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# Angstrom Type Empirical Correlation for Estimating Global Solar Radiation in North-Eastern Nigeria.

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## ABSTRACT

*An accurate knowledge of solar radiation distribution at a particular geographical location is of vital importance for the development of many solar energy devices. In this study, global solar radiation received on horizontal surfaces and sunshine duration for Bauchi, Dutse, Ibitaraba, Maiduguri, Nguru and Yola for the period of fifteen years were analyzed and tabulated. A set of constants for Angstrom-type correlation were obtained to establish the linear regression model capable of generating solar radiation at any given location in North-Eastern, Nigeria. The study resulted in the development of respective Angstrom linear regression models for each of the six meteorological locations, which culminated in the development of the Angstrom model for North-Eastern, Nigeria. Moreover, three sunshine-based models of first, second and third order to estimate annual average global solar radiation has also been obtained employing sunshine hour's data (1990-2005). In general, the three sunshine-based models performed well in terms of their coefficient of determination with $R^2$ = 99.74% given by the linear Angstrom-Prescott (1940) model, for Ogelman et al., (1984) model, $R^2$ =99.89% while the Samuel (1991) model proved to be the best estimator with $R^2$ = 99.93% in North-Eastern, Nigeria. The calculated global solar radiation is in good agreement with the three sunshine based models. This study shows that more than one sunshine-based model can be used to predict solar radiation across the North-Eastern, Nigeria. In order to test for the performance of statistical significance of the models, mean bias error (MBE), root mean square error (RMSE), mean percentage error (MPE) and t-test values were adopted, the results shows that despite overestimation and underestimation of the models, there are fairly good level of significance at both confidence level of 95% and 99%. The results of the coefficient of determination indicate that the calculated clearness index and relative sunshine duration shows excellent data.*

***Keywords:*** *Global Solar Radiation, Sunshine Hours, Regression Constants, North-East, Sunshine-based Models, Nigeria.*

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## I. INTRODUCTION

Solar energy occupies one of the most important places among the various possible alternative energy sources. It is the energy provided by the sun. Nigeria receives abundant solar energy that can be usefully harnessed with an annual average daily solar radiation of about 5250 $Whm^{-2}$ $day^{-1}$. This varies between 3500 $Whm^{-2}$ $day^{-1}$ at the coastal areas and 7000 $Whm^{-2}$ $day^{-1}$ at the northern boundary. The average amount of sunshine hours all over the country is about 6.5 hours (Chineke and Igwiro, 2008,Yakubu and Medugu, 2012).

Accurate quantitative data of the variation of solar radiation reaching the earth surface, together with relevant meteorological parameters are essential requirements for conducting a wide range of scientific studies. Typical examples are found in hydrological studies when calculating soil moisture deficits (Mills, 2000), investigation of biological process (Kudish and Evseev, 2000), climatology (Dissing and Wendler, 1998), thermal design of environmental control of buildings (Agboola, 2011) and quantitative evaluation of eco-physiological system for the determination of irrigation water needs and the potential yield of crops (Tardieu, 2013). The design and estimation of performance of solar heating, cooling and distillation systems also requires detailed knowledge of solar radiation data (Tarawneh, 2007).

According to Augustine and Nnabuchi (2009), Sambo (1985) developed correlation with solar radiation using sunshine hours for Kano with the regression coefficients a= 0.413 and b= 0.241 for all the months between 1980- 1984, Arinze and Obi, (1983) developed a correlation with solar radiation using sunshine hours in Northern Nigeria with regression coefficients a= 0.2 and b= 0.74, Burari *et al*., (2001) developed a model for estimation of global solar radiation in Bauchi with regression coefficients a= 0.24 and b= 0.46. Other workers (e.g. Ojosu, 1984; Fagbenle, 1990; Folayan, 1988; Adebiyi, 1988; Turton, 1987; Bamiro, 1983) developed theoretical and empirical correlations of broad applicability to provide solar data for system design in most Nigeria cities. They observed that the regression coefficients are not universal but depends on the climatic conditions.

In the absence and scarcity of trustworthy solar radiation data, the need for an empirical model to predict and estimate global solar radiation seems inevitable. These models use climatological parameters of the location under study. Among all such parameters, sunshine hours are the most widely and commonly used. The models employing this common and important parameter are called sunshine-based models (Ahmad and Ulfat, 2004).

However, the main objective of this study is to develop empirical correlation model capable of predicting the mean monthly global solar radiation for the North-Eastern Nigeria. Three sunshine-based models of estimation are employed as to develop new constants for the first, second and third order Angstrom type correlations with the view of establishing the most suitable model of prediction in six different locations in the North-Eastern Nigeria. The meteorological data comprises of global solar radiation on horizontal surfaces and sunshine hours used in this study was collected from the Nigeria Meteorogical Agency (NIMET), Abuja, Nigeria for the period of fifteen years (1990-2005).

## II. METHODOLOGY

The original Angstrom-type regression equation is related with the monthly average daily radiation to the clear day radiation at the location and the average fraction of possible sunshine hours (Angstrom, 1924). Page (1961) and others have modified the method using the values of extraterrestrial radiation on a horizontal surface rather than that of clear day radiation (Duffie and Beckman, 1991):

$$\frac{H}{H_o} = a + b\left(\frac{S}{S_o}\right) \tag{1}$$

where $H$ is the monthly average daily global radiation on a horizontal surface (MJ.m$^{-2}$.day$^{-1}$), $H_o$ the monthly average daily extraterrestrial radiation on a horizontal surface (MJ.m$^{-2}$.day$^{-1}$), $S$ the monthly average daily hours of bright sunshine, $S_o$ the monthly average day length, and "a" and "b" values are known as Angstrom constants and they are empirical.

The values of the monthly average daily extraterrestrial irradiation ($H_O$) can be calculated from the following equation (2) (Duffie and Beckman, 1991):

$$H_O = \left(\frac{24}{\pi}\right) I_{SC}\left[1 + 0.033\cos\left(\frac{360n}{365}\right)\right] \times \left[\cos\phi\cos\delta\sin w_s + \left(\frac{2\pi w_s}{360}\right)\sin\phi\sin\delta\right] \tag{2}$$

Where $I_{SC}$ is the solar constant (=1367 W m$^{-2}$), $\phi$ the latitude of the site, δ the solar declination, $w_s$ the mean sunrise hour angle for the given month, and n the number of days of the year starting from the first of January.

The solar declination (δ) and the mean sunrise hour angle ($w_s$) can be calculated by the following equations (3) and (4), respectively in Akinoglu and Ecevit, (1990):

$$\delta = 23.45\sin\left(360\,\frac{284 + n}{365}\right) \tag{3}$$

$$w_S = cos^{-1}\left(-\tan\phi\tan\delta\right) \tag{4}$$

For a given month, the maximum possible sunshine duration (monthly average day length ($S_o$) which is related to $W_s$, the mean sunrise hour angle can be computed by using the following equation (5) (Duffie and Beckman, 1991)

$$S_o = \frac{2}{15} w_s \tag{5}$$

Then, the monthly mean of daily global radiation $H$ was normalized by dividing with monthly mean of daily extraterrestrial radiation $H_O$. We can define clearness index $(K_T)$ as the ratio of the observed/measured horizontal terrestrial solar radiation $(H)$, to the calculated/predicted horizontal/extraterrestrial solar radiation $(H_O)$ (Falayi *et al.*, 2011)

$$K_T = \frac{H}{H_o} \tag{6}$$

In this study, $H_o$ and $S_o$ were computed for each month by using Equations (2) and (5), respectively. The regression coefficients *a* and *b* in Equation (1) was obtained from the graph of $\frac{H_{cal}}{H_o}$ $against$ $\frac{S}{S_o}$. The

values of the monthly average daily global radiation H and the average number of hours of sunshine were obtained from daily measurements covering a period of 15 years. The regression coefficient **a** and **b** has been calculated from the relationship given by (Tiwari *et al*, 1997):

$$a = -0.110 + 0.235\cos\phi + 0.323\left(\frac{S}{S_O}\right) \tag{7}$$

$$b = 1.449 - 0.553\cos\phi - 0.694\left(\frac{S}{S_O}\right) \tag{8}$$

To compute the estimated values of the monthly average daily global radiation $H_{Cal}$, the values of **a** and **b** calculated from equation (7) and (8) were used in Equation (1) (Yakubu and Medugu, 2012).

Three models were selected for this study. They are Angstrom- Prescott (1940), Ogelman *et al.*, (1984) and Samuel (1991) models of estimation of monthly mean of daily horizontal global solar radiation as summarized in the Table below:

**Table 1: Sunshine-based models**

| Model no | Regression equation | Source |
|---|---|---|
| 1 | $\frac{H}{H_o} = a + b\left(\frac{S}{S_o}\right)$ | Angstrom- Prescott (1940) |
| 2 | $\frac{H}{H_O} = a + b\left(\frac{S}{S_o}\right) + c\left(\frac{S}{S_o}\right)^2$ | Ogelman *et al*., (1984) |
| 3 | $\frac{H}{H_o} = a + b\left(\frac{S}{S_o}\right) + c\left(\frac{S}{S_o}\right)^2 + d\left(\frac{S}{S_o}\right)^3$ | Samuel(1991) |

## Sunshine-based models

The most commonly used parameter for estimating global solar radiation is sunshine duration. Sunshine duration for a given period is defined as the sum of that sub-period for which the direct solar irradiance exceeds 120Wm$^{-2}$. Solar radiation intensity is taken as incoming short-wave radiation measured in MJ/m$^2$ /day. Sunshine duration can be easily and reliably measured, and data are widely available at the weather stations. Most of the models for estimating solar radiation that appear in the literature only use sunshine ratio

$(\frac{S}{S_o})$ for prediction of monthly average daily global radiation. The following are the sunshine-based

Models utilized in this study:

1). Angstrom – Prescott model.
The Angstrom – Prescott (1940) model is the most commonly used model as given by:

$$\frac{H}{H_o} = a + b\left(\frac{S}{S_o}\right) \qquad (9)$$

Where H is the global solar radiation, $H_O$ the extraterrestrial solar radiation, S the actual sunshine hour, $S_o$ maximum possible duration, **a** and **b** are empirical coefficients. $H_o$ and $S_o$ were calculated using equation (2) and (5). However, equation (9) may be termed the first order and is linear.

2). Ogelman *et al* model.
Following equation has been presented by Ogelman for estimating global solar radiation (*Ogelman et al*., 1984):

$$\frac{H}{H_O} = a + b\left(\frac{S}{S_o}\right) + c\left(\frac{S}{S_o}\right)^2 \qquad (10)$$

Where a,**b** and **c** are empirical coefficients. However, equation (10) may be termed second order and is quadratic in nature.
3). Samuel model.

Samuel estimated global solar radiation on a horizontal surface by the following equation (*Samuel*, 1991):

$$\frac{H}{H_o} = a + b\left(\frac{S}{S_o}\right) + c\left(\frac{S}{S_o}\right)^2 + d\left(\frac{S}{S_o}\right)^3 \qquad (11)$$

Where a-d are empirical coefficients. Similarly, equation (11) may be termed third order and is polynomial.
In this study, the accuracy of the estimated global solar radiation data was statistically tested by mean bias error (MBE), root mean bias error (RMSE), mean percentage error (MPE%) and t-test (t) and are defined as follows:

$$MBE = \frac{\sum_{i=1}^{n}\left[\left(\bar{H}_{i,cal} - \bar{H}_{i,meas}\right)\right]}{n} \qquad (12)$$

$$RMSE = \left\{\frac{\left[\sum_{i=1}^{n}\left(\bar{H}_{i,cal} - \bar{H}_{i,meas}\right)\right]^2}{n}\right\}^{\frac{1}{2}} \qquad (13)$$

$$MPE = \frac{\sum_{i=1}^{n}\frac{\left(\bar{H}_{i,meas} - \bar{H}_{i,cal}\right)\times 100}{\bar{H}_{i,meas}}}{n} \qquad (14)$$

$$t = \left[\frac{(n-1)-(MBE)^2}{(RMSE)^2-(MBE)^2}\right]^{\frac{1}{2}} \qquad (15)$$

where $\bar{H}_{i,cal}$ and $\bar{H}_{i,meas}$ is the ith calculated and measured global solar radiation values and n is the total number of observations. In general, a low RMSE is desirable. The positive MBE shows overestimation while a negative MBE indicates underestimation (El-Sebaii and Trebea, 2005)

## III. RESULTS AND DISCUSSION

**Table 1: Calculated monthly mean of global solar radiation and input parameters of monthly mean average of global solar radiation for Bauchi (1990 – 2005)**

| Months | Hcal | $H_o$ | S | $S_o$ | Hcal/$H_o$ | S/$S_o$ |
|---|---|---|---|---|---|---|
| Jan | 19.55 | 30.57 | 7.93 | 11.41 | 0.6394 | 0.6951 |
| Feb | 21.09 | 32.76 | 8.23 | 11.55 | 0.6438 | 0.7124 |
| Marc | 21.65 | 34.90 | 7.46 | 11.79 | 0.6204 | 0.6329 |
| Apri | 22.99 | 38.45 | 6.95 | 12.09 | 0.5978 | 0.5750 |
| May | 24.06 | 39.18 | 7.61 | 12.36 | 0.6141 | 0.6157 |
| Jun | 23.36 | 38.01 | 7.75 | 12.56 | 0.6147 | 0.6171 |
| Jul | 20.89 | 37.21 | 6.29 | 12.60 | 0.5613 | 0.4993 |
| Aug | 21.72 | 38.29 | 6.36 | 12.46 | 0.5672 | 0.5105 |
| Sept | 22.65 | 38.92 | 6.59 | 12.20 | 0.5820 | 0.5401 |
| Oct | 23.36 | 36.98 | 7.96 | 11.91 | 0.6319 | 0.6684 |
| Nov | 21.44 | 32.86 | 8.76 | 11.63 | 0.6524 | 0.7534 |
| Dec | 19.25 | 29.56 | 8.54 | 11.44 | 0.6511 | 0.7465 |

**Table 2: Calculated monthly mean of global solar radiation and input parameters of monthly mean average of global solar radiation for Dutse (1990 - 2005)**

| Months | Hcal | $H_o$ | S | $S_o$ | Hcal/$H_o$ | S/$S_o$ |
|---|---|---|---|---|---|---|
| Jan | 18.90 | 29.83 | 7.6 | 11.32 | 0.6335 | 0.6713 |
| Feb | 20.30 | 32.13 | 7.65 | 11.49 | 0.6318 | 0.6660 |
| Marc | 21.41 | 34.49 | 7.43 | 11.75 | 0.6207 | 0.6321 |
| Apri | 23.13 | 38.36 | 7.09 | 12.10 | 0.6029 | 0.5860 |
| May | 25.08 | 39.39 | 8.47 | 12.41 | 0.6367 | 0.6823 |
| Jun | 22.39 | 37.45 | 8.31 | 14.48 | 0.5978 | 0.5739 |
| Jul | 22.22 | 37.65 | 7.07 | 12.68 | 0.5904 | 0.5574 |
| Aug | 22.48 | 38.60 | 6.77 | 12.53 | 0.5825 | 0.5405 |
| Sept | 23.38 | 38.96 | 7.09 | 12.23 | 0.6002 | 0.5797 |
| Oct | 23.66 | 36.69 | 8.49 | 11.90 | 0.6449 | 0.7136 |
| Nov | 21.25 | 32.31 | 9.03 | 11.57 | 0.6578 | 0.7802 |
| Dec | 18.95 | 28.88 | 8.74 | 11.36 | 0.6561 | 0.7696 |

**Table 3: Calculated monthly mean of global solar radiation and input parameters of monthly mean average of global solar radiation for Ibitaraba (1990 – 2005)**

| Months | Hcal | $H_o$ | S | $S_o$ | Hcal/$H_o$ | S/$S_o$ |
|---|---|---|---|---|---|---|
| Jan | 18.74 | 31.61 | 6.51 | 11.53 | 0.5927 | 0.5645 |
| Feb | 19.82 | 33.63 | 6.48 | 11.65 | 0.5891 | 0.5564 |
| Marc | 19.67 | 35.45 | 5.77 | 11.83 | 0.5548 | 0.4877 |
| Apri | 23.07 | 38.55 | 6.97 | 12.07 | 0.5984 | 0.5776 |
| May | 23.09 | 38.83 | 6.99 | 12.28 | 0.5947 | 0.5690 |
| Jun | 21.48 | 37.38 | 6.54 | 12.44 | 0.5746 | 0.5257 |
| Jul | 18.65 | 36.54 | 5.16 | 12.47 | 0.5105 | 0.4138 |
| Aug | 19.53 | 37.81 | 5.23 | 12.36 | 0.5165 | 0.4231 |
| Sept | 20.96 | 38.83 | 5.61 | 12.16 | 0.5399 | 0.4614 |
| Oct | 21.89 | 37.35 | 6.56 | 11.93 | 0.5862 | 0.5499 |
| Nov | 21.58 | 33.62 | 8.29 | 11.71 | 0.6418 | 0.7082 |
| Dec | 19.42 | 30.52 | 7.93 | 11.56 | 0.6361 | 0.6861 |

**Table 4: Calculated monthly mean of global solar radiation and input parameters of monthly mean average of global solar radiation for Maiduguri (1990 - 2005**

| Months | Hcal | $H_o$ | S | $S_o$ | Hcal/$H_o$ | S/$S_o$ |
|---|---|---|---|---|---|---|
| Jan | 19.34 | 29.81 | 8.28 | 11.32 | 0.6490 | 0.7316 |
| Feb | 20.95 | 32.10 | 8.61 | 11.48 | 0.6527 | 0.7497 |
| Marc | 21.94 | 34.48 | 8.01 | 11.75 | 0.6364 | 0.6815 |
| Apri | 23.76 | 38.35 | 7.61 | 12.10 | 0.6196 | 0.6290 |
| May | 24.63 | 39.39 | 8.01 | 12.42 | 0.6252 | 0.6452 |
| Jun | 23.30 | 38.43 | 7.51 | 12.64 | 0.6062 | 0.5940 |
| Jul | 21.63 | 37.66 | 6.65 | 12.69 | 0.5744 | 0.5241 |
| Aug | 22.43 | 38.61 | 6.73 | 12.53 | 0.5809 | 0.5372 |
| Sept | 23.41 | 38.96 | 7.11 | 12.23 | 0.6009 | 0.5813 |
| Oct | 22.95 | 36.68 | 7.69 | 11.90 | 0.6256 | 0.6464 |
| Nov | 21.33 | 32.29 | 9.27 | 11.57 | 0.6606 | 0.8011 |
| Dec | 18.78 | 28.86 | 8.4 | 11.35 | 0.6507 | 0.7398 |

**Table 5: Calculated monthly mean of global solar radiation and input parameters of monthly mean average of global solar radiation for Nguru (1990 - 2005)**

| Months | Hcal | $H_o$ | S | $S_o$ | Hcal/$H_o$ | S/$S_o$ |
|---|---|---|---|---|---|---|
| Jan | 19.09 | 29.28 | 8.35 | 11.26 | 0.6518 | 0.7418 |
| Feb | 20.72 | 31.65 | 8.65 | 11.44 | 0.6546 | 0.7563 |
| Marc | 21.73 | 34.18 | 7.94 | 11.73 | 0.6356 | 0.6768 |
| Apri | 24.11 | 38.27 | 7.97 | 12.11 | 0.6299 | 0.6582 |
| May | 25.35 | 39.53 | 8.68 | 12.45 | 0.6413 | 0.6970 |
| Jun | 24.85 | 38.70 | 8.89 | 12.70 | 0.6420 | 0.6999 |
| Jul | 23.42 | 37.95 | 7.92 | 12.75 | 0.6172 | 0.6212 |
| Aug | 23.89 | 38.81 | 7.76 | 12.58 | 0.6156 | 0.6170 |
| Sept | 24.30 | 38.96 | 7.83 | 12.25 | 0.6236 | 0.6391 |
| Oct | 23.74 | 36.46 | 8.77 | 11.89 | 0.6510 | 0.7378 |
| Nov | 20.94 | 31.90 | 8.85 | 11.53 | 0.6565 | 0.7674 |
| Dec | 18.65 | 28.37 | 8.73 | 11.30 | 0.6574 | 0.7728 |

**Table 6: Calculated monthly mean of global solar radiation and input parameters of monthly mean average of global solar radiation for Yola (1990 – 2005)**

| Months | Hcal | $H_o$ | S | $S_o$ | Hcal/$H_o$ | S/$S_o$ |
|---|---|---|---|---|---|---|
| Jan | 19.82 | 31.12 | 7.89 | 11.47 | 0.6369 | 0.6877 |
| Feb | 21.11 | 33.22 | 7.91 | 11.60 | 0.6353 | 0.6818 |
| Marc | 21.34 | 35.19 | 7.04 | 11.81 | 0.6063 | 0.5961 |
| Apri | 23.00 | 38.51 | 6.94 | 12.08 | 0.5974 | 0.5747 |
| May | 24.42 | 39.00 | 8.02 | 12.32 | 0.6260 | 0.6509 |
| Jun | 21.84 | 37.68 | 6.69 | 12.50 | 0.5795 | 0.5353 |
| Jul | 20.07 | 36.86 | 5.88 | 12.53 | 0.5445 | 0.4692 |
| Aug | 21.18 | 38.04 | 6.09 | 12.41 | 0.5566 | 0.4908 |
| Sept | 22.36 | 38.88 | 6.41 | 12.18 | 0.5751 | 0.5263 |
| Oct | 23.18 | 37.18 | 7.67 | 11.92 | 0.6236 | 0.6435 |
| Nov | 21.84 | 33.26 | 9.15 | 11.67 | 0.6567 | 0.7841 |
| Dec | 19.62 | 30.07 | 8.71 | 11.50 | 0.6525 | 0.7573 |

**Table 7: Calculated annual average of global solar radiation and input parameters**

**For North-Eastern, Nigeria (1990 – 2005)**

| Stations | Hcal | $H_o$ | S | $S_o$ | Hcal/$H_o$ | S/$S_o$ |
|---|---|---|---|---|---|---|
| Bauc | 21.8339 | 35.6404 | 7.5358 | 11.9988 | 0.6147 | 0.6305 |
| Yola | 21.6476 | 35.7516 | 7.3667 | 11.9990 | 0.6075 | 0.6165 |
| Ibitara | 20.6592 | 35.8420 | 6.5050 | 11.9991 | 0.5780 | 0.5437 |
| Dutse | 21.9289 | 35.3942 | 7.8117 | 12.1518 | 0.6213 | 0.6461 |
| Nguru | 22.5650 | 35.3401 | 8.3617 | 11.9985 | 0.6397 | 0.6988 |
| Mai | 22.0379 | 35.4686 | 7.8233 | 11.9986 | 0.6235 | 0.6551 |

**Table 8: Summary of monthly mean average of regression constants, extraterrestrial solar radiation, measured and calculated values, measured and calculated clearness index and relative percentage error for Bauchi (1990 - 2005)**

| Months | A | b | $H_o$ | $H_{meas}$ | $H_{cal}$ | $H_{meas}/H_o$ | Hcal/Ho | Error % |
|---|---|---|---|---|---|---|---|---|
| Jan | 0.35 | 0.42 | 30.57 | 14.79 | 19.55 | 0.4838 | 0.6394 | -32.16 |
| Feb | 0.35 | 0.41 | 32.76 | 16.46 | 21.09 | 0.5025 | 0.6438 | -28.12 |
| Marc | 0.33 | 0.47 | 34.90 | 18.21 | 21.65 | 0.5218 | 0.6204 | -18.90 |
| Apri | 0.31 | 0.51 | 38.45 | 17.47 | 22.99 | 0.4543 | 0.5978 | -31.59 |
| May | 0.32 | 0.48 | 39.18 | 16.87 | 24.06 | 0.4306 | 0.6141 | -42.62 |
| Jun | 0.32 | 0.48 | 38.01 | 14.64 | 23.36 | 0.3852 | 0.6147 | -59.58 |
| Jul | 0.28 | 0.56 | 37.21 | 13.88 | 20.89 | 0.3730 | 0.5613 | -50.47 |
| Aug | 0.29 | 0.55 | 38.29 | 13.86 | 21.72 | 0.3619 | 0.5672 | -56.71 |
| Sept | 0.30 | 0.53 | 38.92 | 15.39 | 22.65 | 0.3954 | 0.5820 | -47.19 |
| Oct | 0.34 | 0.44 | 36.98 | 16.36 | 23.36 | 0.4425 | 0.6319 | -42.81 |
| Nov | 0.36 | 0.38 | 32.86 | 16.88 | 21.44 | 0.5137 | 0.6524 | -27.00 |
| Dec | 0.36 | 0.39 | 29.56 | 14.39 | 19.25 | 0.4868 | 0.6511 | -33.77 |

**Table 9: Summary of monthly mean average of regression constants, extraterrestrial solar radiation, measured and calculated values, measured and calculated clearness index and relative percentage error for Dutse (1990 - 2005)**

| Months | A | b | $H_o$ | Hmeas | Hcal | Hmeas/Ho | Hcal/Ho | Error % |
|---|---|---|---|---|---|---|---|---|
| Jan | 0.34 | 0.44 | 29.83 | 23.07 | 18.90 | 0.7734 | 0.6335 | 18.09 |
| Feb | 0.34 | 0.45 | 32.13 | 24.99 | 20.30 | 0.7779 | 0.6318 | 18.77 |
| Marc | 0.32 | 0.47 | 34.49 | 26.51 | 21.41 | 0.7685 | 0.6207 | 19.24 |
| Apri | 0.31 | 0.50 | 38.36 | 26.05 | 23.13 | 0.6791 | 0.6029 | 11.22 |
| May | 0.34 | 0.43 | 39.39 | 24.06 | 25.08 | 0.6109 | 0.6367 | -4.22 |
| Jun | 0.31 | 0.51 | 37.45 | 22.24 | 22.39 | 0.5938 | 0.5978 | -0.66 |
| Jul | 0.30 | 0.52 | 37.65 | 20.05 | 22.22 | 0.5326 | 0.5904 | -10.85 |
| Aug | 0.29 | 0.53 | 38.60 | 19.26 | 22.48 | 0.4989 | 0.5825 | -16.74 |
| Sept | 0.31 | 0.51 | 38.96 | 20.63 | 23.38 | 0.5296 | 0.6002 | -13.35 |
| Oct | 0.35 | 0.41 | 36.69 | 22.7 | 23.66 | 0.6187 | 0.6449 | -4.22 |
| Nov | 0.37 | 0.37 | 32.31 | 23.78 | 21.25 | 0.7360 | 0.6578 | 10.63 |
| Dec | 0.37 | 0.37 | 28.88 | 22.63 | 18.95 | 0.7836 | 0.6561 | 16.27 |

**Table 10: Summary of monthly mean average of regression constants, extraterrestrial solar radiation, measured and calculated values, measured and calculated clearness index and relative percentage error for Ibitaraba (1990 - 2005)**

| Months | a | b | $H_o$ | Hmeas | Hcal | Hmeas/Ho | Hcal/Ho | Error % |
|---|---|---|---|---|---|---|---|---|
| Jan | 0.30 | 0.51 | 31.61 | 22.51 | 18.74 | 0.7121 | 0.5927 | 16.77 |
| Feb | 0.30 | 0.52 | 33.63 | 23.52 | 19.82 | 0.6993 | 0.5891 | 15.75 |
| Marc | 0.28 | 0.56 | 35.45 | 22.91 | 19.67 | 0.6463 | 0.5548 | 14.16 |
| Apri | 0.31 | 0.50 | 38.55 | 21.51 | 23.07 | 0.5580 | 0.5984 | -7.23 |
| May | 0.31 | 0.51 | 38.83 | 19.56 | 23.09 | 0.5038 | 0.5947 | -18.06 |
| Jun | 0.29 | 0.54 | 37.38 | 18.38 | 21.48 | 0.4917 | 0.5746 | -16.85 |
| Jul | 0.26 | 0.61 | 36.54 | 17.54 | 18.65 | 0.4800 | 0.5105 | -6.35 |
| Aug | 0.26 | 0.61 | 37.81 | 17.5 | 19.53 | 0.4629 | 0.5165 | -11.58 |
| Sept | 0.27 | 0.58 | 38.83 | 18.03 | 20.96 | 0.4644 | 0.5399 | -16.27 |
| Oct | 0.30 | 0.52 | 37.35 | 18.33 | 21.89 | 0.4908 | 0.5862 | -19.44 |
| Nov | 0.35 | 0.41 | 33.62 | 20.92 | 21.58 | 0.6223 | 0.6418 | -3.14 |
| Dec | 0.34 | 0.43 | 30.52 | 22.18 | 19.42 | 0.7267 | 0.6361 | 12.46 |

**Table 11: Summary of monthly mean average of regression constants, extraterrestrial solar radiation, measured and calculated values, measured and calculated clearness index and relative percentage error for Maiduguri (1990 - 2005)**

| Months | a | b | $H_o$ | Hmeas | Hcal | Hmeas/Ho | Hcal/Ho | Error % |
|---|---|---|---|---|---|---|---|---|
| Jan | 0.36 | 0.40 | 29.81 | 26.48 | 19.34 | 0.8884 | 0.6490 | 26.95 |
| Feb | 0.36 | 0.39 | 32.10 | 27.44 | 20.95 | 0.8547 | 0.6527 | 23.64 |
| Marc | 0.34 | 0.43 | 34.48 | 27.13 | 21.94 | 0.7868 | 0.6364 | 19.11 |
| Apri | 0.32 | 0.47 | 38.35 | 24.68 | 23.76 | 0.6435 | 0.6196 | 3.71 |
| May | 0.33 | 0.46 | 39.39 | 21.23 | 24.63 | 0.5389 | 0.6252 | -16.01 |
| Jun | 0.31 | 0.50 | 38.43 | 19.18 | 23.30 | 0.4991 | 0.6062 | -21.46 |
| Jul | 0.29 | 0.54 | 37.66 | 17.56 | 21.63 | 0.4663 | 0.5744 | -23.19 |
| Aug | 0.29 | 0.53 | 38.61 | 17.76 | 22.43 | 0.4600 | 0.5809 | -26.29 |
| Sept | 0.31 | 0.50 | 38.96 | 20.52 | 23.41 | 0.5267 | 0.6009 | -14.09 |
| Oct | 0.33 | 0.46 | 36.68 | 24.35 | 22.95 | 0.6639 | 0.6256 | 5.77 |
| Nov | 0.38 | 0.35 | 32.29 | 26.58 | 21.33 | 0.8232 | 0.6606 | 19.75 |
| Dec | 0.36 | 0.39 | 28.86 | 26.3 | 18.78 | 0.9114 | 0.6507 | 28.60 |

**Table 12: Summary of monthly mean average of regression constants, extraterrestrial solar radiation, measured and calculated values, measured and calculated clearness index and relative percentage error for Nguru (1990 - 2005)**

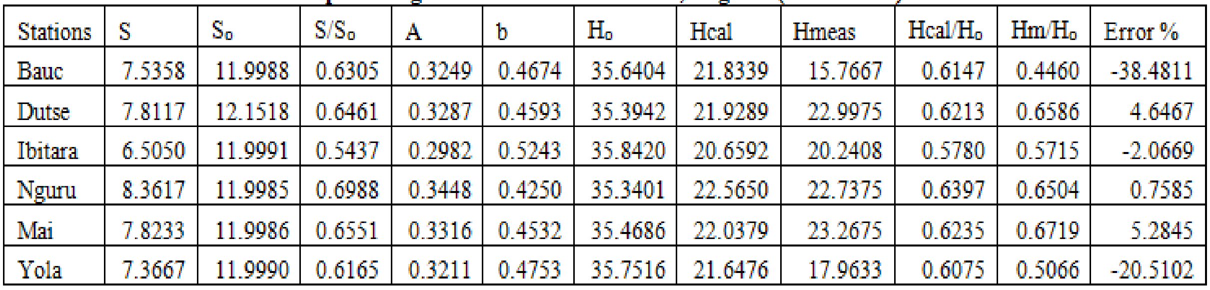
| Months | a | b | $H_o$ | Hmeas | Hcal | Hmeas/Ho | Hcal/Ho | Error % |
|---|---|---|---|---|---|---|---|---|
| Jan | 0.36 | 0.40 | 29.28 | 21.52 | 19.09 | 0.7349 | 0.6518 | 11.31 |
| Feb | 0.36 | 0.39 | 31.65 | 24.07 | 20.72 | 0.7604 | 0.6546 | 13.92 |
| Marc | 0.34 | 0.44 | 34.18 | 25.89 | 21.73 | 0.7574 | 0.6356 | 16.07 |
| Apri | 0.33 | 0.45 | 38.27 | 26.49 | 24.11 | 0.6921 | 0.6299 | 8.99 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| May | 0.34 | 0.43 | 39.53 | 25.33 | 25.35 | 0.6408 | 0.6413 | -0.07 |
| Jun | 0.35 | 0.42 | 38.70 | 23.62 | 24.85 | 0.6103 | 0.6420 | -5.20 |
| Jul | 0.32 | 0.48 | 37.95 | 20.76 | 23.42 | 0.5470 | 0.6172 | -12.82 |
| Aug | 0.32 | 0.48 | 38.81 | 19.83 | 23.89 | 0.5109 | 0.6156 | -20.50 |
| Sept | 0.33 | 0.47 | 38.96 | 20.77 | 24.30 | 0.5331 | 0.6236 | -16.98 |
| Oct | 0.36 | 0.40 | 36.46 | 22.1 | 23.74 | 0.6061 | 0.6510 | -7.40 |
| Nov | 0.37 | 0.38 | 31.90 | 21.74 | 20.94 | 0.6816 | 0.6565 | 3.68 |
| Dec | 0.37 | 0.37 | 28.37 | 20.73 | 18.65 | 0.7306 | 0.6574 | 10.02 |

**Table 13: Summary of monthly mean average of regression constants, extraterrestrial solar radiation, measured and calculated values, measured and calculated clearness index and relative percentage error for Yola (1990 - 2005)**

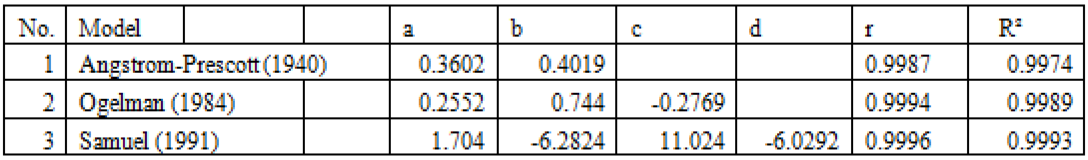
| Months | a | b | $H_o$ | Hmeas | Hcal | Hmeas/Ho | Hcal/Ho | Error % |
|---|---|---|---|---|---|---|---|---|
| Jan | 0.34 | 0.43 | 31.12 | 13.74 | 19.82 | 0.4415 | 0.6369 | -44.25 |
| Feb | 0.34 | 0.43 | 33.22 | 20.48 | 21.11 | 0.6165 | 0.6353 | -3.06 |
| Marc | 0.31 | 0.49 | 35.19 | 21.31 | 21.34 | 0.6055 | 0.6063 | -0.12 |
| Apri | 0.31 | 0.50 | 38.51 | 20 | 23.00 | 0.5194 | 0.5974 | -15.02 |
| May | 0.33 | 0.45 | 39.00 | 18.64 | 24.42 | 0.4779 | 0.6260 | -30.99 |
| Jun | 0.29 | 0.53 | 37.68 | 16.97 | 21.84 | 0.4503 | 0.5795 | -28.68 |
| Jul | 0.27 | 0.58 | 36.86 | 15.21 | 20.07 | 0.4126 | 0.5445 | -31.97 |
| Aug | 0.28 | 0.56 | 38.04 | 14.9 | 21.18 | 0.3916 | 0.5566 | -42.11 |
| Sept | 0.29 | 0.54 | 38.88 | 16.64 | 22.36 | 0.4280 | 0.5751 | -34.37 |
| Oct | 0.33 | 0.46 | 37.18 | 18.65 | 23.18 | 0.5017 | 0.6236 | -24.30 |
| Nov | 0.38 | 0.36 | 33.26 | 19.96 | 21.84 | 0.6001 | 0.6567 | -9.43 |
| Dec | 0.37 | 0.38 | 30.07 | 19.06 | 19.62 | 0.6339 | 0.6525 | -2.94 |

**Table 14: Summary of annual average of regression constants, extraterrestrial solar radiation, measured and calculated values, measured and calculated clearness indexes and relative percentage error for North-Eastern, Nigeria (1990 - 2005)**



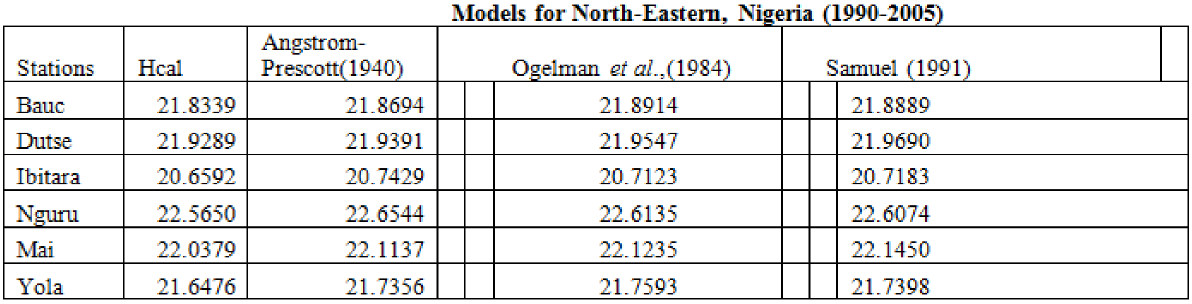
| Stations | S | $S_o$ | $S/S_o$ | A | b | $H_o$ | Hcal | Hmeas | $Hcal/H_o$ | $Hm/H_o$ | Error % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Bauc | 7.5358 | 11.9988 | 0.6305 | 0.3249 | 0.4674 | 35.6404 | 21.8339 | 15.7667 | 0.6147 | 0.4460 | -38.4811 |
| Dutse | 7.8117 | 12.1518 | 0.6461 | 0.3287 | 0.4593 | 35.3942 | 21.9289 | 22.9975 | 0.6213 | 0.6586 | 4.6467 |
| Ibitara | 6.5050 | 11.9991 | 0.5437 | 0.2982 | 0.5243 | 35.8420 | 20.6592 | 20.2408 | 0.5780 | 0.5715 | -2.0669 |
| Nguru | 8.3617 | 11.9985 | 0.6988 | 0.3448 | 0.4250 | 35.3401 | 22.5650 | 22.7375 | 0.6397 | 0.6504 | 0.7585 |
| Mai | 7.8233 | 11.9986 | 0.6551 | 0.3316 | 0.4532 | 35.4686 | 22.0379 | 23.2675 | 0.6235 | 0.6719 | 5.2845 |
| Yola | 7.3667 | 11.9990 | 0.6165 | 0.3211 | 0.4753 | 35.7516 | 21.6476 | 17.9633 | 0.6075 | 0.5066 | -20.5102 |

Table 15: Summary of Model parameters for the average variation of relative sunshine duration (S/So) and clearness index (Hcal/Ho) for North-Eastern, Nigeria (1990 - 2005)



| No. | Model | | | a | b | c | d | r | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Angstrom-Prescott (1940) | | | 0.3602 | 0.4019 | | | 0.9987 | 0.9974 |
| 2 | Ogelman (1984) | | | 0.2552 | 0.744 | -0.2769 | | 0.9994 | 0.9989 |
| 3 | Samuel (1991) | | | 1.704 | -6.2824 | 11.024 | -6.0292 | 0.9996 | 0.9993 |

**Table 16: Estimation of annual average daily global solar radiation from three sunshine-based Models for North-Eastern, Nigeria (1990-2005)**



| Stations | Hcal | Angstrom-Prescott(1940) | Ogelman *et al.*,(1984) | Samuel (1991) |
|---|---|---|---|---|
| Bauc | 21.8339 | 21.8694 | 21.8914 | 21.8889 |
| Dutse | 21.9289 | 21.9391 | 21.9547 | 21.9690 |
| Ibitara | 20.6592 | 20.7429 | 20.7123 | 20.7183 |
| Nguru | 22.5650 | 22.6544 | 22.6135 | 22.6074 |
| Mai | 22.0379 | 22.1137 | 22.1235 | 22.1450 |
| Yola | 21.6476 | 21.7356 | 21.7593 | 21.7398 |

**Table 17: The equation with regression and statistical indicators for North-Eastern, Nigeria (1990 - 2005)**

| Stations | a | b | MBE | RMSE | MPE | R | $R^2$ | t |
|---|---|---|---|---|---|---|---|---|
| Bauc | 0.389 | 0.358 | 6.0672 | 6.2661 | 6.0672 | 0.9899 | 0.9799 | 12.849 |
| Dutse | 0.417 | 0.316 | -1.0686 | 3.1439 | -1.0686 | 0.9890 | 0.9781 | 1.199 |
| Ibitara | 0.334 | 0.449 | 0.4160 | 2.8534 | 0.4160 | 0.9882 | 0.9766 | 0.489 |
| Nguru | 0.453 | 0.268 | -0.1725 | 2.6664 | -0.1725 | 0.9943 | 0.9887 | 0.215 |
| Mai | 0.416 | 0.317 | -1.2296 | 4.8486 | -1.2296 | 0.9850 | 0.9702 | 0.869 |
| Yola | 0.386 | 0.36 | 3.6843 | 4.3163 | 3.6843 | 0.9846 | 0.9695 | 5.434 |

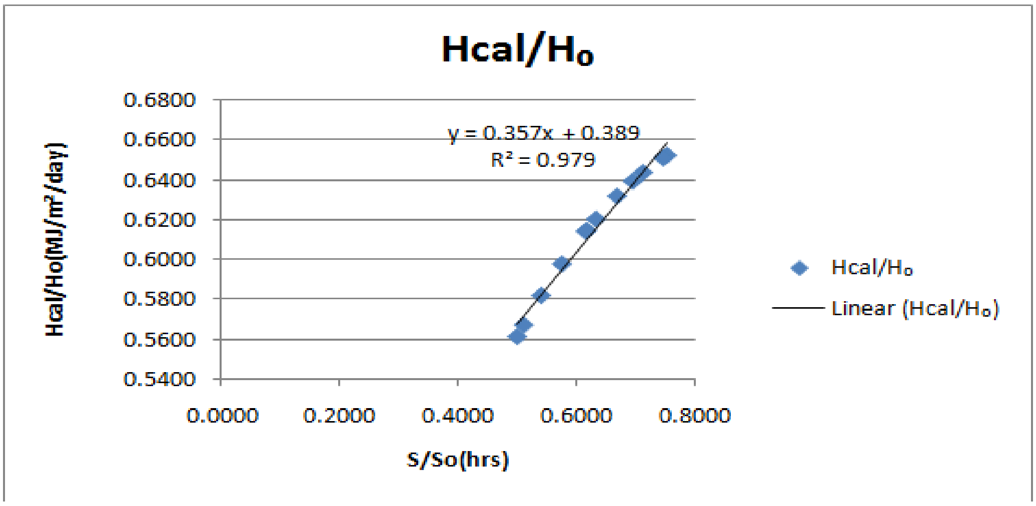


**Figure 1: Variation of clearness index with respect to sunshine hours for Bauchi (1990 - 2005)**

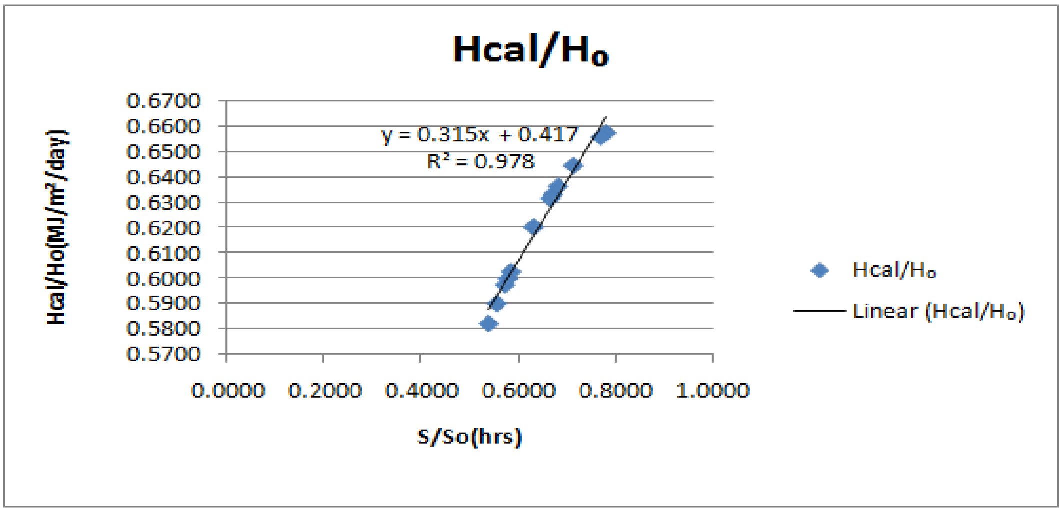


**Figure 2: Variation of clearness index with respect to sunshine hours for Dutse (1990 - 2005)**

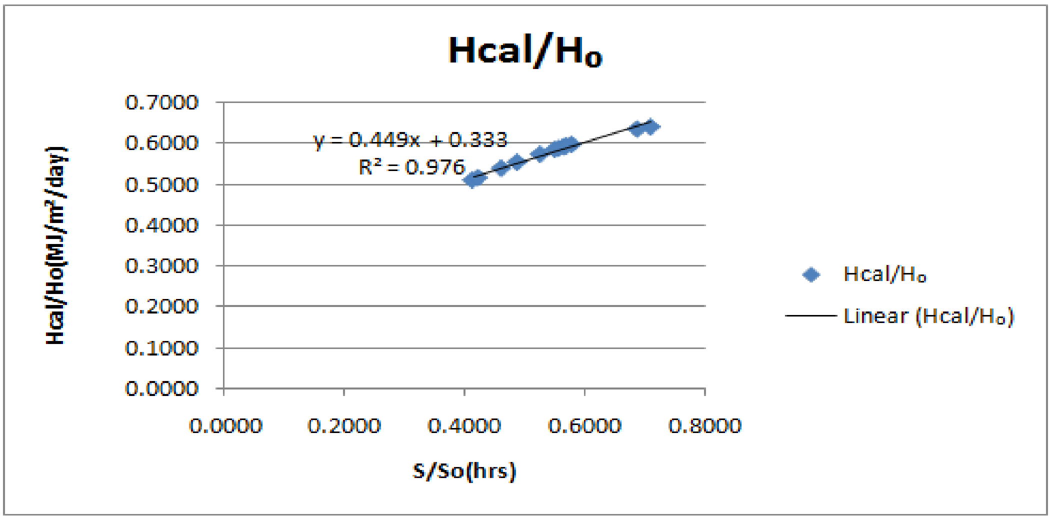


**Figure 3: Variation of clearness index with respect to sunshine hours for Ibitaraba (1990 - 2005)**

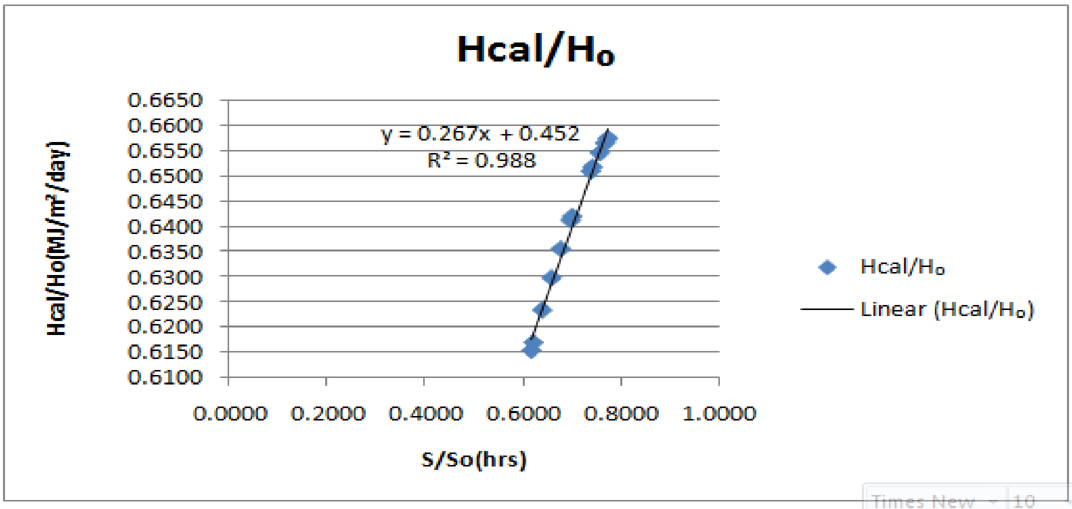


**Figure 4: Variation of clearness index with respect to sunshine hours for Nguru (1990 - 2005)**

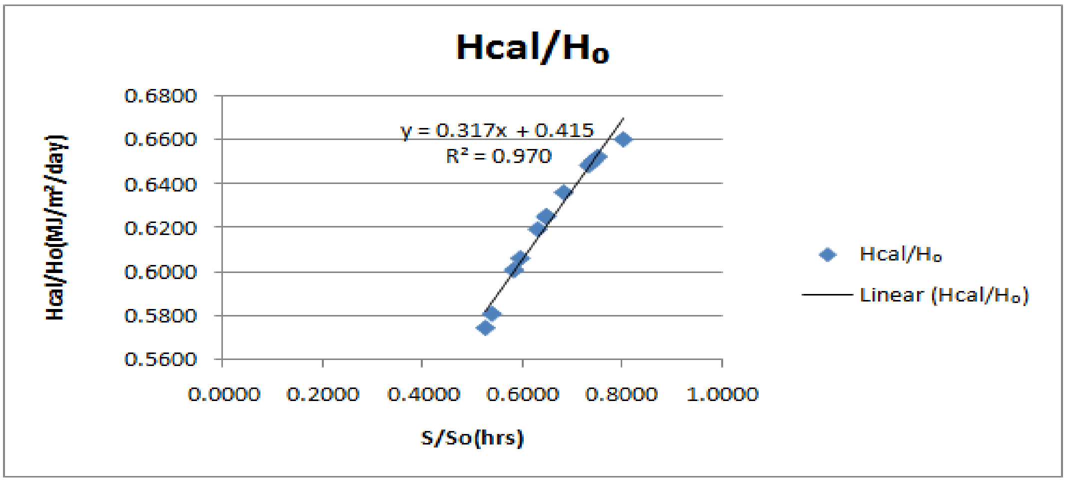


**Figure 5: Variation of clearness index with respect to sunshine hours for Maiduguri (1990 - 2005)**

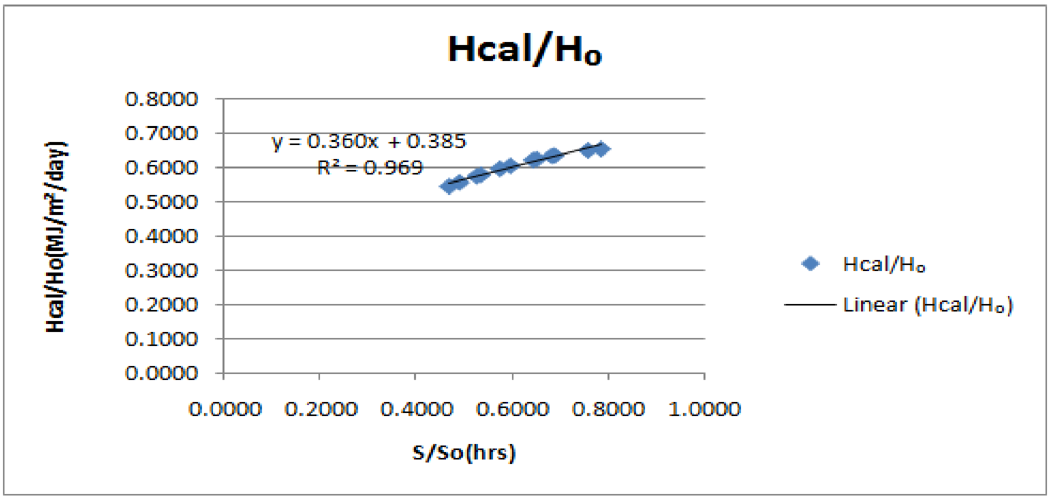


**Figure 6: Variation of clearness index with respect to sunshine hours for Yola (1990 - 2005)**

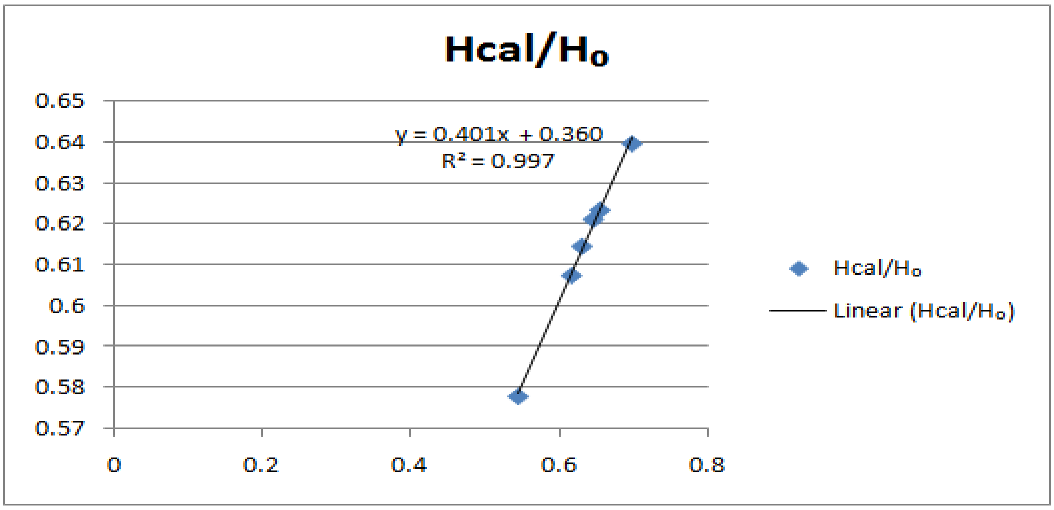


**Figure 7: Variation of clearness index with respect to sunshine hours for North-Eastern, Nigeria (1990 - 2005)**

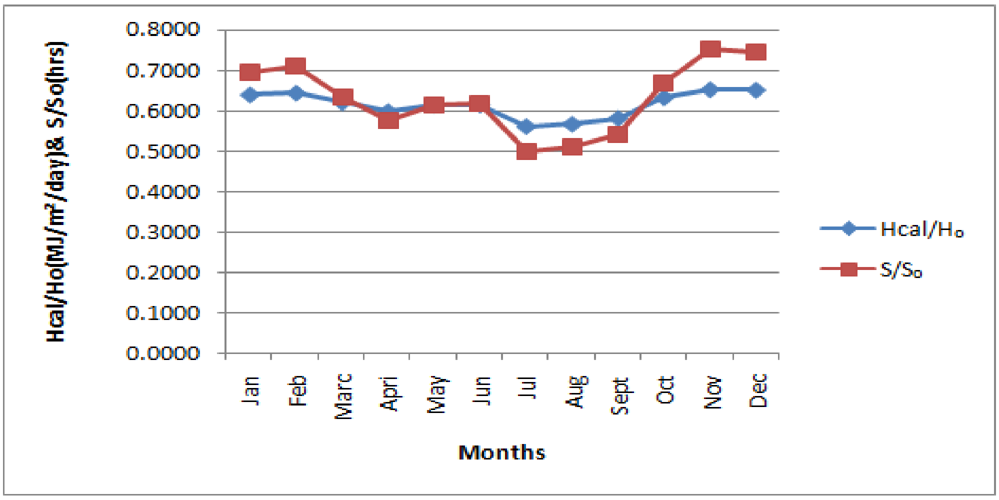


**Figure 8:Correlation of monthly variation of Hcal/Ho and S/So for Bauchi (1990 - 2005)**

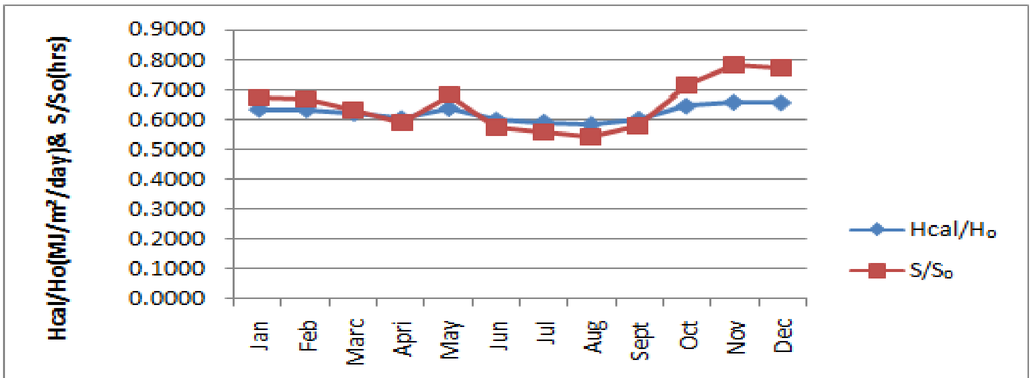


**Figure 9:Correlation of monthly variation of Hcal/Ho and S/So for Dutse (1990 - 2005)**

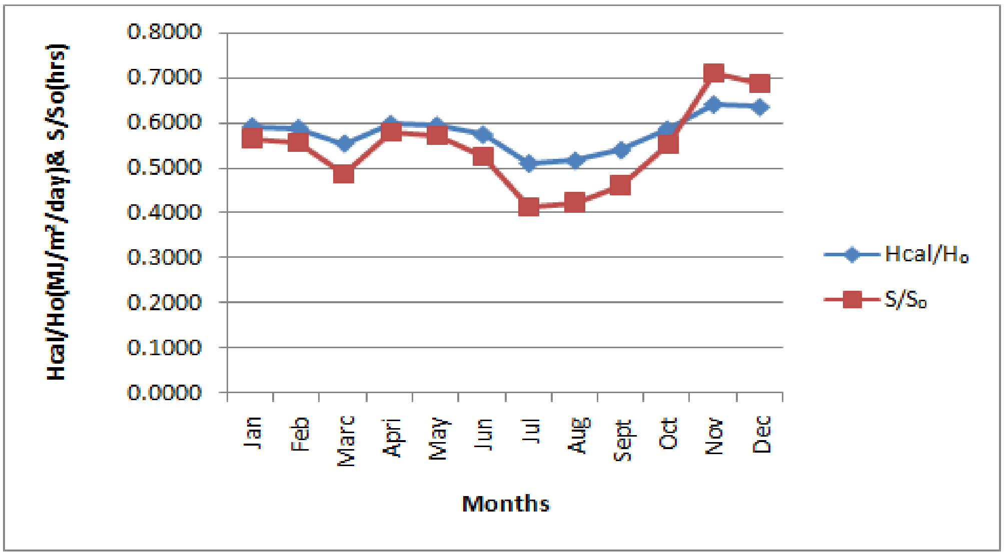


**Figure 10: Correlation of monthly variation of Hcal/Ho and S/So for Ibitaraba (1990 - 2005)**

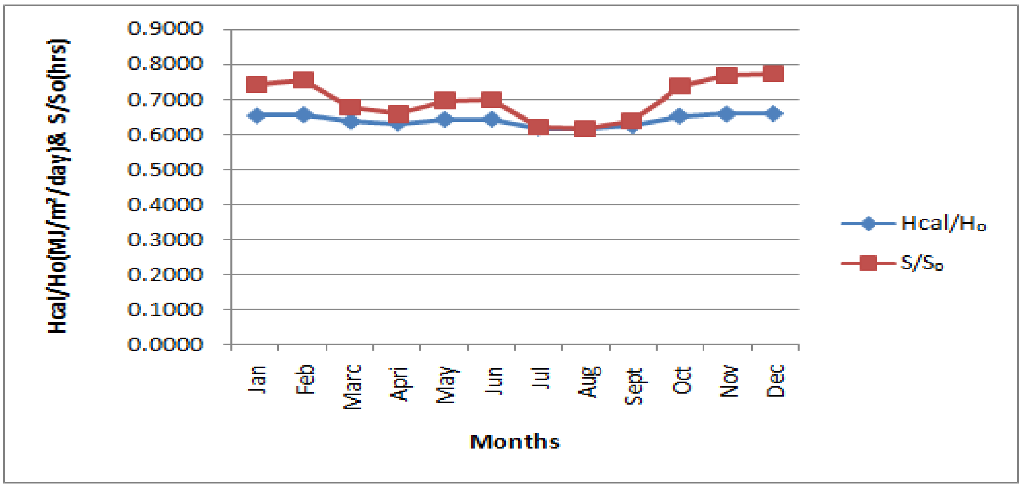


**Figure 11:Correlation of monthly variation of Hcal/Ho and S/So for Nguru (1990 - 2005)**

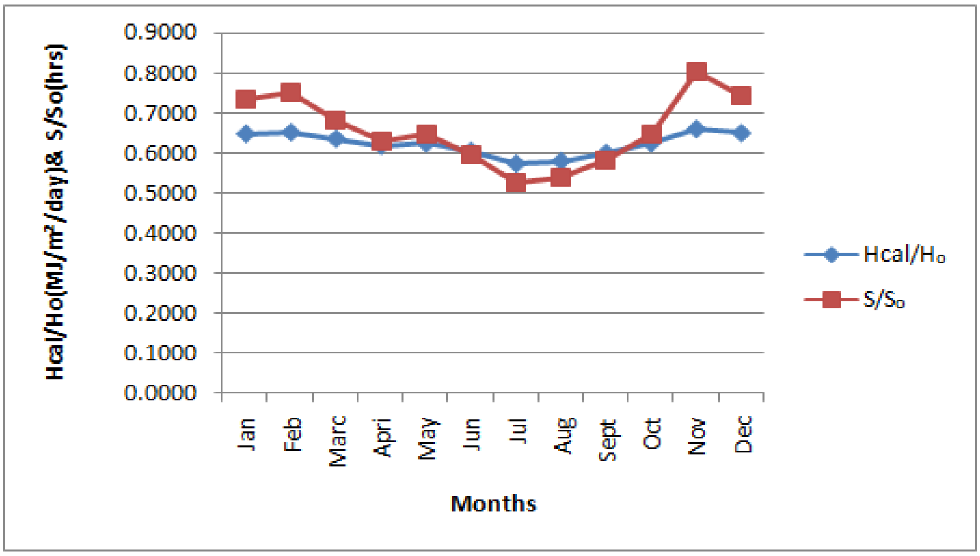


**Figure 12:Correlation of monthly variation of Hcal/Ho and S/So for Maiduguri (1990 - 2005)**

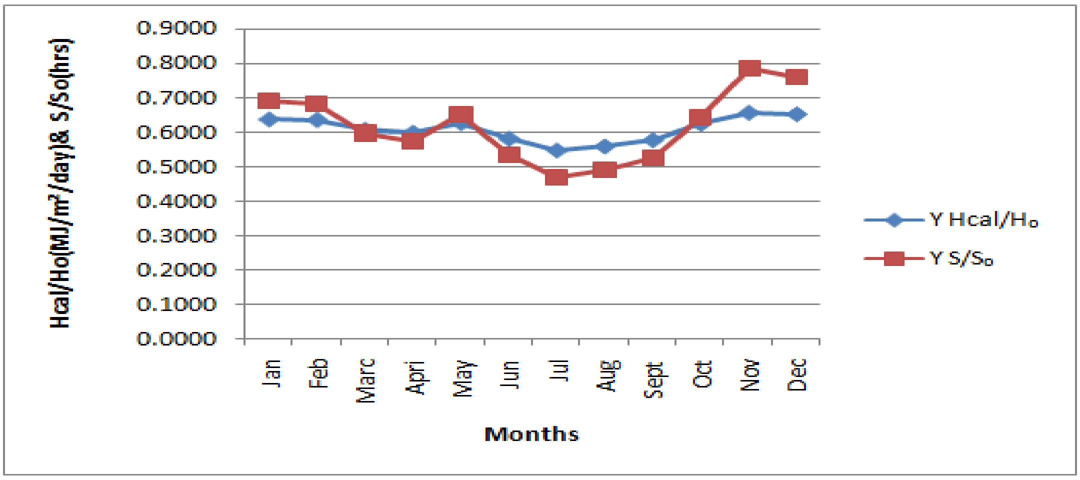


**Figure 13:Correlation of monthly variation of Hcal/Ho and S/So for Yola (1990 - 2005)**

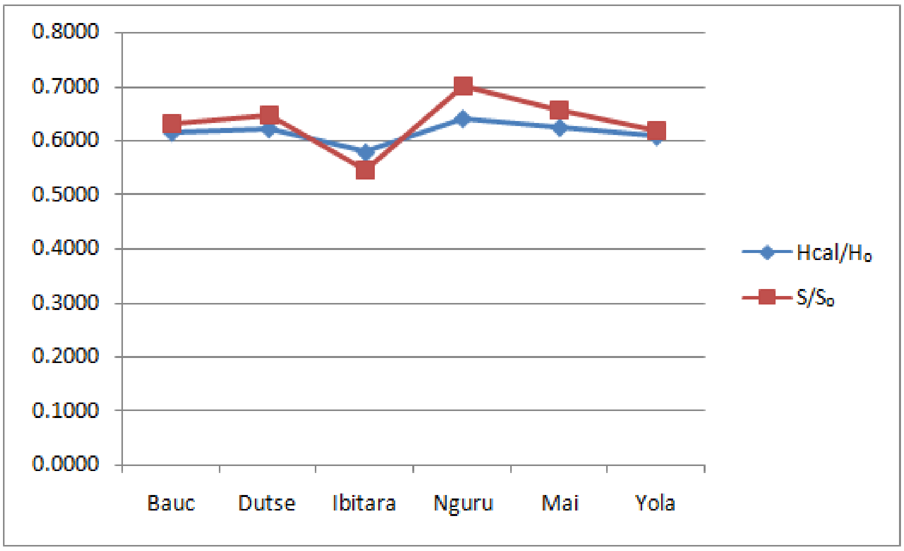


**Figure 14:**Correlation of annual average variation of Hcal/Ho and S/So for North- Eastern, Nigeria (1990 - 2005)

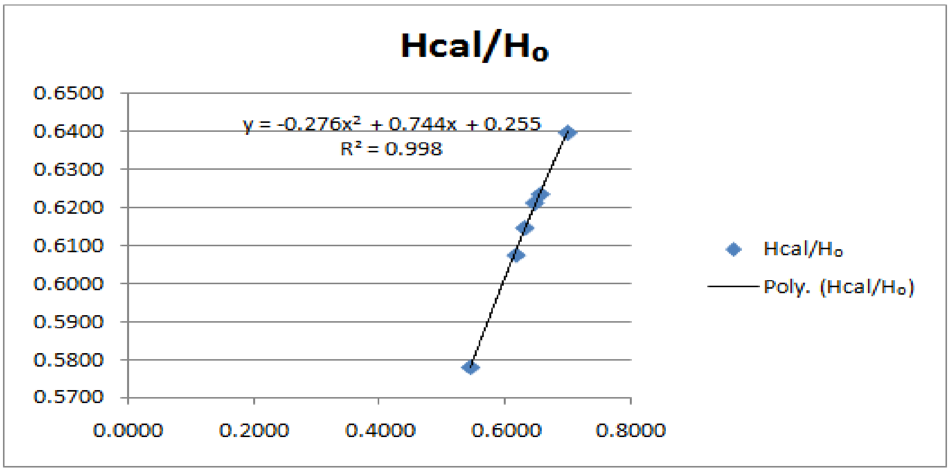


**Figure 16:** Ogelman *et al.*, (1984) model fitting for variation of clearness index (Hcal/Ho) Versus relative sunshine duration(S/So) for North-Eastern, Nigeria(1990 - 2005)

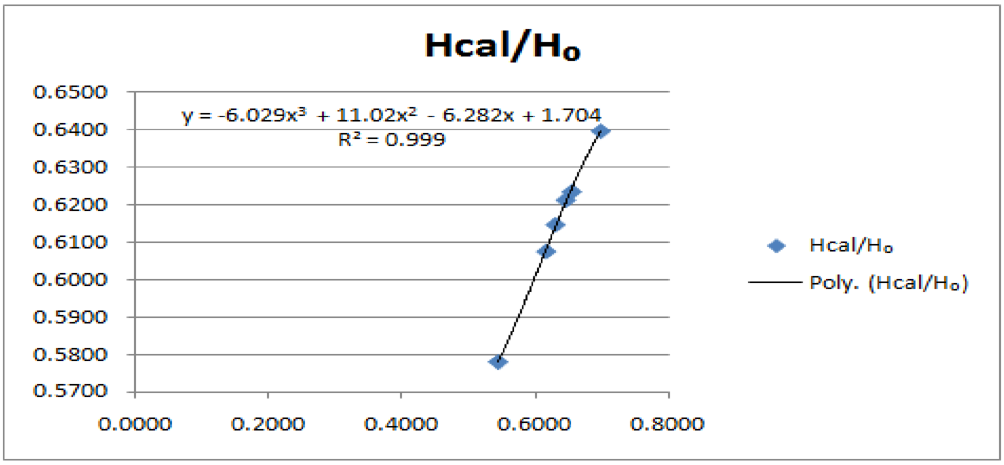


**Figure 17:** Samuel (1991) model fitting for variation of clearness index (Hcal/Ho) versus relative sunshine duration(S/So) for North-Eastern, Nigeria(1990 - 2005)

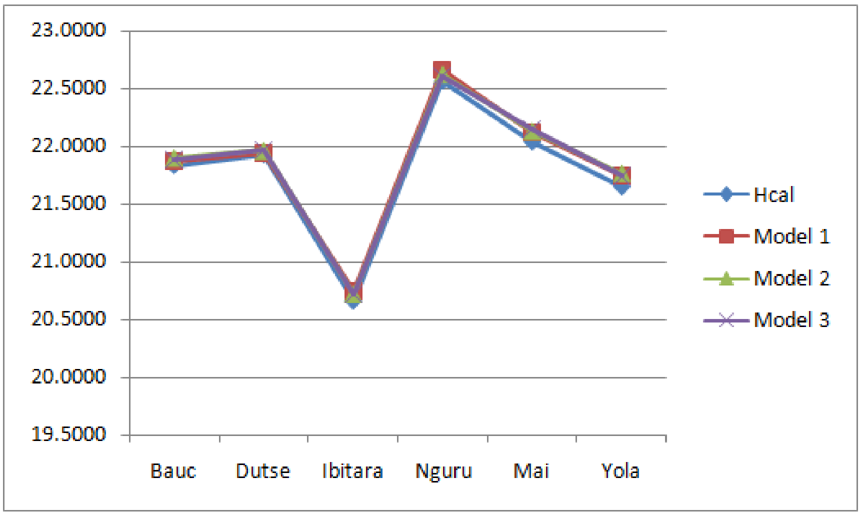


**Figure 18: Comparison of the calculated value of annual average daily global solar radiation from various models, compared with the calculated value for North-Eastern, Nigeria (1990-2005).**

The extraterrestrial solar radiation, $H_O$ (MJ/m$^2$/day), and the monthly day length, $S_O$ (hr), were computed for each location using equations (2) and (5), the input parameters for the calculation of the mean monthly global solar radiation for six locations and as well as North- Eastern, Nigeria (1990- 2005) are shown in the Tables 1-7 respectively. It was observed from table 7 that the approximate values of sunshine durations for the six locations and the North-Eastern, Nigeria for the period of fifteen years under study are Bauchi=63%, Dutse=65%, Ibitaraba=54%, Maiduguri=66%, Nguru=70% and Yola=62% throughout the year while for the North-East as a whole was 63%. However, using these parameters, the regression constant ‘a’ and ‘b’ are evaluated, as a = 0.318 and b = 0.470. Using these parameters, the regression constants for each location ‘a’ and ‘b’ is being evaluated respectively. Substituting these values into equation (1), we now established the Angstrom- type empirical correlations for the estimation developed for six locations as follow:

1. Bauchi

$$\frac{H_{cal}}{H_o} = 0.389 + 0.358\left(\frac{S}{S_o}\right) \tag{16}$$

2. Dutse

$$\frac{H_{cal}}{H_o} = 0.417 + 0.316\left(\frac{S}{S_o}\right) \tag{17}$$

3. Ibitaraba

$$\frac{H_{cal}}{H_o} = 0.334 + 0.449\left(\frac{S}{S_o}\right) \tag{18}$$

4. Maiduguri

$$\frac{H_{cal}}{H_o} = 0.416 + 0.317\left(\frac{S}{S_o}\right) \tag{19}$$

5. Nguru

$$\frac{H_{cal}}{H_o} = 0.453 + 0.268\left(\frac{S}{S_o}\right) \tag{20}$$

6. Yola

$$\frac{H_{cal}}{H_o} = 0.386 + 0.360\left(\frac{S}{S_o}\right) \tag{21}$$

In Bauchi, the value of $\frac{H_{cal}}{H_o}$ **(**= 0.5613) correspond to the lowest value of $\frac{S}{S_o}$ (= 0.4993) and Hcal (20.89MJ/m$^2$/day) in the month of July, the value of $\frac{H_{cal}}{H_o}$ **(**= 0.5825) corresponding to the lowest value of $\frac{S}{S_o}$ (= 0.5405) and Hcal (22.48MJ/m$^2$/day) in the month of August for Dutse, also, the value of $\frac{H_{cal}}{H_o}$ **(**= 0.5105) corresponding to the lowest value of $\frac{S}{S_o}$ (= 0.4138) and Hcal (18.65MJ/m$^2$/day) in the month of July for Ibitaraba, for Maiduguri, the value of $\frac{H_{cal}}{H_o}$ **(**= 0.5744) also correspond to the lowest value of $\frac{S}{S_o}$ (= 0.5241) and Hcal (21.63MJ/m$^2$/day) in the month of July, likewise, the value of $\frac{H_{cal}}{H_o}$ **(**= 0.6156) corresponding to the lowest value of $\frac{S}{S_o}$ (= 0.6170) and Hcal (23.89MJ/m$^2$/day) in the month of August for Nguru and lastly, the value of $\frac{H_{cal}}{H_o}$ **(**= 0.5445) correspond to the lowest value of $\frac{S}{S_o}$ (= 0.4692) and Hcal (20.07MJ/m$^2$/day) in the month of July for Yola which are indication of poor sky condition. These conditions correspond to the general wet or rainy season (June – September) observed in Nigeria, during which there is much cloud cover.

It is observed from Equations (16) **-** (21) that neither a nor b vary with latitude or altitude in any systematic manner. However, the values of the sum of the regression constants a + b, which represent the maximum Clearness Index ($\frac{S}{S_o} \square 1$), averaged over the period of analysis, are found to be almost equal for the six meteorological stations, El-Sebaii and Trabea(2005) and Salima and Chavula (2012). The values of (a + b) obtained for six locations in North-Eastern, Nigeria are 0.75, 0.73, 0.78, 0.73, 0.72 and 0.75 respectively.
Averaged results for the linear regression models for the six locations were used in developing the linear regression model for estimating global solar radiation in North- East, (1990-2005):

$$\frac{H_{cal}}{H_o} = 0.3602 + 0.4019\left(\frac{S}{S_o}\right) \tag{22}$$

With coefficient of determination R$^2$ (99.74) and maximum clearness index equals 0.6397 at Nguru.
The regression constants (Table 8-13), **a** and **b** of different months were evaluated from equations (7) and (8) for the six locations. To compute the calculated values of the mean monthly average of global solar radiation $H_{cal}$, the values of **a** and **b** were inserted into equation (1) and the correlation may be used to compute $H_{cal}$ at other locations having the same altitude. Looking at these values of measured and calculated clearness indexes; it is observed that some locations had the lowest values in the months of July or August.

The lowest clear index $\frac{H_{meas}}{H_o}$ (= 0.3730), $\frac{H_{cal}}{H_o}$ (= 0.5613) with $H_{meas}$ (=13.88MJ/m$^2$/day), $H_{cal}$ (= 20.89MJ/m$^2$/day) and $\frac{H_{meas}}{H_o}$ (= 0.3619), $\frac{H_{cal}}{H_o}$( = 0.5672) with $H_{meas}$ (=13.86 MJ/m$^2$/day) , $H_{cal}$ (= 21.72MJ/m$^2$/day) for Bauchi was observed in the months of July and August, $\frac{H_{meas}}{H_o}$ (= 0.4989), $\frac{H_{cal}}{H_o}$ (=

0.5825) with $H_{meas}$ (19.26 MJ/m$^2$/day) and $H_{cal}$ (= 22.48 MJ/m$^2$/day) for Dutse occurred in the month of August, $\frac{H_{meas}}{H_o}$ = (0.4800), $\frac{H_{cal}}{H_o}$ = 0.5105} with $H_{meas}$ (=17.54MJ/m$^2$/day), $H_{cal}$ (= 18.65MJ/m$^2$/day) and $\frac{H_{meas}}{H_o}$ (= 0.4629), $\frac{H_{cal}}{H_o}$ (= 0.5165) with $H_{meas}$ (=17.50MJ/m$^2$/day) , $H_{cal}$ (= 19.53MJ/m$^2$/day) for Ibitaraba was also observed in both July and August, $\frac{H_{meas}}{H_o}$ = 0.4663, $\frac{H_{cal}}{H_o}$ = 0.5744} with $H_{meas}$ (=17.56 MJ/m$^2$/day), $H_{cal}$ (= 21.63 MJ/m$^2$/day) and $\frac{H_{meas}}{H_o}$ (= 0.4600), $\frac{H_{cal}}{H_o}$ (= 0.5809) with ($H_{meas}$ =17.76 MJ/m$^2$/day , $H_{cal}$ = 22.43 MJ/m$^2$/day) for Maiduguri was seen in the months July- August, while for Nguru, $\frac{H_{meas}}{H_o}$ (= 0.5109), $\frac{H_{cal}}{H_o}$ (= 0.6156) with $H_{meas}$ (19.83 MJ/m$^2$/day) and $H_{cal}$ (= 23.89 MJ/m$^2$/day) was observed in the month of August, and $\frac{H_{meas}}{H_o}$ (= 0.4126), $\frac{H_{cal}}{H_o}$ (= 0.5445) with $H_{meas}$ (=15.21MJ/m$^2$/day), $H_{cal}$ (= 20.07MJ/m$^2$/day) and $\frac{H_{meas}}{H_o}$ (= 0.3916), $\frac{H_{cal}}{H_o}$ (= 0.5566) with $H_{meas}$ ( =14.90MJ/m$^2$/day) , $H_{cal}$ (= 21.18MJ/m$^2$/day) for Yola was observed in July- August which can all be traced to the meteorological conditions of locations.

The results presented in Table 14 shows that the values of calculated clearness index corresponding to the relative sunshine hour for the six locations seems to be good throughout the fifteen years (1990-2005) with their respective calculated and measured values of global solar radiation with Nguru having higher annual average sunshine hour of about 8.36hours which is an indication of clear sky condition in the North-Eastern, Nigeria (Ojosu, 1987). The relative percentage error for these locations and North-Eastern, Nigeria were estimated and their error ranged between the following minima and maxima values:(-59.58%, -18.90%) for Bauchi, (-16.74%, 19.24%) for Dutse, (-19.44%, 16.77%) for Ibitaraba, (-26.29%, 28.60%) for Maiduguri, (-20.50%, 16.07%) for Nguru, (-44.25%, -2.94%) for Yola and for the North-Eastern, Nigeria (-38.48%, 5.28%). Figures (1-6) and 7 shows regression constants and regression of determination ($R^2$) for all the locations and as well as North-Eastern, Nigeria (1990-2005). It was observed that models for these locations and North-East have excellent fits for the data.

The correlations of monthly variation of calculated clearness index and sunshine fraction for the period of fifteen years are shown in Figures (8-13) for the six locations. Though there is a similarity in both patterns, however, there is significance difference in the values of both parameters for these locations. It is clearly observed that there is a defined trough in the curves for the months of July – August at six locations. This is an indication that the atmospheric condition over these locations and their environs were at a poor state in which the sky were not clear. The value of the clearness index and the relative sunshine fractions for the six locations were observed to be as follow: 0.5613 and 0.4993 in the month of July for Bauchi, 0.5825 and 0.5405 in the month of August for Dutse, 0.5105 and 0.4138 in the month of July for Ibitaraba, at Maiduguri, it was 0.5744 and 0.5241 in the month of July, 0.6156 and 0.6170 in the month of August for Nguru and that of Yola were 0.5445 and 0.4692 in the month of July respectively. The results suggest that the rainfalls at these locations are at peak during the months of July – August when the sky is cloudy and the solar radiation is fairly low. However, just immediately after the August minimum, the clearness index and the relative sunshine fraction increased remarkably with the cloud cover crossing over the clearness index. The values of the clearness index and relative sunshine fraction for the six locations which reached peaks at the months of November are Bauchi (0.6524 and 0.7534), Dutse (0.6578 and 0.7802), Ibitaraba (0.6418 and 0.7082), Maiduguri (0.6606 and 0.8011), Yola (0.6567 and 0.7841) while in December reached peaks at 0.6574 and 0.7728 for Nguru. This implies that a clear sky will obviously fell within the dry season and hence a high solar radiation is experienced. Obviously, this is generally the dry season period in Nigeria.

Moreover, the correlation of annual average variation of calculated clearness index and sunshine fraction for North-Eastern, Nigeria for the period of fifteen years (1990 - 2005) is shown in Figure 14. Although there is similarity in both patterns, however, a significant difference between clearness index and sunshine fraction is also observed. There is a dip in the curves in Ibitaraba. This is an indication that the state of the atmosphere in Ibitaraba and its surroundings was at poor condition in which the sky was not clear. The values of the clearness index and the relative sunshine duration were observed to be 0.5780 and 0.5437 respectively. However, just immediately after the minimum at Ibitaraba, the clearness index and the relative sunshine duration increased remarkably with the cloud cover crossing over the clearness index and Nguru reached peaks at 0.6397 and 0.6988 respectively. This implies that a clear sky will obviously be met within the dry season and hence a high solar radiation is experienced.

In the sunshine-based models proposed for this study, three models were used to show the validation of relative sunshine duration and clearness index for North-Eastern, Nigeria for the period of fifteen years (1990 - 2005). Figures 15-17 show the results of the performance of each model in terms of regression of coefficient ($R^2$), correlation coefficient (r). The empirical correlation models were also developed for the three sunshine-based models for North-Eastern, Nigeria (1990 - 2005). The results for the three sunshine-based models were summarized below:

1. The empirical correlation for Angstrom-Prescott (1940) model in equation (9) was

$$\frac{H_{cal}}{H_o} = 0.3602 + 0.4019\left(\frac{S}{S_o}\right) \tag{23}$$

The coefficient of determination, $R^2$ (99.74%) obtained for this analysis shows that the model is excellently fits for the data.

2. The empirical correlation model for Ogelman *et al.*, (1984) model in equation (10) was

$$\frac{H_{cal}}{H_o} = 0.2552 + 0.744\left(\frac{S}{S_o}\right) - 0.2769\left(\frac{S}{S_o}\right)^2 \tag{24}$$

The coefficient of determination, $R^2$ (99.89%) obtained for this analysis shows that the model is excellently fits for the data.

3. The empirical correlation model for Samuel (1991) model in equation (11) was

$$\frac{H_{cal}}{H_o} = 1.704 - 6.2824\left(\frac{S}{S_o}\right) + 11.024\left(\frac{S}{S_o}\right)^2 - 6.0292\left(\frac{S}{S_o}\right)^3 \tag{25}$$

The coefficient of determination, $R^2$ (99.93%) obtained for this analysis shows that the model is excellently fits for the data.

The obtained values of the regression constants of Eqs. (23)- (25), coefficients of determination ($R^2$) along with correlation of coefficient (r) for six locations are summarized in Table 14. From the results of Table 14, it is obvious that for all these locations, the values of both coefficient of determination ($R^2$) and correlation of coefficient (r) are higher than 0.95 which indicate excellent fitting between the clearness index $\frac{H_{cal}}{H_o}$ and the relative sunshine duration $\frac{S}{S_o}$ .

The values of annual average daily global solar radiation estimated by equation (1) for the six locations are shown in Table 16. The values were plotted and then compared with the three sunshine-based models in Figure 18. The development of the Angstrom-type correlation of the first, second and third order will enable the solar energy researchers to use the estimated data with confidence, because of its good agreement. These correlations will also be useful for the places with similar climatic conditions and having no facilities of recording the global solar radiation data.

The validating of the calculated data was tested by calculating the MBE, MPE and t-test along with their coefficient of determination ($R^2$) and correlation of coefficient (r) for each of the locations with standard techniques. The values of these statistical indicators are shown in Table 17. Statistical results from this table show that both values of ($R^2$) and (r) are higher than 0.90 and 0.95 respectively for the six locations. This is an indication that the models are both significant.

The values of RMSE are found to be in the range 2.67 – 6.27(MJ/$m^2$/day) i.e. $2.67 \leq \text{RMSE} \leq 6.27$ indicating fairly good agreement between the measured and the calculated values of global solar radiation. The negative and positive values of MBE and MPE show overestimation and underestimation respectively of global solar radiation.

The six locations are statistically tested at the (1- α) confidence levels of significance of 95% and 99%. For the critical t-value, i.e., at α level of significance and (n-1) degree of freedom, the calculated t-value must be less than the critical value ($t_{critical}$= 2.20, df=11, $P < 0.05$). It is shown that at $t_{cal} < t_{critical}$ value, the model of calculated t-values for Dutse, Ibitaraba, Nguru and Maiduguri was significant at the degree of freedom to the t-$_{critical}$ value and insignificant at the model of calculated t-values for both Bauchi and Yola. Furthermore, the critical value, i.e., at α level of significance and (n-1) degree of freedom, the calculated t-value must be less than the critical value ($t_{critical}$ = 3.12, df=11, $P < 0.01$). It is observed that at tcal < $t_{critical}$ value for both Bauchi and Yola, t-values are insignificant at the degree of freedom to the t-$_{critical}$ value while for Dutse, Ibitaraba, Nguru and Maiduguri was significant at the degree of freedom to the t-$_{critical}$ value.

## IV. CONCLUSION

The need for radiation data covering entire areas led to the development of radiation models that allow the calculation of radiation parameters within certain margins of error. These models grew particularly important in connection with the use of solar energy. The study resulted in the development of respective Angstrom linear regression models for each of the six meteorological locations, which culminated in the development of the Angstrom model for North-Eastern, Nigeria given by equation (22).

North-Eastern, Nigeria is endowed appreciable with solar radiation and large rural dwellers lived in villages without proper infrastructure to develop an electricity grid, the use of photo voltaic (PV) is seen as attractive alternative because of its modular features, namely, its ability to generate electricity at the point of use, its low maintenance requirements and its non-polluting characteristics. In general, the three sunshine-based models performed well in terms of their coefficient of determination with $R^2$ = 99.74% given by the linear Angstrom-Prescott (1940) model, for Ogelman *et al.*, (1984) model, $R^2$ =99.89% while the Samuel (1991) model proved to be the best estimator with $R^2$ = 99.93% in North-Eastern, Nigeria.

Based on the result shown in Table 15, we can conclude that more than one sunshine-based model can be used to predict the global solar radiation across the North-Eastern, Nigeria.

Looking at statistical analysis of the models, we also observed that despite overestimation and underestimation of the models, there are fairly good level of significance at both confidence level of 95% and 99%.

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