

## Metric Projections in State Estimation in Electric Power Systems

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### Abstract

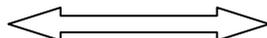
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The identification of weak nodes and participation factors in branches have been analyzed with different technical of analysis as: sensitivities, modal and of the singular minimum value, leaving of the Jacobian matrix of load flows. In this work shows up the application of metric projections for the identification of weak nodes and of the branches with more participation.

**Keywords** - Euclidean distance, Jacobian matrix, metric projections, metric spaces, state estimation.

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### 1. INTRODUCTION

Identifying weak nodes in electric power systems is a problem of great interest because the electrical system can reach up voltage instability and voltage collapse [1]-[3], if no relevant action is taken. Hence the importance of identifying the nodes of the system before contingencies or demand growth. Identifying stress peaks of the system to different scenarios of power system. Knowing weak nodes and branches with strong participation can take action to improve the reactive power support, the margin of stability and capacity of transmission lines. This work involves the application of metric projections [4] to the proximity of the minimum and maximum distances from a given scenario with cutoff values thus identifying weak nodes and branches with the strong participation. Traditionally the identification of weak nodes or branches are included in techniques such as sensitivities analysis, modal analysis and the minimum singular value, based on the analysis of the Jacobian matrix [5]-[7]. The proposed technique performs the Jacobian analysis but from metric distances, presenting a faster computer processing and identifying nodes weak and branches and participation. The analysis of the distances in matrix form has been used to calculate the distances between cities, locating the distances in matrix form and the comparison between arrays was performed. Where the measurement of distances is performed based on the Euclidean norm [8]. It has also been used to identify leverage points in the state estimation in electric power systems [4], [9], [10].

It is observed that there is a relationship between leverage points and sensitive nodes and branches as they are generated by electrical parameters and topology of the electrical system, [4], [11], [12]. And their structural characteristics are related to the parameters of electrical system (transmission lines, transformers). On one side identifying atypical natures of suspicious points and on the other side weak nodes and sensitive branches are sought. In both cases, the distance of each point with respect to the total points are calculated in an n-dimensional system.

### 2. METRIC SPACES

A metric space [13]-[16] is a pair  $(X, d)$  where  $X$  is a nonempty set and  $d$  is a nonreal function defined on  $X \times X$ , called distance or metric, and satisfies the following axioms:

i. Non-negative:

$$d(x, y) > 0 \text{ if } x \neq y \quad \forall x, y \in X$$

ii. Identity of indiscernibles:

$$d(x, y) = 0 \Leftrightarrow x = y \quad \forall x, y \in X$$

iii. Symmetry:

$$d(x, y) = d(y, x) \quad \forall x, y \in X$$

iv. Triangle inequality:

$$d(x, z) \leq d(x, y) + d(y, z) \quad \forall x, y \in X$$

For a given set  $X$  may define more than one metric. When the metric of the space is required, we simply speak about the metric space  $X$  although we know that it really is a pair  $(X, d)$ . The elements of the call point  $X$  metric space.

### 2.A DISCRETE METRIC SPACE

Given a nonempty set  $X$ , we define any discrete metric  $d$  on  $X$  by

$$d(x, y) = \begin{cases} 1 & x \neq y \\ 0 & x = y \end{cases}$$

It's easily verified that  $(X, d)$  is a metric space.

### 2.B THE REAL LINE $\mathfrak{R}$

Let  $X = \mathfrak{R}$ ,  $d(x, y) = |x - y|$  for every  $x, y \in \mathfrak{R}$ . The metric axioms are true. The set of complex numbers  $\mathbb{C}$  with the distance function  $d(z, w) = |z - w|$  is also a metric space.

### 2.C EUCLIDEAN DISTANCE

There are many different ways to define the distance between two points. The distance between two points is the length of the path connecting them. In the plane, the distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by the Pythagorean Theorem.

Let  $X = \mathfrak{R}^n$ , the set of all  $n$  of real numbers. If  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$  are elements of  $X$ , we define the distance:

$$d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2} \quad (1)$$

The above formula is known as the Euclidean Distance [25]-[28], it is the shortest distance between two points, and it's also known as the "standard" distance between two vectors. The first three metric axioms are check and it can be easily verified. The triangular inequality is described as

$$d(x, z) = \sqrt{\sum_{i=1}^n |x_i - z_i|^2} \leq \sqrt{\sum_{i=1}^n |x_i - y_i|^2} + \sqrt{\sum_{i=1}^n |y_i - z_i|^2} \quad (2)$$

If we replace the earlier inequality  $x_k - z_k = a_k$  and  $y_k - z_k = b_k$ , therefore  $x_k - y_k = a_k + b_k$ , and the inequality is described as

$$\sqrt{\sum_{k=1}^n (a_k + b_k)^2} \leq \sqrt{\sum_{k=1}^n a_k^2} + \sqrt{\sum_{k=1}^n b_k^2} \quad (3)$$

This last inequality is derived from the Cauchy-Schwarz-Buniakovsky inequality (CBS)

$$\left( \sum_{k=1}^n a_k b_k \right)^2 \leq \sum_{k=1}^n a_k^2 \cdot \sum_{k=1}^n b_k^2 \quad (4)$$

Indeed, using the inequality CBS we get

$$\begin{aligned} \sum_{k=1}^n (a_k + b_k)^2 &= \sum_{k=1}^n a_k^2 + 2 \sum_{k=1}^n a_k b_k + \sum_{k=1}^n b_k^2 \leq \sum_{k=1}^n a_k^2 + 2 \sqrt{\sum_{k=1}^n a_k^2 \sum_{k=1}^n b_k^2} + \sum_{k=1}^n b_k^2 \\ &= \left( \sqrt{\sum_{k=1}^n a_k^2} + \sqrt{\sum_{k=1}^n b_k^2} \right)^2 \quad (5) \end{aligned}$$

## 2.D THE SPACE $(\mathfrak{R}^n, d_p)$

Let  $X = \mathfrak{R}^n$ , the set of all the n-pairs of real numbers. If  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$  are elements of  $X$ , we define the distance  $d_p$  between  $x$  and  $y$  by

$$d_p(x, y) = \left( \sum_{k=1}^n |x_k - y_k|^p \right)^{\frac{1}{p}} \quad (6)$$

where  $p$  is a fixed number greater or equal to 1. The metric axioms are true. To verify the triangle inequality we make the same replacement, and then we show the Minkowski inequality.

[Minkowski]

$$\left( \sum_{k=1}^n |a_k + b_k|^p \right)^{\frac{1}{p}} \leq \left( \sum_{k=1}^n |a_k|^p \right)^{\frac{1}{p}} + \left( \sum_{k=1}^n |b_k|^p \right)^{\frac{1}{p}} \quad (7)$$

For  $p = 1$  the inequality is trivial, for  $p > 1$  the proof is based on Hölder inequality, which is a generalized version of CBS:

[Hölder]

$$\left( \sum_{k=1}^n |a_k b_k| \right) \leq \left( \sum_{k=1}^n |a_k|^p \right)^{\frac{1}{p}} \cdot \left( \sum_{k=1}^n |b_k|^q \right)^{\frac{1}{q}} \quad (8)$$

where the numbers  $p > 1$  and  $q > 1$  satisfy the condition

$$\frac{1}{p} + \frac{1}{q} = 1 \quad (9)$$

To prove (8), consider the function  $y(t) = t^\alpha$  with  $\alpha > 0$ . Since  $y'(t) = \alpha t^{\alpha-1} > 0$ ,  $y(t)$  is an increasing function for positive  $t$ . For those same  $t$  the inverse function  $t = y^{1/\alpha}$  is defined. If we'll chart the function  $y$ , choosing two positive real numbers  $a$  y  $b$ , and marking the corresponding points in  $t$  and  $y$  axes, respectively, and drawing straight parallel lines to the axes.

We'll obtain two "triangles", limited by the lines, the axes and the  $y$  curve, whose areas are

$$A_1 = \frac{a^{\alpha+1}}{\alpha + 1} \quad A_2 = \frac{b^{1/\alpha}}{\frac{1}{\alpha} + 1}$$

Furthermore, it is clear that meets  $A_1 A_2 \geq ab$ . We write  $\alpha + 1 = p$  and  $\frac{1}{\alpha} + 1 = q$ , then

$$\frac{1}{p} + \frac{1}{q} = 1$$

Therefore, for any positive real  $a$  and  $b$ , and conjugate pair  $p, q$  we have

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q} \quad (10)$$

Substituting in (10)

$$a = \frac{|a_k|}{(\sum_{i=1}^n |a_i|^p)^{1/p}} \quad b = \frac{|b_k|}{(\sum_{i=1}^n |b_i|^q)^{1/q}}$$

And summing over the index  $k$  have Hölder inequality(8).

Now we show the Minkowski inequality. Consider the identity

$$(|a| + |b|)^p = (|a| + |b|)^{p-1}|a| + (|a| + |b|)^{p-1}|b| \quad (11)$$

Replace  $a = a_k, b = b_k$  and add over the index  $k$

$$\sum_{k=1}^n (|a_k| + |b_k|)^p = \sum_{k=1}^n (|a_k| + |b_k|)^{p-1}|a_k| + \sum_{k=1}^n (|a_k| + |b_k|)^{p-1}|b_k| \quad (12)$$

Apply to each of the sums on the right of the Hölder inequality and we consider that  $(p - 1)q = p$ , we find

$$\sum_{k=1}^n (|a_k| + |b_k|)^p = \left( \sum_{k=1}^n (|a_k| + |b_k|)^p \right)^{1/q} = \left[ \left[ \sum_{k=1}^n (|a_k|)^p \right]^{1/p} + \left[ \sum_{k=1}^n (|b_k|)^p \right]^{1/p} \right] \quad (13)$$

Dividing both sides by

$$\left( \sum_{k=1}^n (|a_k| + |b_k|)^p \right)^{1/q}$$

We get

$$\left( \sum_{k=1}^n (|a_k| + |b_k|)^p \right)^{1/q} \leq \left[ \sum_{k=1}^n (|a_k|)^p \right]^{1/p} + \left[ \sum_{k=1}^n (|b_k|)^p \right]^{1/p} \quad (14)$$

and from this it follows immediately Minkowski inequality.

If in the equation (6)  $p = 2$  we obtain the Euclidean distance.

## 2.E MANHATTAN DISTANCE

The Manhattan distance [17]-[19] estimate the distance to be traveled to get from one point to another as if it were a grid map. The Manhattan distance between two points is the sum of the differences in these points. The formula for this distance between a point  $x = (x_1, x_2, \dots, x_n)$  and a point  $y = (y_1, y_2, \dots, y_n)$ , it's obtained from equation (6) if  $p = 1$ :

$$d(x, y) = \sum_{i=1}^n |x_i - y_i| \quad (15)$$

The Manhattan distance is measured in "the streets" rather than a straight line. Instead of walking directly from point A to point B, with the Manhattan distance you cannot walk through the buildings, but you walk the streets. The Manhattan distance is also known as the distance "city-blocks" or distance "taxi-cab". It is named

because it is the shortest distance that a car would travel in a city moving through the streets, as the Manhattan's streets (taking into account that in Manhattan there is only one-way streets and oblique streets and the real streets only exist in the corners of the blocks).

### 3. LEAST-SQUARES STATE ESTIMATION

The least-squares state estimator [20]-[23] for alternating current (AC) is based on a nonlinear model measurements

$$z = h(x) + e \quad (16)$$

where:

$z$ : measurement vector of dimension  $m$ ,

$x$ : state vector of dimension  $n$ , where  $n < m$ ,

$h(\cdot)$ : vector of the nonlinear function that relates the measurements with state vector,

$e$ : measurement error vector of dimension  $m$ ,

$m, n$ : number of measurements and state variables respectively.

The elements of  $e$  are assumed to have mean equal to zero and the corresponding variance matrix is given by  $R_z$ . The optimality conditions are applied to the performance of  $J(x)$ , which is expressed by

$$J(x) = \frac{1}{2} \sum_{j=1}^m \left( \frac{z_j - h_j(x)}{\sigma_j} \right)^2 \quad (17)$$

where:

$J(x)$ : Measurement residue.

From equation (17) we'll have to find the best estimate of the state vector  $\hat{x}$  of the system, which it consist to resolve the weighted least squares problem, that is, minimize the amount of residuals squared measures, whose objective function can be rewritten as:

$$J(x) = [z - h(x)]^T W [z - h(x)] = \sum_{j=1}^m \frac{[z_j - h_j(x)]^2}{\sigma_j^2} \quad (18)$$

where  $\sigma_j$  is the element  $(j, j)$  of the covariance matrix,  $R_z$ . The optimality condition of first order for this model can be written as:

$$\frac{\partial J(x)}{\partial x} = 0 \rightarrow H^T(x) W [z - h(x)] = 0 \quad (19)$$

where

$$H(x) = \frac{\partial h(x)}{\partial x}$$

It's the Jacobian matrix of vector  $h(x)$ , of dimension  $m \times n$ . It's about finding the value of  $\hat{x}$  that satisfies the linear equation (19). The most effective way to solve this equation is using the iterative method of Newton-Raphson. Neglecting terms where second derivatives appear from  $h(x)$ , the linear system of  $n$  equations to be solved at each iteration is the following:

$$(H^T(x^v) W_z H(x^v)) \Delta \hat{x}^v = H^T(x) W_z \Delta z(x^v) \quad (20)$$

$$\hat{x}^{v+1} = \hat{x}^v + \Delta \hat{x}^v \quad (21)$$

where:

$$R = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix} \quad (22)$$

$$W = R^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & & & \\ & \frac{1}{\sigma_2^2} & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_n^2} \end{bmatrix} \quad (23)$$

$$G = H^T(x^v)W_zH(x^v) \quad (24)$$

where:  
 $\sigma_i^2$  = the measurement error variance.

The variance  $\sigma_i^2$  provides the accuracy of a particular measurement. A larger variance indicates that the corresponding measurement is not very accurate, so it is desirable to have small variance in measurements.

#### 4.RESULTS

Consolidating the results from the implementation of metrics in the Jacobian matrix of the state estimator for a test system of 5 nodes [24], with the increase of reactive power. Voltage results. We present the results of the behavior of the voltage of each of the nodes or the nodes with higher voltage abatement present for each of the cases, for the increased inductive reactive power, until the last convergence point for each of the cases. At the end the most sensitive nodes for each case are shown. Metrics projections results. We present metrics projections results using the Jacobian matrix of the state estimator derived from power flows measurements and power injections measurements. They show the results using the elements  $\frac{\partial P}{\partial \delta}$  and  $\frac{\partial Q}{\partial V}$  from the Jacobian matrix of the state. In the last part, we present the minimum metrics projections (MMP) for each case.

Fig. 1 shows the voltage behavior at each node as the increase of the inductive reactive power in node 3.

Fig. 2 and 3 show the minimum metrics projections by nodes of the elements  $\frac{\partial P}{\partial \delta}$  and  $\frac{\partial Q}{\partial V}$  of the Jacobian matrix state estimator considering the power flow measurements.

Fig. 4 and 5 show the minimum metrics projections by nodes of the elements  $\frac{\partial P}{\partial \delta}$  and  $\frac{\partial Q}{\partial V}$  of the Jacobian matrix state estimator considering power injections measurements.

The results that provide metrics projections should be noted that these values are normalized with the base case. For all cases voltage profile (VP) considered for cutoff values (CV) is 0.8. All metrics projections are obtained from the Jacobian matrix of the state estimator.

#### 4 .A VOLTAGE RESULTS

It shows the variation of the magnitude of the voltage in the system with the increase of the inductive reactive power at node 3.

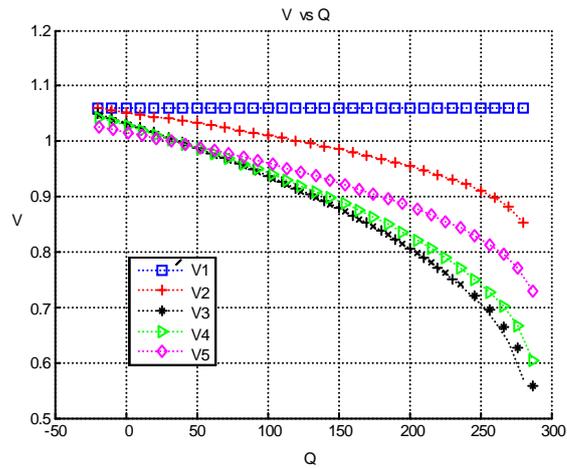


Figure 1 Voltage variation with the increase of the inductive reactive power inductive at node 3.

**4.B METRICS PROJECTIONS**

1. We present the results of the minimum metrics projections by nodes of the elements  $\frac{\partial P}{\partial \delta}$  of the Jacobian matrix state estimator considering the active power flow measurements.

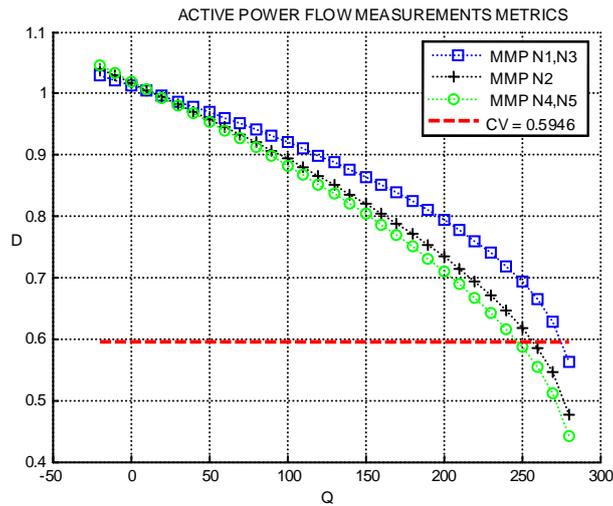


Figure 2 Metrics projections by nodes behavior with the increase of the reactive power considering active power flow measurements.

2. We present the results of the minimum metrics projections by nodes of the elements  $\frac{\partial Q}{\partial V}$  of the Jacobian matrix state estimator considering the reactive power flow measurements.

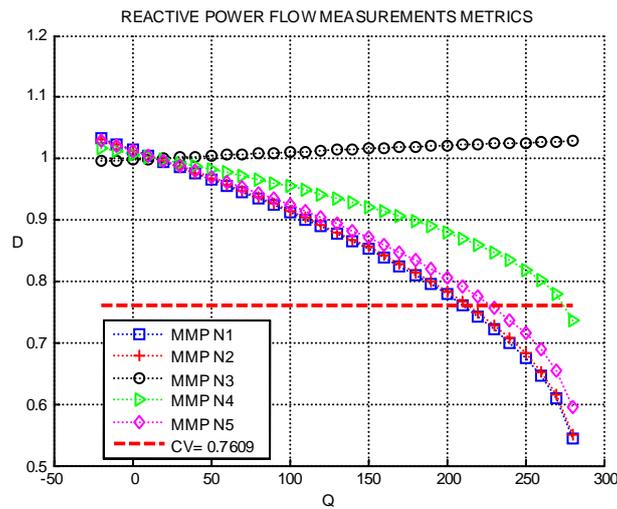


Figure 3 Metrics projections by nodes behavior with the increase of the reactive power considering reactive power flow measurements.

3. We present the results of the minimum metrics projections by nodes of the elements  $\frac{\partial P}{\partial \delta}$  of the Jacobian matrix state estimator considering the active power injections measurements.

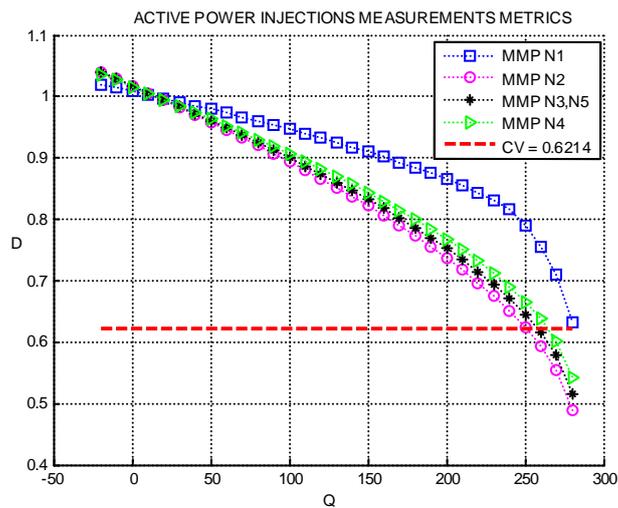


Figure 4 Metrics projections by nodes behavior with the increase of the reactive power considering active power injections measurements.

4. We present the results of the minimum metrics projections by nodes of the elements  $\frac{\partial Q}{\partial V}$  of the Jacobian matrix state estimator considering the reactive power injections measurements.

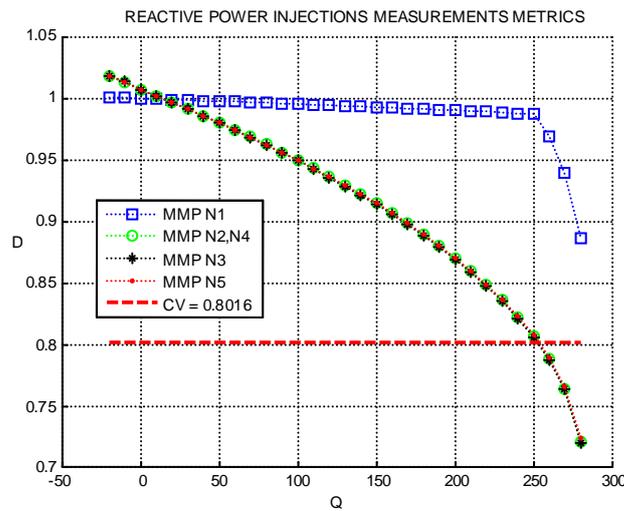


Figure 5 Metrics projections by nodes behavior with the increase of the reactive power considering reactive power injections measurements.

#### 4.C ANALYSIS

Fig. 1 shows the voltage behavior at the five nodes of the system with the increase of the inductive reactive power in the node 3, where the point of maximum power transfer is 280 MVAR, beyond this value the system does not converge and hence, the program gives incorrect estimates. In this figure we can identify nodes 3 and 4 as the nodes that have higher voltage depression, being near 0.6 PU at node 4, while node 3 is the most affected reaching a value of 0.5706 PU.

Fig. 2 shows the behavior of the minimum metrics projections by nodes using flow measurements considering the elements  $\frac{\partial P}{\partial \delta}$  of the Jacobian matrix. The CV is exceeded by all metrics projections, the minimum metrics projections are regarding nodes 4 and 5 with a value of 0.4410 for both cases, performing these projections in line 4-5 in both nodes.

Fig.3 shows the behavior of the minimum metrics projections by nodes using flow measurements considering the elements  $\frac{\partial Q}{\partial v}$  of the Jacobian matrix. The VC is exceeded at 200 MVAR, the minimum metrics projections are regarding nodes 1 and 2 with a value of 0.5443, performing these projections in line 1-3 in node 1 and a value of 0.5510 in line 2-3.

Fig.4 shows the behavior of the minimum metrics projections by nodes using power injections measurements considering the elements  $\frac{\partial P}{\partial \delta}$  of the Jacobian matrix. The CV is 0.6214 with a VP of 0.8, the CV is exceeded at 250 MVAR, and the minimum metrics projections are regarding nodes 2, 3 and 5 with a value of 0.4886 in line 5-4 at node 2 and with a value of 0.5151 in line 5-4 at nodes 3 and 5.

Fig.5 shows the behavior of the minimum metrics projections by nodes using power injections measurements considering the elements  $\frac{\partial Q}{\partial v}$  of the Jacobian matrix. The VC is exceeded at 250 MVAR, the minimum metric projection is regarding node 3 with a value of 0.7203, performing these projection in line 5-4.

### 1.CONCLUSIONS

Metric projections have a similar behavior to voltage with the increasing of reactive power in one or more nodes; it allows us to identify weak nodes of the system in a fast and reliable way, including the branches involved. Because of the metric projections are obtained from the Jacobian matrix of the state estimator, this allows us to take into account all the parameters of the system when metrics are calculated. The metric projections, as the state estimator, can be calculated in real time as the computational requirements by the metrics are minimal and therefore their calculation is fast.

As metric projections are calculated between the rows of the Jacobian matrix of the state estimator including compensator node, it let us analyse all the nodes in the system and they may alarm us in case of a disturbance.

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## Biographies

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