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Double-Diffusive Convection-Radiation Interaction On Unsteady Mhd Flow Of A Micropolar Fluid Over A Vertical Moving Porous Plate Embedded In A Porous Medium With Heat Generation And Soret Effects

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Abstract
This paper investigation is concerned with the first-order homogeneous chemical reaction and thermal radiation on hydromagnetic free convection heat and mass transfer for a micropolar fluid past a semi-infinite vertical moving porous plate in the presence of thermal diffusion and heat generation. The fluid is considered to be a gray, absorbing-emitting but non-scattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The plate moves with constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. A uniform magnetic field acts perpendicular to the porous surface, which absorbs the fluid with a suction velocity varying with time. Numerical results of velocity profiles of micropolar fluids are compared with the corresponding flow problems for a Newtonian fluid. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. The effects of various parameters on the velocity, microrotation, temperature and concentration fields as well as the skin-friction coefficient, Nusselt number and the Sherwood number are presented graphically and in tabulated forms.

Keywords: Thermal radiation; MHD; Micropolar; Heat generation; Chemical reaction.

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I Introduction

Modeling and analysis of the dynamics of micropolar fluids has been the subject of many research papers in recent years. This stems from the fact that these types of fluids may have many engineering and industrial applications. Micropolar fluids are defined as fluids consisting of randomly oriented molecules whose fluid elements undergo translational as well as rotational motions. Analysis of physical problems using these types of fluids has revealed several interesting phenomena and microscopic effects arising from local structure and micro-rotation of fluid elements not found in Newtonian fluids. The theory of micropolar fluids and thermo-micropolar fluids was developed by Eringen [1, 2] in an attempt to explain the behavior of certain fluids containing polymeric additives and naturally occurring fluids such as the phenomenon of the flow of colloidal fluids, real fluid with suspensions, exotic lubricants, liquid crystals, human and animal blood. Ahmadi [3] presented solutions for the flow of a micropolar fluid past a semi-infinite plate taking into account microinertia effects. Soundalgekar and Takhar [4] studied the flow and heat transfer past a continuously moving plate in a micropolar fluid.

The effect of radiation on MHD flow and heat transfer problems has become industrially more important. At high operating temperatures, radiation effect can be quite significant. The effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation has been studied by Seddeek [5]. The same author investigated [6] thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature-dependent viscosity. Ghaly and Elbarbary [7] have investigated the radiation effect on MHD free convection flow of a gas at a stretching surface with a uniform free stream. Pal and Chatterjee [8] performed analysis for heat and mass in MHD non-Darcian flow of a micropolar fluid over a stretching sheet embedded in a porous media with non-uniform heat source and thermal radiation. In all the above investigations only steady state flows over a semi-infinite vertical plate have been studied. The unsteady free convection flows over vertical plate were studied by

Raptis [9], Kim and Fedorov [10], Raptis and Perdikis [11], etc. The radiation effects on MHD free-convection flow of a gas past a semi-infinite vertical plate is studied by Takhar et al. [12]. Ramachandra Prasad and Bhaskar Reddy [13] investigated radiation and mass transfer effects on unsteady MHD free convection flow past a heated vertical plate in a porous medium with viscous dissipation. Sankar Reddy et al. [14] presented unsteady MHD convective heat and mass transfer flow of a micropolar fluid past a semi-infinite vertical moving porous plate in the presence radiation. The study of the MHD Oscillatory flow of a micropolar fluid over a semi-infinite vertical moving porous plate through a porous medium with thermal radiation is considered by Sankar Reddy et al. [15].

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Vajravelu and Hadjinicolaou [16] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Chamkha [17] investigated unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption. Alam et al. [18] studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of magnetic field and heat generation. Hady et al. [19] investigated the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect. Rahman and Sattar [20] presented magnetohydrodynamic convective flow of a micropolar fluid past a vertical porous plate in the presence of heat generation/absorption. Sharma et al. [21] investigated combined effect of magnetic field and heat absorption on unsteady free convection and heat transfer flow in a micropolar fluid past a semi-infinite moving plate with viscous dissipation. Sankar Reddy et al. [22] investigated radiation effects on MHD mixed convection flow of a micropolar fluid past a semi infinite plate in a porous medium with heat absorption.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and the mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries, For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions. A homogeneous reaction is one that occurs uniformly throughout a given phase. The species generation in a homogeneous reaction is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Deka et al. [23] studied the effect of the first-order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Muthucumaraswamy and Ganesan [24, 25] studied the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate and the effects of suction on heat and mass transfer along a moving vertical surface in the presence of a chemical reaction. Chamkha [26] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation / absorption and a chemical reaction. Seddeek et al. [27] analyzed the effects of chemical reaction, radiation and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media. Ibrahim et al. [28] analyzed the effects of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Demesh et al. [29] investigated combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flow over a uniformly stretched permeable surface.

However, the problem of unsteady MHD double-diffusive free convection for a heat generating micropolar fluid with thermal radiation and chemical reaction has received little attention. Hence, the object of the present chapter is to study the effect of a first-order homogeneous chemical reaction, thermal radiation, heat source and thermal diffusion on an unsteady MHD double-diffusive free convection flow of a micropolar fluid past a vertical porous plate in the presence of mass blowing or suction. It is assumed that the plate moves with a constant velocity in the flow direction in the presence of a transverse applied magnetic field. It is also assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time. The equations of continuity, linear momentum, angular momentum, energy and diffusion, which govern the flow field, are solved by using a regular perturbation method. The behavior of the velocity, microrotation, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the physical parameters.

II Mathematical Analysis

We consider a two dimensional unsteady flow of a laminar, incompressible, electrically conducting, radiating and micropolar fluid past a semi-infinite vertical moving porous plate embedded in a uniform porous medium in the presence of a pressure gradient with double-diffusive free convection and chemical reaction. The x' - axis is taken along the porous plate in the upward direction and y' - axis normal to it. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. The radiative heat flux in the x'-direction is considered negligible in comparison with that in the y'-direction [30]. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small and hence the induced magnetic field is negligible [31]. It is assumed that there is no applied voltage of which implies the absence of an electric filed. A homogeneous first-order chemical reaction between the fluid and the species concentration is considered. The fluid properties are assumed to be constant except that the influence of density variation with temperature and concentration has been considered in the body-force term (Boussinesq's approximation). Since the plate is of infinite length, all the flow variables are functions of y' and time t' only. Now, under the above assumptions, the governing boundary layer equations are

$$\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$\begin{aligned}\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=-\frac{1}{\rho}\frac{dp'}{dx'}+\left(\nu+\nu_r\right)\frac{\partial^2 u'}{\partial y'^2}+g\beta_f\left(T^*-T_\infty\right)+g\beta_c\left(C^*-C_\infty\right)\\ -\left(\frac{\sigma B_0^2}{\rho}+\frac{\nu}{K'}\right)u'+2\nu_r\frac{\partial\omega'}{\partial y'}\end{aligned} \tag{2}$$

$$\rho j'\left(\frac{\partial\omega'}{\partial t'}+v'\frac{\partial\omega'}{\partial y'}\right)=\gamma\frac{\partial^2\omega'}{\partial y'^2} \tag{3}$$

$$\frac{\partial T}{\partial t'}+v'\frac{\partial T}{\partial y'}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y'^2}-\frac{1}{\rho c_p}\frac{\partial q_r'}{\partial y'}+\frac{Q_0}{\rho c_p}\left(T-T_\infty\right) \tag{4}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D_M'\frac{\partial^2 C'}{\partial y'^2}+D_T'\frac{\partial^2 T'}{\partial y'^2}-K_r'\left(C'-C_\infty\right) \tag{5}$$

where x', y' and t' are the dimensional distances along and perpendicular to the plate and dimensional time, respectively. u' and v' are the components of dimensional velocities along x' and y' directions, ρ is the fluid density, μ is the viscosity, c_p is the specific heat at constant pressure, σ is the fluid electrical conductivity, B_0 is the magnetic induction, K' - the permeability of the porous medium, j' is the micro inertia density, ω' is the component of the angular velocity vector normal to the $x'y'$-plane, γ is the spin gradient viscosity, T' is the dimensional temperature, D_M' is the coefficient of chemical molecular diffusivity, D_T' is the coefficient of thermal diffusivity, C' is the dimensional concentration, *k* is the thermal conductivity of the fluid, *g* is the acceleration due to gravity, and q_r' , K_r' are the local radiative heat flux, the reaction rate constant respectively. The term $Q_0(T'-T_\infty')$ represents the amount of heat generated or absorbed per unit volume, Q_0 being a constant, which may take on either positive or negative values. When the wall temperature T' exceeds the free stream temperature T_∞', the heat source term Q_0 >0 and heat sink when Q_0 <0. The second and third terms on the right hand side of the energy Eq. (4) represents thermal radiation and heat absorption effects, respectively. Also, the second and third terms on the right hand side of the concentration Eq. (5) represents Sorret and chemical reaction effects, respectively.

It is assumed that the porous plate moves with a constant velocity u'_p in the direction of fluid flow, and the free stream velocity U'_∞ follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time.

Under these assumptions, the appropriate boundary conditions for the velocity, microrotation, temperature, and concentration fields are

$$u' = u'_P,\ T = T_w + \varepsilon(T_w - T_\infty)e^{n't'},\ \omega' = -\frac{\partial u'}{\partial y'},\ C' = C'_w + \varepsilon(C'_w - C'_\infty)e^{n't'}\ \ at\ y' = 0$$
$$u' \to U'_\infty = U_0\left(1 + \varepsilon e^{n't'}\right),\ \omega' \to 0,\ T \to T_\infty,\ C \to C_\infty\ \ as\ \ y' \to \infty \tag{6}$$

where, u'_P, C'_w and T_w are the wall dimensional velocity, temperature and concentration, respectively. C'_∞ and T_∞ are the free stream dimensional concentration and temperature, respectively, U_0 and n' are constants.

From the continuity Eq. (1), it is clear that the suction velocity normal to the plate is either a constant or a function of time. Hence it is assumed in the form:

$$v' = -V_0(1 + \varepsilon A e^{n't'}) \tag{7}$$

where, A is a real positive constant, ε and εA are very small (less than unity), and V_0 is a scale of suction velocity which has non-zero positive constant. The negative sign indicates that the suction is towards the plate.

Outside the boundary layer, Eq. (2) gives

$$-\frac{1}{\rho}\frac{dp'}{dx'} = \frac{dU'_\infty}{dt'} + \frac{\sigma}{\rho}B_0^2 U'_\infty. \tag{8}$$

By using the Rosseland approximation (Brewster [32]), the radiative heat flux q_r is given by

$$q'_r = -\frac{4\sigma_s \partial T^4}{3k_e \partial y'} \tag{9}$$

where, σ_s and k_e are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty{}^3 T - 3T_\infty{}^4 \tag{10}$$

By substituting Eqs. (6) and (7) in Eq. (4), we get

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q_0}{\rho c_p}\left(T' - T_\infty\right) + \frac{16\sigma_s T'^3_\infty}{3\rho c_p k'_s}\frac{\partial q'_r}{\partial y'} \tag{11}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$u = \frac{u'}{U_0},\quad v = \frac{v'}{V_0},\quad y = \frac{V_0 y'}{\upsilon},\ U_p = \frac{u'_p}{U_0},\ n = \frac{n'\nu}{V_0^2},\ \omega = \frac{\nu}{U_0 V_0}\omega',$$
$$t = \frac{t' V_0^2}{\nu},\ \theta = \frac{T^* - T_\infty}{T_w - T_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ j = \frac{V_0^2}{\nu^2}j',\ M = \frac{\sigma B_0^2 \upsilon}{\rho V_0^2},$$
$$K = \frac{K' V_0^2}{\nu^2},\ \mathrm{Pr} = \frac{\nu\rho C_p}{k} = \frac{\mu C_p}{k},\ R = \frac{k k_e}{4\sigma_s T_\infty^3},\ \phi = \frac{\nu Q_0}{\rho c_p V_0^2},\ Sc = \frac{\nu}{D'}, \tag{12}$$
$$Gr = \frac{\nu\beta_f g(T_w - T_\infty)}{U_0 V_0^2},\ Gc = \frac{\nu\beta_c g(C'_w - C_\infty)}{U_0 V_0^2},\ K_r = \frac{K'_r \nu}{V_0^2},\ S_0 = \frac{D_T^*(T_w - T_\infty)}{\left(C_w - C_\infty\right)}$$

Furthermore, the spin-gradient viscosity γ which gives some relationship between the coefficients of viscosity and micro-inertia, is defined as

$$\gamma = (\mu + \frac{\Lambda}{2})\, j' = \mu\, j'\left(1 + \frac{1}{2}\beta\right), \beta = \frac{\Lambda}{\mu} \tag{13}$$

where β denotes the dimensionless viscosity ratio, in which Λ is the coefficient of gyro-viscosity (or vertex viscosity).

In view of Eqs. (7), (8), (12) and (13), the governing Eqs. (2), (3), (5) and (11) reduce to the following dimensionless form:

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \left(1 + \beta\right)\frac{\partial^2 u}{\partial y^2} + G_r\theta + G_c C + N(U_\infty - u) + 2\beta\frac{\partial \omega}{\partial y} \tag{14}$$

$$\frac{\partial \omega}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \omega}{\partial y} = \frac{1}{\eta}\frac{\partial^2 \omega}{\partial y^2} \tag{15}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \theta}{\partial y} = \frac{1}{\Gamma}\frac{\partial^2 \theta}{\partial y^2} + \phi\theta \tag{16}$$

$$\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + S_0\frac{\partial^2 T}{\partial y^2} - K_r C \tag{17}$$

where $N = M + \frac{1}{K}$, $\eta = \frac{\mu j}{\gamma} = \frac{2}{2+\beta}$, $\Gamma = \left(1 - \frac{4}{3R+4}\right)\Pr$

and Gr, Gc M, K $\Pr$, R, ϕ, S_0, K_r and Sc denote thermal Grashof number, the modified Grashof number, magnetic field parameter, permeability parameter, Prandtl number, radiation parameter, heat generation/absorption parameter, Soret number, chemical reaction parameter and the Schmidt number, respectively.

The boundary conditions (6) are then given by the following dimensionless equations:

$$\begin{aligned} &u = U_p,\ \omega = -\frac{\partial u}{\partial y}, \theta = 1 + \varepsilon e^{nt}, C = 1 + \varepsilon e^{nt}\ \ at\ \ y = 0 \\ &u = U_\infty \to \left(1 + \varepsilon e^{nt}\right), \omega \to 0, \theta \to 0, C \to 0\ \ as\ \ y \to \infty \end{aligned} \tag{18}$$

III Solution Of The Problem

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we may represent the translational velocity, microrotation, temperature and concentration in the neighbourhood of the plate as

$$\begin{aligned} u &= u_0(y) + \varepsilon e^{nt}u_1(y) + O(\varepsilon^2) + ... \\ \omega &= \omega_0(y) + \varepsilon e^{nt}\omega_1(y) + O(\varepsilon^2) + ... \\ \theta &= \theta_0(y) + \varepsilon e^{nt}\theta_1(y) + O(\varepsilon^2) + \ldots \\ C &= C_0(y) + \varepsilon e^{nt}C_1(y) + O(\varepsilon^2) + \ldots \end{aligned} \tag{19}$$

Substituting equation (19) into equations (14)-(17), and equating the harmonic and non-harmonic terms, and neglecting the higher-order terms of O(ε^2), we obtain the following pairs of equations for $(u_0, \omega_0, \theta_0, C_0)$ and $(u_1, \omega_1, \theta_1, C_1)$.

$$(1+\beta)u_0'' + u_0' - NU_0 = -N - G_r\theta_0 - G_c C_0 - 2\beta\omega_0' \tag{20}$$

$$(1+\beta)u_1'' + u_1' - (N+n)u_1 = -(N+n) - Au_0' - G_r\theta_1 - G_c C_1 - 2\beta\omega_1' \tag{21}$$

$$\omega_0'' + \eta\omega_0' = 0 \tag{22}$$

$$\omega_1'' + \eta\omega_1' - n\eta\omega_1 = -A\eta\omega_1' \tag{23}$$

$$\theta_0'' + \Gamma\theta_0' - \Gamma\phi\theta_0 = 0 \tag{25}$$

$$\theta_1'' + \Gamma\theta_1' - \Gamma(n+\phi)\theta_1 = -A\Gamma\theta_0' \tag{26}$$

$$C_0'' + ScC_0' - k_r ScC_0 = -S_0 Sc\theta_0'' \tag{27}$$

$$C_1'' + ScC_1' - (n+k_r)ScC_1 = -AScC_0' - S_0 Sc\theta_1'' \tag{28}$$

The corresponding boundary conditions can be written as

$$\begin{aligned} &u_0 = U_p, u_1 = 0, \omega_0 = -u_0', \omega_1 = -u_1', \ \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \ \ at\ y = 0 \\ &u_0 = 1, u_1 = 1, \omega_0 \to 0, \omega_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0 \ \ as\ \ y \to \infty \end{aligned} \tag{29}$$

Solving equations (19) - (26) under the boundary conditions (27), we get

$$u_0(y) = 1 + a_1 e^{-m_1 y} + a_3 e^{-m_3 y} + a_5 e^{-\eta y} + a_6 e^{-m_6 y}$$

$$u_1(y) = 1 + b_1 e^{-m_1 y} + b_2 e^{-m_2 y} + b_3 e^{-m_3 y} + b_4 e^{-m_4 y} + b_5 e^{-m_5 y} + b_6 e^{-m_6 y} + b_7 e^{-Scy} + b_8 e^{-\eta y}$$

$$\omega_0(y) = k_1 e^{-\eta y}$$

$$\omega_1(y) = k_2 e^{-m_5 y} - \frac{A\eta}{n} k_1 e^{-\eta y}$$

$$\theta_0(y) = e^{-m_4 y}$$

$$\theta_1(y) = e^{-m_4 y} + \frac{Am_3}{n}\left(e^{-m_4 y} - e^{-m_3 y}\right)$$

$$C_0(y) = (1+c_1)e^{-m_6 y} - c_1 e^{-m_3 y}$$

$$C_1(y) = c_3 e^{-m_3 y} + c_4 e^{-m_4 y} + c_6 e^{-m_6 y} + c_7 e^{-m_7 y}$$

In view of the above solutions, the velocity, microrotation, temperature and concentration distributions in the boundary layer become

$$\begin{aligned} u(y,t) = {} & 1 + a_1 e^{-m_1 y} + a_3 e^{-m_3 y} + a_5 e^{-\eta y} + a_6 e^{-m_6 y} \\ & + \varepsilon e^{nt}\left\{1 + b_1 e^{-m_1 y} + b_2 e^{-m_2 y} + b_3 e^{-m_3 y} + b_4 e^{-m_4 y} + b_5 e^{-m_5 y} + b_6 e^{-m_6 y} + b_7 e^{-Scy} + b_8 e^{-\eta y}\right\} \end{aligned}$$

$$\omega(y,t) = k_1 e^{-\eta y} + \varepsilon e^{nt}\left\{k_2 e^{-m_5 y} - \frac{A\eta}{n} k_1 e^{-\eta y}\right\}$$

$$\theta(y,t) = e^{-m_4 y} + \varepsilon e^{nt}\left\{e^{-m_4 y} + \frac{Am_3}{n}\left(e^{-m_4 y} - e^{-m_3 y}\right)\right\}$$

$$C(y,t) = (1+c_1)e^{-m_6 y} - c_1 e^{-m_3 y} + \varepsilon e^{nt}\left\{c_3 e^{-m_3 y} + c_4 e^{-m_4 y} + c_6 e^{-m_6 y} + c_7 e^{-m_7 y}\right\}$$

where

$$m_1 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4N(1+\beta)}\right], \ m_2 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4(N+n)(1+\beta)}\right]$$

$$m_3 = \frac{\Gamma}{2}\left[1 + \sqrt{1 + \frac{4\phi}{\Gamma}}\right], \ m_4 = \frac{\Gamma}{2}\left[1 + \sqrt{1 + \frac{4(n+\phi)}{\Gamma}}\right], \ m_5 = \frac{\eta}{2}\left[1 + \sqrt{1 + \frac{\delta}{\eta}}\right]$$

$$m_6 = \frac{Sc}{2}\left[1 + \sqrt{1 + \frac{Kr}{Sc}}\right], \ m_7 = \frac{Sc}{2}\left[1 + \sqrt{1 + \frac{4(n+K_r)}{Sc}}\right]$$

and the remaining constants are given in Appendix.

From the engineering point of view, the most important characteristics of the flow are the skin friction coefficient C_f, couple stress coefficient C_m, Nusselt number Nu and Sherwood number Sh, which are discussed below

Knowing the velocity field in the boundary layer, we can calculate the skin-friction coefficient C_f at the porous plate, which in the non dimensional form is given by

$$C_f = \frac{2\tau_w^*}{\rho U_0 V_0}, \text{ where } \tau_w^* = (\mu+\Lambda)\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} + \Lambda\omega^*\big|_{y^*=0}$$

$$= 2\left[1+(1-n)\beta\right] u'(0)$$
$$= -2\left[1+(1-n)\beta\right]\begin{Bmatrix}[a_1m_1+a_3m_3+a_5\eta+a_6m_6] \\ +\varepsilon e^{nt}[b_1m_1+b_2m_2+b_3m_3+b_4m_4+b_5m_6+b_6m_6+b_7m_7+b_8\eta]\end{Bmatrix}$$

Knowing the microrotation in the boundary layer, we can calculate the couple stress coefficient C_m at the porous plate, which in the non dimensional form is given by

$$C_m = \frac{M_w}{\mu j U_0}, \text{ where } M_w = \gamma\left.\frac{\partial\omega^*}{\partial y^*}\right|_{y^*=0}$$
$$= \left(1+\frac{1}{2}\beta\right)\omega'(0) = \left(1+\frac{1}{2}\beta\right)\left(\frac{\partial\omega_0}{\partial y}+\varepsilon e^{nt}\frac{\partial\omega_1}{\partial y}\right)_{y=0}$$
$$= -\left(1+\frac{1}{2}\beta\right)\left(k_1\eta+\varepsilon e^{nt}\left\{k_2m_5+\frac{4A\eta^2k_1}{\delta}\right\}\right)$$

Knowing the temperature field in the boundary layer, we can calculate the heat transfer coefficient at the porous plate, which in non-dimensional form in terms of the Nusselt number Nu is given by

$$Nu_x = x\frac{\left(\partial T/\partial y^*\right)\big|_{y^*=0}}{T_w - T_\infty}$$
$$Nu_x\,\mathrm{Re}_x^{-1} = -\theta'(0) = -\left(\frac{\partial\theta_0}{\partial y}+\varepsilon e^{nt}\frac{\partial\theta_1}{\partial y}\right)_{y=0}$$
$$= m_4+\varepsilon e^{nt}\left[m_5\left(1+\frac{Am_3}{n}\right)-\frac{Am_4^2}{n}\right]$$

Knowing the concentration field in the boundary layer, we can calculate the mass transfer coefficient at the porous plate, which in non-dimensional form in terms of Sherwood number Sh is given by

$$Sh_x = \frac{j_w x}{D^*(C_w^*-C_\infty^*)}., \text{ where } j_w = -D^*\left.\frac{\partial C^*}{\partial y^*}\right|_{y^*=0}$$
$$Sh_x\,\mathrm{Re}_x^{-1} = -C'(0) = -\left(\frac{\partial C_0}{\partial y}+\varepsilon e^{nt}\frac{\partial C_1}{\partial y}\right)_{y=0}$$
$$= (1+c_1)m_6 - c_1m_3 + \varepsilon e^{nt}\left\{c_3m_3+c_4m_4+c_6m_6+c_7m_7\right\}$$

where $\mathrm{Re}_x = \frac{V_0 x}{\nu}$ is the Reynolds number.

IV Results And Discussion

The formulation of the problem that accounts for the effects of chemical reaction, thermal diffusion, heat source and thermal radiation on MHD convective mass transfer flow of an incompressible, micropolar fluid along an infinite vertical porous moving plate embedded in a porous medium is performed in the preceding sections. This enables us to carry out the numerical calculations for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the thermophysical parameters. In the present study we have chosen A= 0.5, t =1.0, n = 0.1, and ε = 0.01, while *Up, M, K, Gc, Gr, Sc, Kr, Pr,* ϕ, *R* and *So* are varied over a range, which are listed in the figure legends.

The effect of viscosity ratio β on the velocity and microrotation is presented in Fig. 1. It is seen that as β increases, the velocity gradient near the porous plate decreases, and then approaches to the free stream velocity. Also, the velocity distribution across the boundary layer is lower for a Newtonian fluid (β =0) for the same flow conditions and fluid properties, as compared with that of a micropolar fluid. Further, the magnitude of microrotation decreases, as β increases.

Fig. 2 illustrates the variation of velocity and microrotation distribution across the boundary layer for various values of the plate velocity U_p. It is observed that as U_p increases, the translational velocity increases near the porous plate and it decreases far away from the porous plate. Also, it is observed that the microrotation increases, as the plate moving velocity increases.

For different values of the magnetic field parameter M, the translational velocity and microrotation profiles are plotted in Fig 3. It is observed that as M increases, the velocity distribution across the boundary layer decreases, whereas the microrotation increases.

For various values of the permeability parameter K, the profiles of the translational velocity and microrotation across the boundary layer are shown in Fig.4. It is observed that as K increases, the velocity across the boundary layer increases, whereas the microrotation decreases.

The translational velocity and the microrotation profiles against spanwise coordinate y for different values of Grashof number Gr and modified Grashof number Gc are described in Fig. 5. It is observed that an increase in Gr or Gc leads to a rise in the velocity and a fall in the microrotation. Here the positive values of Gr corresponds to a cooling of the surface by natural convection.

Fig. 6 shows the translational velocity and the microrotation profiles across the boundary layer for different values of Prandtl number $\Pr$ and radiation parameter *R*. It is observed that as $\Pr$ or *R* increases, the translational velocity decreases, whereas the magnitude of microrotation increases.

The translational velocity and microrotation profiles against y for different values of R are displayed in Fig. 7. It is observed that as R increases, the translational velocity decreases, whereas the microrotation increases.

Fig. 8 displays the effect of the heat generation coefficient ϕ on the velocity and microrotation across the boundary layer. It is observed that as ϕ increases, there is a rise in both the velocity and microrotation.

Fig. 9 displays results for the velocity and microrotation distributions across the boundary layer for different values of chemical reaction parameter *Kr*. It is noticed that as the chemical reaction parameter *Kr* increases, the velocity increases, whereas the microrotation decreases. The effect of Soret number So on the velocity and microrotation distribution is shown in Fig.10. It is observed that as So increases, the velocity increases, whereas the microrotation decreases.

Typical variation of the temperature along the span wise coordinate *y* is shown in Fig. 11 for different values of the Prandtl number Pr and radiation parameter R. It is observed that as Pr or R increases, the temperature distribution across the boundary layer decreases. Fig.12 shows the variation of the temperature for different values of ϕ. It is seen that as the heat generation parameter ϕ increases, the temperature decreases.

For different values of the chemical reaction parameter *Kr*, the concentration profiles are plotted in Fig.13. It is observed that as the *Kr* increases, the concentration distribution across the boundary layer decreases.Fig.14 represents the concentration profiles for different values of Soret number *So*. It is noticed that the concentration decreases, as *So* increases.

Tables 1-5 depict the effects of the magnetic parameter M, the radiation parameter R, the heat generation coefficient ϕ, the chemical reaction parameter Kr and Sorret number *So* on the skin-friction coefficient C_f, the couple stress coefficient C_m, Nusselt number Nu and Sherwood number Sh, respectively. From Table 1, it is observed that both the skin-friction and couple stress decrease, as M increases. From the analytical results, it can be seen that the rate of heat transfer depends on the radiation parameter and heat generation coefficient and the rate of mass transfer depends on the chemical reaction and Sorret number. From Tables 2 and 3, it is observed that as R or ϕ increases, there is decrease in the skin-friction, wall couple stress and Sherwood number, and there is a rise in the absolute values of the rate of heat transfer. Finally, form Tables 4 and 5, it is noticed that as Kr increases, there is an increase in the skin-friction, couple stress and Sherwood number. Also, as *So* increases, there is a decrease in the skin-friction, couple stress and Sherwood number. But the Nusselt number remains unchanged with the variation of *Kr* and *So*.

V Conclusions

The present paper deals with the analysis of the effects of chemical reaction, thermal diffusion, heat source and thermal radiation on MHD convective flow and mass transfer of an incompressible, micropolar fluid along a semi infinite vertical porous moving plate in a porous medium. The porous plate was assumed to move with a constant velocity in the direction of the fluid flow. The governing equations were developed and transformed into a system of nonlinear ordinary differential equations by a regular perturbation technique. From the present numerical study the following conclusions can be drawn.

- Velocity increases with increase in the viscosity ratio β, thermal Grashof number *Gr*, modified Grashof number *Gr*, heat generation coefficient ϕ, chemical reaction parameter *Kr* and Soret number *So*, but reverse trend is seen by increasing the magnetic field parameter *M*, Prandtl number *Pr*, radiation parameter *R* and Schmidt number *Sc*.
- Microrotation decreases with increase in the viscosity ratio β, Prandtl number *Pr*, radiation parameter *R*, heat generation coefficient ϕ, chemical reaction parameter *Kr* and Soret number *So*, but reverse trend is seen by increasing the magnetic field parameter *M* and Schmidt number *Sc*.
- Temperature decreases with increase in the value of the Prandtl number *Pr*, radiation parameter *R* and heat generation coefficient ϕ.
- Concentration decreases with increase in the value of chemical reaction parameter *Kr* and Soret number *so*.

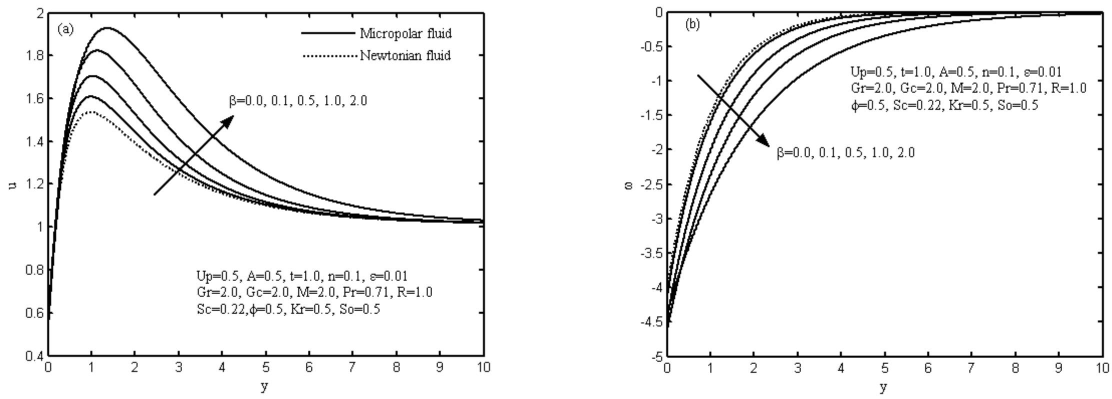

Fig.1 Velocity and microrotation profiles for different values of β

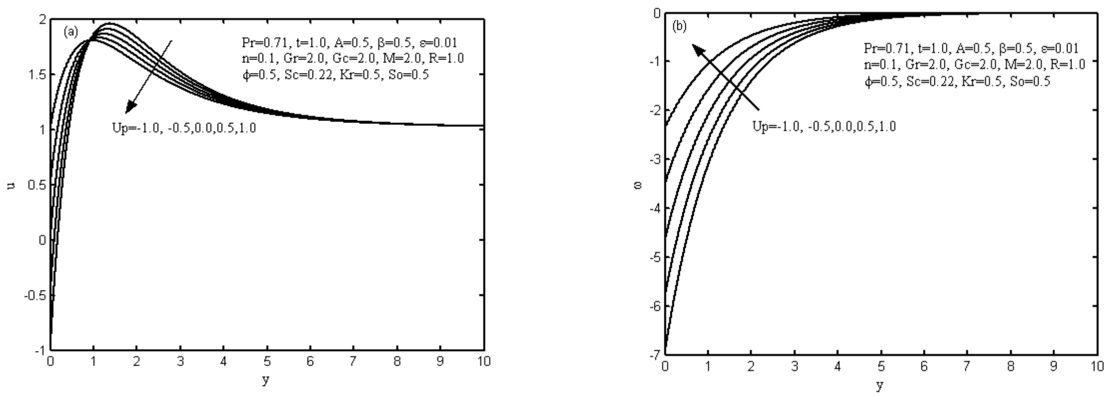

Fig.2Velocity and microrotation profiles for different values of Up

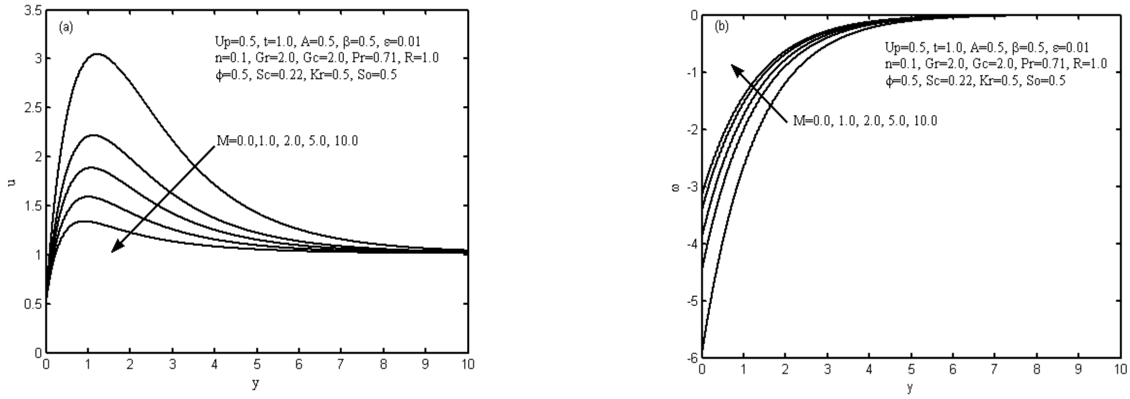

Fig.3 Velocity and microrotation profiles for different values of M

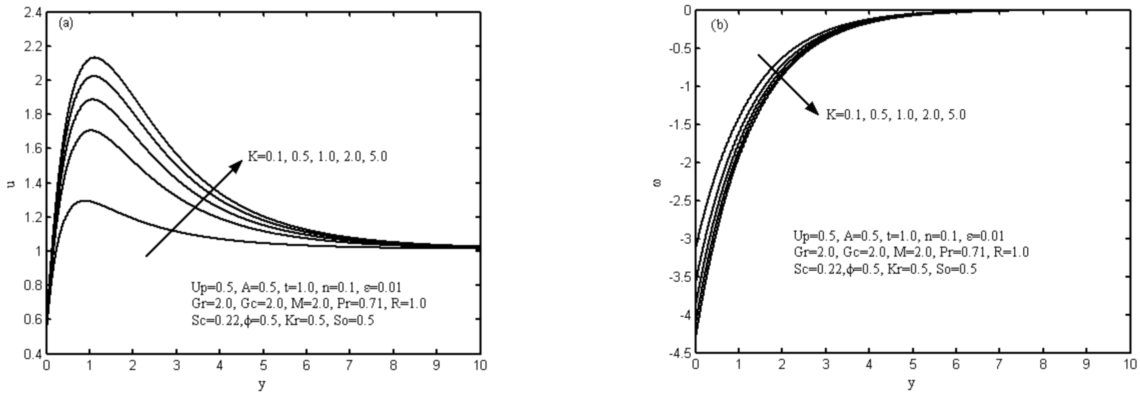

Fig.4 Velocity and microrotation profiles for different values of K

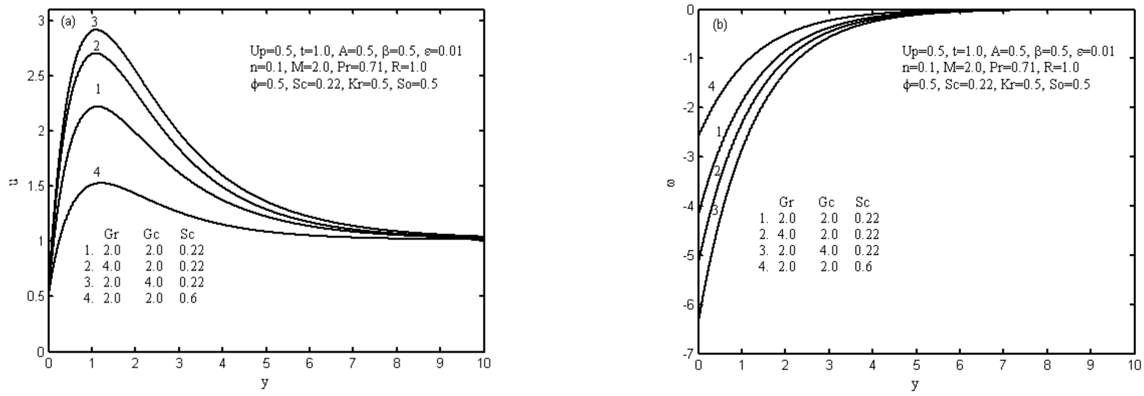

Fig.5 Velocity and microrotation profiles for different values of *Gr, Gc & Sc*

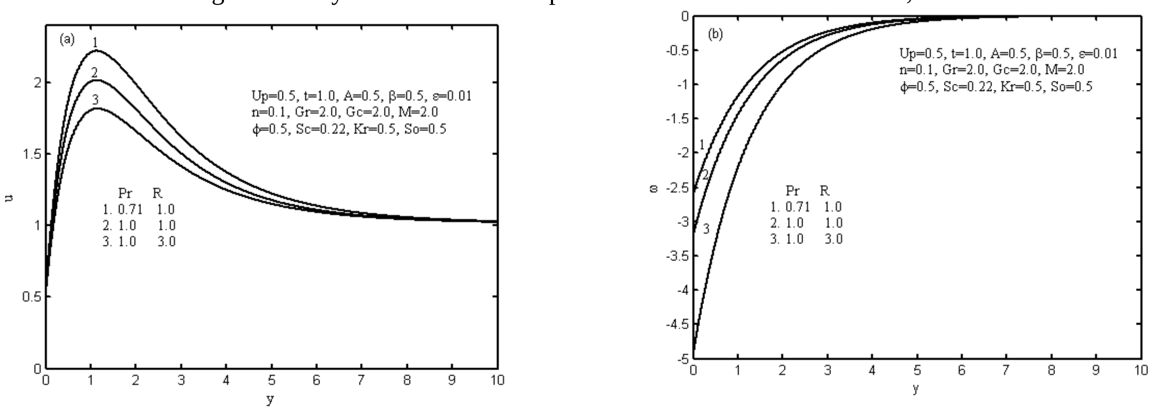

Fig.6 Velocity and microrotation profiles for different values of *Pr & R*

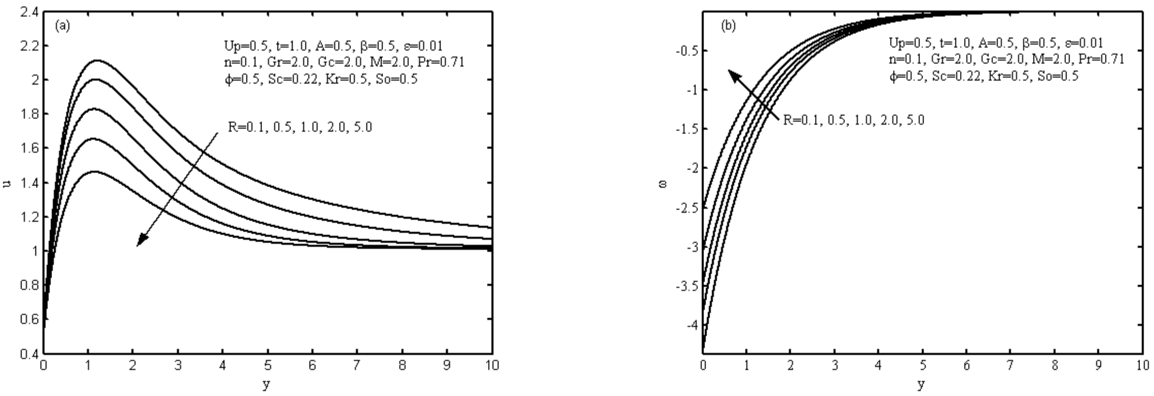

Fig.7 Velocity and microrotation profiles for different values of *R*

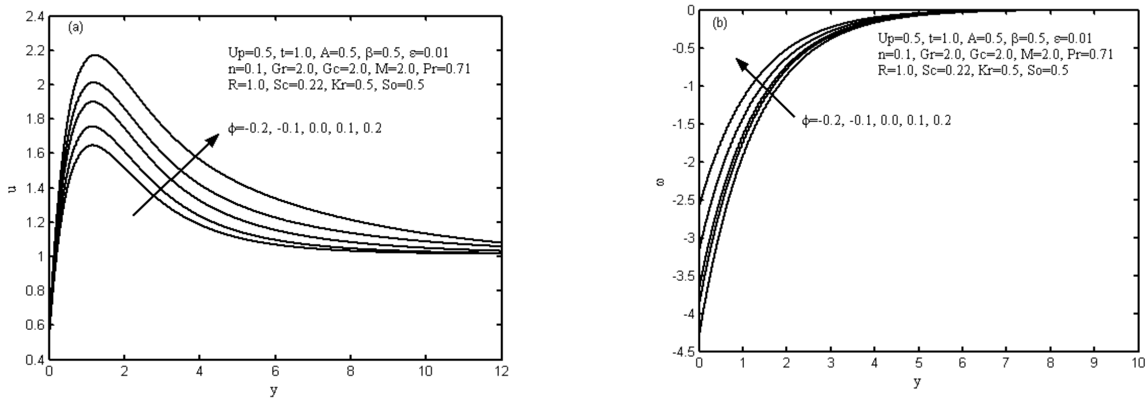

Fig. 8 Velocity and microrotation profiles for different values of ϕ

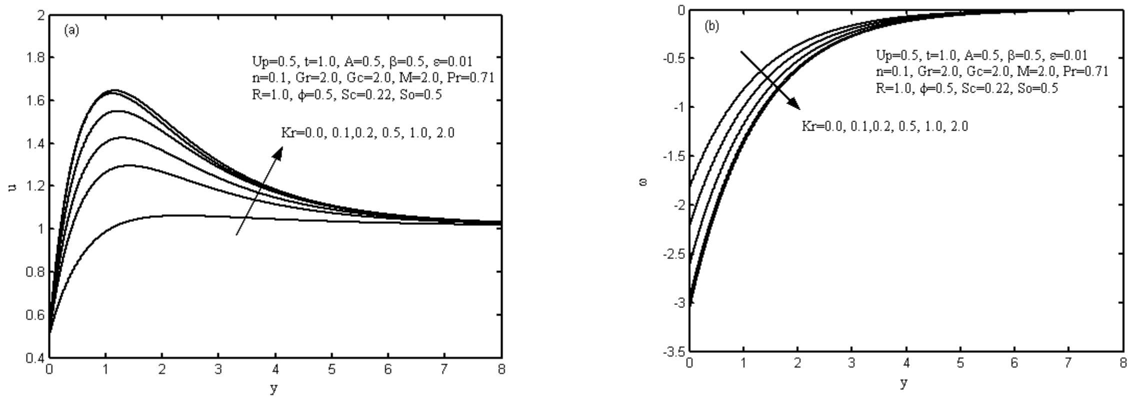

Fig. 9 Velocity and microrotation profiles for different values of ϕ

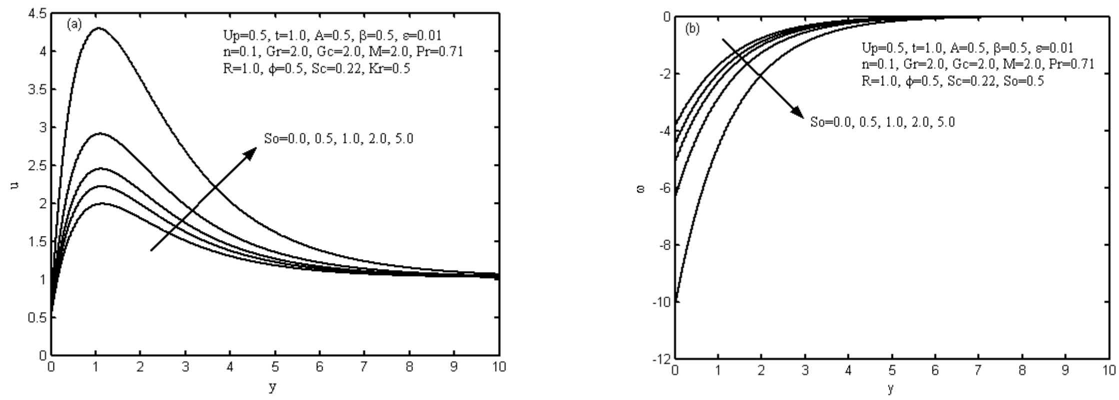

Fig. 10 Velocity and microrotation profiles for different values of *So*

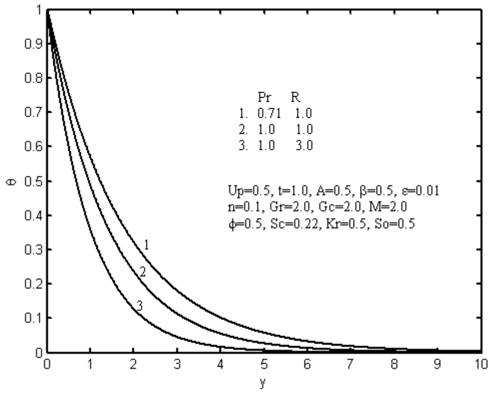

Fig. 11 Temperature profiles for different values of *Pr & R*

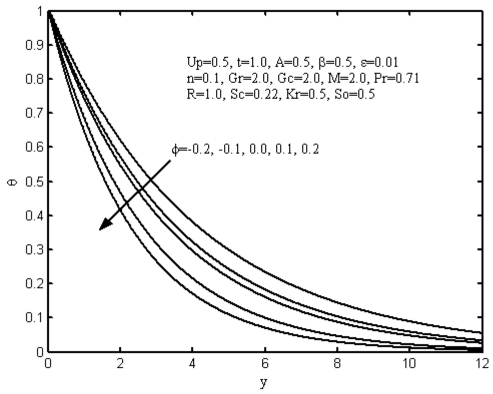

Fig. 12 Temperature profiles for different values of ϕ

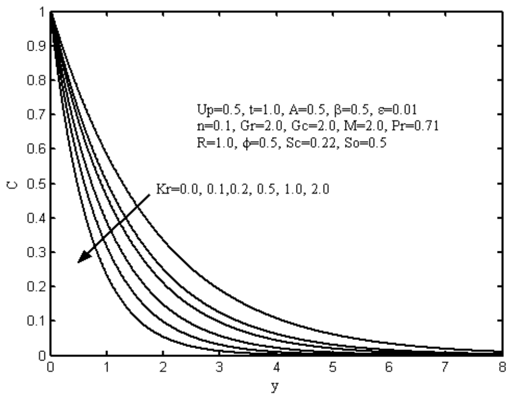

Fig . 13 Concentration profiles for different values of *Kr*

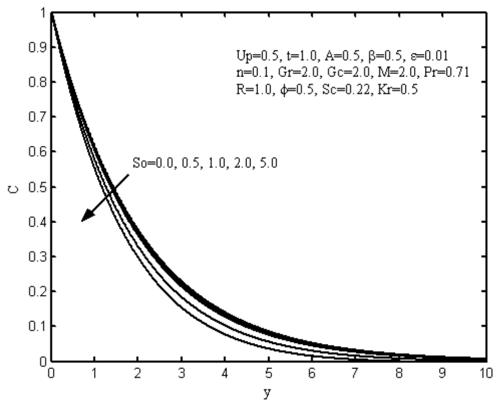

Fig . 14 Concentration profiles for different values of *So*.

Table 1: Effects of M on skin-friction, couple wall stress, Nusselt number and Sherwood number for reference values in figs. 3(a) and 3(b).

M	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
0.0	7.6182	7.7006	0.5786	0.9704
1.0	3.0872	3.1180	0.5786	0.9704
2.0	2.6121	2.6362	0.5786	0.9704
5.0	2.3688	2.3892	0.5786	0.9704
10.0	2.4618	2.4821	0.5786	0.9704

Table 2: Effects of R on skin-friction and couple wall stress, Nusselt number and Sherwood number for reference values in figs. 10(a) and 10(b).

R	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
0.1	4.0977	4.1332	0.1864	0.9832
0.5	3.2679	3.2970	0.4283	0.9784
1.0	2.6121	2.6362	0.5786	0.9704
2.0	1.5744	1.5910	0.7313	0.9498
5.0	0.4586	0.4670	0.8103	0.9202

Table 3: Effects of ϕ on skin-friction and couple wall stress, Nusselt number and Sherwood number for reference values in figs. 11 and 12.

ϕ	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
-0.5	4.3164	4.3532	0.1547	0.9867
-0.2	4.2368	4.2733	0.1550	0.9842
0.0	3.7002	3.7324	0.3099	0.9812
0.2	3.1909	3.2193	0.4486	0.9775
0.5	2.6121	2.6362	0.5786	0.9704

Table 4: Effects of Kr on skin-friction, couple wall stress, Nusselt number and Sherwood number for reference values in figs. 8 and 14.

Kr	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
0.0	1.0866	1.2686	0.5786	0.6800
0.1	1.2544	1.0996	0.5786	0.6640
0.5	2.6121	2.6362	0.5786	0.9704
1.0	2.7197	2.7447	0.5786	1.1762
2.0	2.7074	2.7338	0.5786	1.4785

Table 5: Effects of So on skin-friction, couple wall stress, Nusselt number and Sherwood number for reference values in figs. 9 and 15.

So	C_f	C_m	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
0.0	3.0276	3.0545	0.5786	0.9897
0.5	2.6121	2.6362	0.5786	0.9704
1.0	2.1965	2.2180	0.5786	0.9512
2.0	1.3655	1.3815	0.5786	0.9128
5.0	0.5344	0.5450	0.5786	0.8744

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APPENDIX

$$a_1 = U_p - 1 - a_3 - a_5 - a_6,\ a_2 = Gcc_2 - Gr,\ a_3 = \frac{a_2}{(1+\beta)m_3^2 - m_3 - N},\ a_4 = -Gc(1+c_1)$$

$$a_5 = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta - N} k_1 = \theta_1 k_1,\ a_6 = \frac{a_4}{(1+\beta)m_6^2 - m_6 - N},$$

$$b_1 = \frac{Am_1 a_1}{(1+\beta)m_1^2 - m_1 - (N+n)},\ b_2 = -(1 + b_1 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8),$$

$$b_3 = \frac{Am_3 a_3 + \frac{AGr\, m_3}{n} - Gcc_3}{(1+\beta)m_3^2 - m_3 - (N+n)},\ b_4 = \frac{-Gr\left(1 + \frac{Am_3}{n}\right) + Gcc_4}{(1+\beta)m_4^2 - m_4 - (N+n)},$$

$$b_5 = \frac{2\beta m_5}{(1+\beta)m_5^2 - m_5 - (N+n)} k_2 = \theta_2 k_2,\ b_6 = \frac{Aa_6 m_6 - Gcc_7}{(1+\beta)m_6^2 - m_6 - (N+n)}$$

$$b_7 = \frac{-Gc\, c_7}{(1+\beta)m_7^2 - m_7 - (N+n)},\ b_8 = \frac{Aa_5\eta - \frac{2\beta\eta^2 k_1}{n}}{(1+\beta)\eta^2 - \eta - (N+n)},$$

$$k_1 = \frac{n}{1 - n\theta_1(\eta - m_1)}\left\{(U_p - 1)m_1 + a_3(m_3 - m_1) + a_6(m_6 - m_1)\right\},$$

$$k_2 = \frac{k_3 + nk_4 m_2}{1 + n\theta_2(m_2 - m_5)},\ k_3 = k_1\frac{A\eta}{\delta} + n(b_1 m_1 + b_3 m_3 + b_4 m_4 + b_6 m_6 + b_7 Sc + b_8\eta),$$

$$k_4 = -(1 + b_1 + b_4 + b_5 + b_6 + b_7 + b_8).$$

$$c_1 = \frac{S_0 Sc m_3^2}{(1+\beta)m_3^2 - m_3 Sc - K_r Sc},\ c_2 = Am_3 Sc\left(\frac{Am_3}{n} - c_1\right),\ c_3 = \frac{c_2}{m_3^2 - m_3 Sc - (n + K_r)Sc}$$

$$c_4 = \frac{c_5}{m_4^2 - m_4 Sc - (n + K_r)Sc},\ c_5 = -S_0 Sc m_4^2\left(1 + \frac{Am_3}{n}\right),\ c_6 = \frac{A(1+c_1)m_6}{-n},$$

$$c_7 = 1 - (c_3 + c_4 + c_6)$$