4] (Properties of Product of Soft Matrices)
Let $M=\left[a_{ij}\right],\ N=\left[b_{ik}\right]\in SM(U)$. Then the following hold:

(i) $(M\vee N)^0=M^0\wedge N^0;\ (M\wedge N)^0=M^0\vee N^0$ (De Morgan's laws)

(ii) $(M\veebar N)^0=M^0\barwedge N^0;\ (M\barwedge N)^0=M^0\veebar N^0$ (De Morgan's laws)

***Proof**:* The proofs follow from definitions.

Example 3.2
Let $M=\left[a_{ij}\right],\ N=\left[b_{ik}\right]\in SM(U)_{4\times 4}$ given by

$$M=\begin{bmatrix}0&1&0&1\\0&1&1&1\\0&1&1&0\\0&1&0&0\end{bmatrix}\text{ and }N=\begin{bmatrix}0&0&1&1\\1&0&0&1\\0&0&1&1\\0&0&1&1\end{bmatrix}$$

Then

$$M\wedge N=\left[a_{ij}\right]\wedge\left[b_{ik}\right]=\left[c_{ip}\right]_{4\times 16}=\begin{bmatrix}0&0&0&0&0&0&1&1&0&0&0&0&0&0&1&1\\0&0&0&0&1&0&0&1&1&0&0&1&1&0&0&1\\0&0&0&0&0&0&1&1&0&0&1&1&0&0&0&0\\0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0\end{bmatrix}$$

Similarly, the other products $M\vee N$, $M\barwedge N$ and $M\veebar N$ can be found.
Also,

$$(M \wedge N)^{\circ} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$M^{\circ} \vee N^{\circ} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Thus, $(M \wedge N)^0 = M^0 \vee N^0$.

Note that the commutative laws are not valid for products of soft matrices.

IV. Application Of Soft Matrices In A Decision Making Problem

We give the following definitions which are used to construct *soft max- min decision making method* (*SMmDMM* for short) and finally, its application to a decision making problem is demonstrated.

Definition 4.1[5]

Let $[c_{ip}] \in SM(U)_{m\times n^2}$, $I_k = \{p:\exists_I, c_{ip}\neq 0, (k-1)\, n<p\leq kn\}$ for all $k \in I = \{1, 2, \ldots, n\}$. Then Soft max-min decision function denoted SMmDF, is defined as follows:

$$\text{SMmDF} : SM(U)_{m\times n^2} \to SM(U)_{m\times 1}, \text{ such that}$$

$\text{SMmDF}([c_{ip}]) = [\max\{t_{ik}\}]$ where $t_{ik} = \{\min\{ c_{ip} \}$, if $I_k\neq\emptyset$, or 0, if $I_k=\emptyset\}$

The one-column soft matrix SMmDF($[c_{ip}]$) is called max-min decision soft matrix.

Definition 4.2[5]

Let $U = \{u_1, u_2, \ldots, u_m\}$ be an initial universe set and *SMmDF*($[c_{ip}]$) = $_{di1}$. Then the optimum subset of U with respect to $[_{di1}]$, denoted Opt$[_{di1}]$ (U) is a subset of U defined by: Opt$[_{di1}]$(U) = $\{u_i : u_i \in U, {}_{di1} = 1, i=1,2,3,\ldots,m\}$.

Construction of soft max-min decision making problem

Now, with the aforesaid definitions, a soft min-max decision making method(SMmDMM) can be constructed using the following steps :

Step 1: Choose feasible subsets of the set of parameters.
Step 2: Construct the soft matrix for each set of parameters.
Step 3: Find a convenient product of the soft matrices.
Step 4: Find a max-min decision soft matrix.
Step 5: Find an optimum set of *U.*

Similarly, we can construct soft min-max, soft min-min and soft max-max decision making methods which may be denoted by SmMDM, SmmDM, SMMDM respectively. One of them may be more suitable than the others, depending on the nature of the problems to be solved.

Example 4.1(Application)

Let a car dealer have a set of different types of cars : $U = \{u_1, u_2, u_3, u_4\}$ which may be characterized by a set of parameters : $E = \{e_1, e_2, e_3, e_4\}$, where e_i(i= 1,2,3,4) stands for *cheap, fuel efficient, modern, portable,* respectively.

Suppose a man and his wife contacted the dealer for purchasing a car and each partner has to consider his/her choice of parameters. Then we select a car by using SMmDMM as follows :

Step 1. Let us assume that the man and his wife choose the sets of parameters : $A=\{e_2, e_3, e_4\}$ and $B=\{e_1, e_3, e_4\}$ respectively.

Step 2. Let us also assume that the following soft matrices are obtained according to their parameters :

$$M=\begin{bmatrix}0 & 1 & 1 & 1\\ 0 & 0 & 1 & 1\\ 0 & 1 & 0 & 0\\ 0 & 1 & 1 & 0\end{bmatrix} \quad \text{and} \quad N=\begin{bmatrix}1 & 0 & 0 & 1\\ 1 & 0 & 1 & 1\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & 1\end{bmatrix}$$

Step 3. We next find the ∧ - product of M and N, since the choices of both the man and his wife have to be considered conjunctively as follows :

$$M \wedge N=\begin{bmatrix}0&0&0&0&1&0&0&1&1&0&0&1&1&0&0&1\\ 0&0&0&0&0&0&0&0&1&0&1&1&1&0&1&1\\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&1&0&0&1&1&0&0&0&0\end{bmatrix}$$

Step 4. We find a max-min decision soft matrix D as:

$$D=\begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix}$$

Step 5. Finally, we find an optimum set of U, with respect to D given by:

$$Opt_D(U) = \{u_2\}.$$

Hence u_2 is identified as the car of optimum choice on which both the man and his wife opted to agree to purchase Note that the optimal set of U may contain more than one element.

V. Conclusion

We describe soft matrices which are representatives of soft sets and discuss soft matrix operations and their basic properties. We finally present an application of soft matrices to a decision making problem, using soft max-min decision making method. Similar and suitable methods can be used depending on the nature of the problems to be solved.

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