

Effect of Arrhenius activation energy and dual stratifications on the MHD flow of a Maxwell nanofluid with viscous heating

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ABSTRACT

Exploration of the coupled action of Arrhenius function, viscous heating, radiative thermal energy and dual stratifications on the mixed convection flow of a Maxwell based nano fluid over a linearly stretched surface with effecting a magnetic force and considering the impact of Darcy–Forchheimer and porous medium is the focus of this investigation. The transport equations of the problem are deduced to a set of nonlinear conjugated equations with the aid of suitable scaling analysis. Application of the BVC45 method to these equations conceded the computational solution. Maxwell variable dwindled down the flow. Activation energy as well as parameter pertaining to temperature difference augmented concentration.

KEYWORDS:- Activation energy, Dual stratifications, non-Newtonian based nanofluid, Viscous heating

I. INTRODUCTION

The innovative efforts of Choi [1] in developing nanofluids brought a revolution in the thermal energy transport. They are extensively used for every effective cooling in various industries such as microelectronics, transportation, manufacture etc., and for removal of hotspots in various components used in computers, power electronics, and telecommunication [2]. In space technology and nuclear system nanofluids are used to have high temperatures and very high cooling [3, 4].

Stratification involves layer formation as a consequent of fluctuate temperatures, solute concentration or with fluids having variant density. Dual stratification is noticed when both processes pertaining to thermal and solute take place at the same instant. Srinivasacharya and Ram Reddy [5] addressed the impact of both stratification on transport phenomena in a micromaterial effecting the porous matrix. Sreelakshmi and Sarojamma [6] examined the two stratification in stagnation point flow related to Maxwell material with radiative heat. In a subsequent paper [7] they investigated the doubly stratified slip flow considering the effects of microrotation and radiation.

Mass transfer takes place due to the concentration differences occurring in the species. Mass transmission along with any chemical action is of significant importance due its wide range of its diligences in chemical engineering, food litigated industries, etc. Further, activation of energy (AE) is said to be the least possible energy essential to engender a chemical reaction (CR). The pioneering study involving Arrhenius AE was carried out by Bestman [8] to examine the rate of mass transmission in a viscous fluid flowing in a tube placed in a vertical direction through a porous medium. Recently, Zeeshan et al [9] discussed the interaction of AE and CR in a flow between two horizontal channel implementing the convective conditions on the wall for temperature and nano particle volume fraction. Rashidi et al [10] analysed the 3-D flow in a rotating frame considering the effect of non-Darcy porous medium as well as the AE. Girisha et al [11] investigated the dispersion of nano particles in a Casson fluid to realise the impact of binary CR and AE.

In this research work, we analysed the oncoming of AE in a non-Darcy Maxwell based nanofluid flow driven by a elongated sheet implementing the of dual stratification, thermal radiation, magnetic force and viscous dissipation.

II. MATHEMATICAL FORMULATON

We have cogitated the Arrhenius AE in the MHD non-Darcy flow of an incompressible Maxwell nanofluid accomplished due to a surface stretched with a linear velocity $u_w = bx$. Fig.1 designates the geometry corresponds to flow. The coupling effect of thermal and solutal stratification and radiative heat transfer is incorporated in the investigation. The influence of a magnetic force of intensity B_0 applied in the transverse direction on the flow is considered.

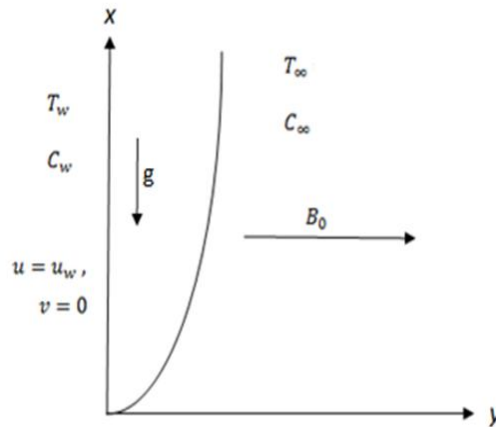


Fig.1 Flow geometry

Following Buongiorno model [12], the equations of the flow are given below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left(u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \left(u + \lambda v \frac{\partial u}{\partial y} \right) + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \frac{v}{k} u - Fu^2, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \frac{\partial^2 T}{\partial y^2} + \frac{\rho^* c_p^*}{\rho c_p} \left(D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right) + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_r^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^n e^{\left(\frac{E_a}{k^* T} \right)}, \quad (4)$$

Boundary conditions:

$$u = u_w, v = 0, T = T_w = T_0 + m_1 x, C = C_w = C_0 + n_1 x \text{ at } y = 0, \quad (5)$$

$$u \rightarrow 0, T \rightarrow T_\infty = T_0 + m_2 x, C \rightarrow C_\infty = C_0 + n_2 x \text{ as } y \rightarrow \infty, \quad (6)$$

III. METHOD OF SOLUTION

The scaling transformations given below are inclosed [13]:

$$u = bxf'(\eta), v = -\sqrt{bv}f(\eta), \eta = \sqrt{\frac{b}{v}}y, \theta(\eta) = \frac{T-T_\infty}{T_w-T_0}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_0}, \quad (7)$$

Substitution of equation (7) into the governing equations (2) – (4) afford the non-linear ordinary differential equations mentioned below:

$$f''' + ff'' - (1 + F_r)f'^2 + \beta(2ff'f'' - f^2f''') - M(f' - \beta ff'') + R(\theta + N\phi) - K_p f' = 0, \quad (8)$$

$$\left(1 + \frac{4}{3}Nr\right)\theta'' + Pr(f\theta' - f'\theta - \varepsilon_1 f' + Nb\theta'\phi' + Nt\theta'^2 + Ec f'^2) = 0, \quad (9)$$

$$\phi'' + LePr(f\phi' - f'\phi - \varepsilon_2 f') + \frac{Nt}{Nb}\theta'' - LePr\Lambda(1 + \delta\theta)^n \phi e^{\left(\frac{E}{1+\delta\theta}\right)} = 0. \quad (10)$$

The boundary conditions (5) and (6) become:

$$f(0) = 0, f'(0) = 1, \theta(0) = 1 - \varepsilon_1, \phi(0) = 1 - \varepsilon_2, \quad (11)$$

$$f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0. \quad (12)$$

The three quantities, coefficients of solutal and thermal transmission and local wall friction are itemized as

$$Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, C_f = \frac{2\tau_w}{\rho u_w^2}, \quad (13)$$

where the wall shear stress τ_w , the surface heat flux q_w and mass flux q_m are given by

$$\tau_w = \mu(1 + \beta) \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left(\frac{16 T_\infty^3 \sigma^*}{3 k^* k} + 1 \right) \left(\frac{\partial T}{\partial y} \right)_{y=0},$$

$$q_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}, \quad (14)$$

Equations (14) and (13), result in

$$\frac{1}{2} C_f Re_x^{1/2} = (1 + \beta) f''(0), \quad (15)$$

$$Nu_x Re_x^{-1/2} = - \left(1 + \frac{4}{3} Nr \right) \left(\frac{1}{1 - \varepsilon_1} \right) \theta'(0), \quad (16)$$

$$Sh_x Re_x^{-1/2} = -\left(\frac{1}{1-\varepsilon_2}\right) \phi'(0), \tag{17}$$

Where $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

The equations in (8) - (10) are highly non-linear coupled ODEs and solving them for analytical solutions is not possible. Hence, they are tackled along with BC given in (11) and (12) by the standard Runge-Kutta method coupled with the elegant shooting procedure. Utilizing MATLAB software their computational solutions are derived. The validity of the procedure is ensured by collating the values of $-f''(0)$ obtained by Hayat et al. [14], Sadiq and Hayat [15] when $F_r = K_p = R = N = 0$ and for the particular case of a Newtonian material for distinct values of magnetic field parameter (M). The data tabulated in Table 1 reveals that the current outcomes are extremely close to them.

M	Hayat et al. [14]	Sadiq and Hayat [15]	Present results
0.0	1.00000	1.00000	1.00000
0.2	1.01980	1.01980	1.01980
0.5	1.11803	1.11803	1.11803
0.8	1.28063	1.28063	1.28063
1.0	1.41421	1.41421	1.41421
1.2	1.56205	1.56205	1.56205
1.5	1.80303	1.80303	1.80303

Table 1 Comparison values of $-f''(0)$ for different values of M when $F_r = k_p = R = N = 0$ in Newtonian case.

IV. RESULTS AND DISCUSSION

The objective of this communication is to examine the response of the flow variables of the Darcy-Forchheimer Maxwell nanofluid to the action of Arrhenius function (AF), dual stratification, magnetic force and radiative thermal energy.

Fig. 2 illustrates the response of velocity to the Maxwell parameter (β) and magnetic force parameter (M). As β is directly related to relaxation time, elevated values of β reflect more time of relaxation which causes the fluid heavily viscous leading to the dwindling in velocity. In the nonmagnetic case, velocity steadily falls. Attrition of velocity is noticed prominently close to the boundary on account of resisting nature of the Lorentz force.

Fig. 3 is the velocity graphs displaying its relationship with the Forchheimer number (F_r) and permeability parameter (K_p). In the porous medium ($F_r = 0.0$), the velocity drops rapidly from its preassigned value near the surface and a fall as we move away from the boundary and eventually merging with value of free stream. When $F_r = 0.5$, the velocity pattern is alike. The increased drag offered by the porous matrix fosters a reduction in speed. As F_r rises more reduction in the velocity persists. A declination is found in velocity for higher values of K_p owing to the Darcy resistance. Increased permeability parameter resists the flow and thus velocity is curtailed.

Fig. 4 reveals that in the absence of viscous heating ($Ec = 0$) a small undershoot of temperature is found for $\varepsilon_1 = 0.4$ owing to extreme thermal variation. This may be due to the fact that as the free stream temperature T_∞ rises in the current stream direction, the flow approaching from below tends to have temperature lesser than its local value and thus negative temperature occurs. Increased values of ε_1 lead to significant reduction of temperature closer to the surface. When $Ec = 1$ the temperature increases for all values of ε_1 and an overshoot of temperature is noticed near the surface.

It is clear from Fig. 5 that the nanoparticle concentration shows a subjugation with an increment in ε_2 . In a similar fashion to that of the effect of ε_1 on temperature, an undershoot in nanoparticle concentration takes place when $\varepsilon_2 = 0.7$ and this undershoot improves with ε_2 . The reaction of nanoparticle concentration in respect of Λ is to show a rapid diminishing action considering Λ . Excess dilution in concentration prevails with augmented Λ . Due to the extreme CR rate the nanoparticle concentration is diminished.

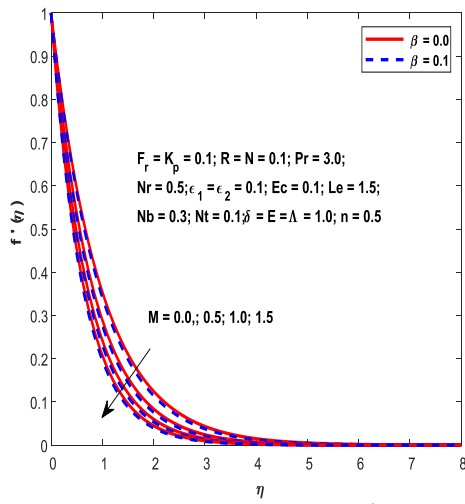


Fig. 2: Variation of M and β on $f'(\eta)$

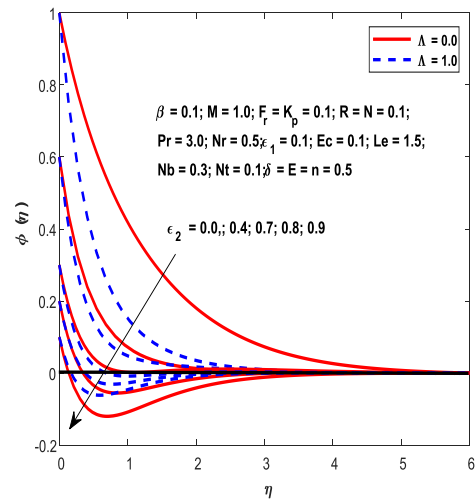


Fig. 5: Variation of ϵ_2 and Λ on $\phi(\eta)$

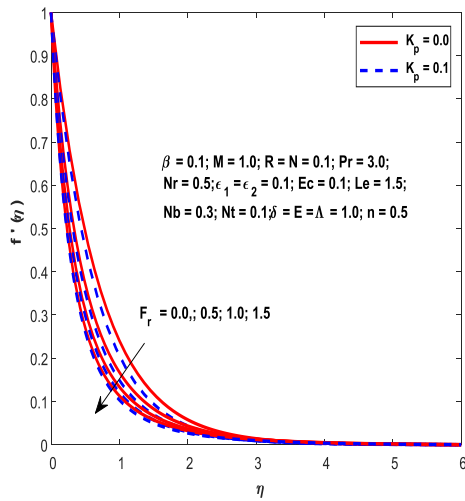


Fig. 3: Variation of F_r and K_p on $f'(\eta)$

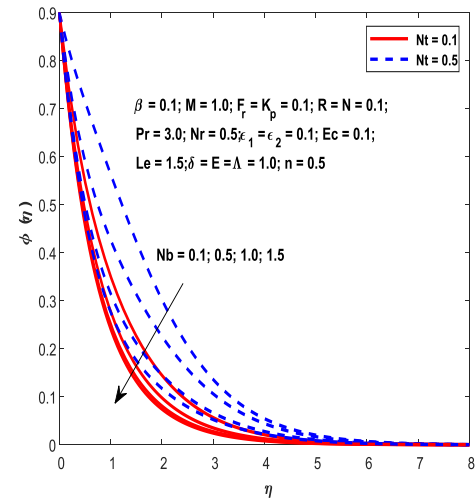


Fig. 6: Variation of N_b and N_t on $\phi(\eta)$

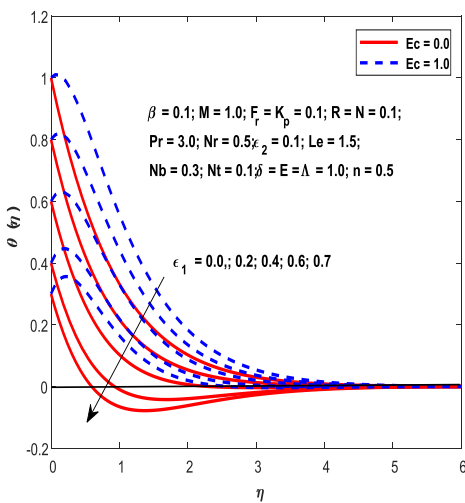


Fig. 4: Variation of ϵ_1 and Ec on $\theta(\eta)$

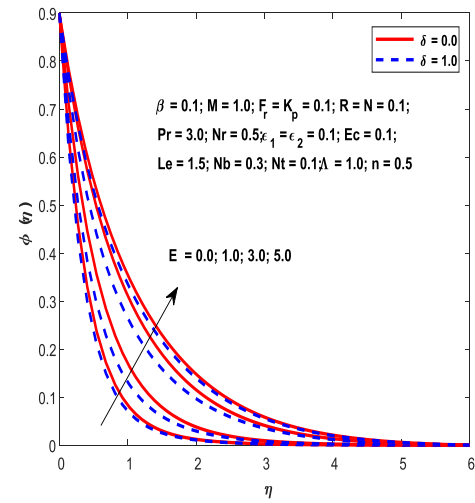


Fig. 7: Variation of E and δ on $\phi(\eta)$

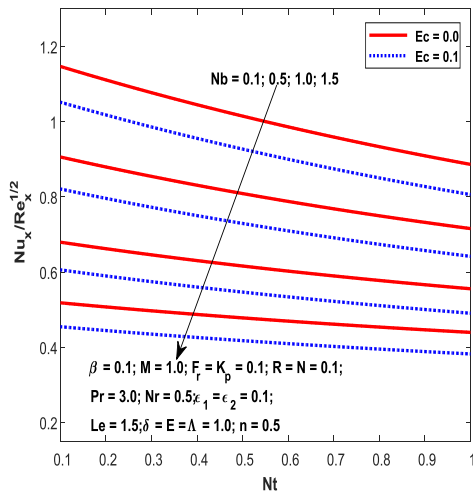


Fig. 8: Variation of Nt , Nb and Ec on $Nu_x Re_x^{-1/2}$

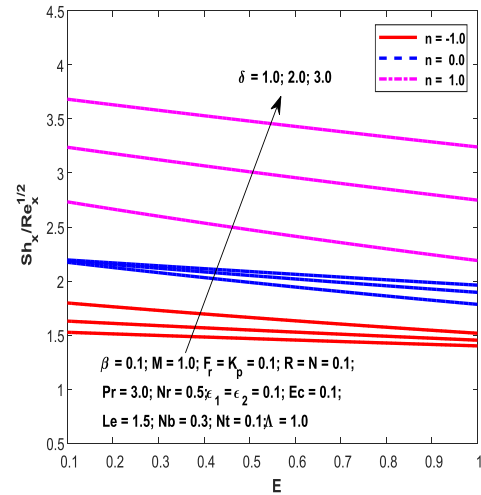


Fig. 9: Variation of δ , n and E on $Sh_x Re_x^{-1/2}$

Fig. 6 reveals the response of nanoparticle concentration to Brownian motion (Nb) and thermophoresis (Nt) parameters. It is clear that increased values of both Nb and Nt are lower the nanoparticle concentration.

Fig. 7 displays the reaction of nanoparticle concentration to AE (E) and variable of temperature difference (δ). Large δ solicits a dilution in the concentration. The AE (E) concentrates more as a result of function of Arrhenius. Increased values of E enhance the concentration nominally close to surface and a significant augmentation persists in far away from wall ultimately satisfying the conditions at far infinity. This may be concluded that the activation of AF is responsible for the stimulation of the generative CR.

From Fig. 8 it may be concluded that the Nusselt number linearly diminishes with thermophoresis variable. Nb and Eckert number show a diminishing action on Nusselt number. Fig. 9 projects the response of Sherwood number varying the parameter of AE (E) from 0.1 to 1.0 when $n = 0$ and altering δ from 1-3. When $n = -1$ the Sherwood number decreases linearly with E . As δ increases Sherwood number augments. When $n = 0$ the reduction in Sherwood number is noticed for $E \geq 0.2$. When $n \geq 1$ the reduction is more, for lesser values of E .

V. CONCLUSION

Mixed convective flow of the Darcy-Forchheimer Maxwell nanofluid flow driven by the elongated sheet including the influence of double stratification and activation energy is addressed. The study indicates that mellow values of Maxwell and magnetic parameters brought a diminution in velocity. Undershoot of temperature (concentration) occurs for stronger thermal (solutal) stratification while Eckert number has a reversal effect. Thermophoresis and microconvection parameters lower the concentration while activation energy has an opposite effect. Brownian motion, thermophoresis parameters and Eckert number cause a reduction in Nusselt number. Sherwood number decreases with AE variable while it increases with temperature difference variable.

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