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Analytical Study on Subgrade Soil Reactions to Octagonal Foundations of Industrial Pole-like Structures

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ABSTRACT

Based on the common assumption that the soil bearing pressure under a spread footing foundation is linearly distributed over the area where the compressive stress is kept between the foundation bottom surface and the subgrade soil, the governing equationsare established for the soil bearing pressure under an octagonal spread foundation with eccentric loads. By choosing an appropriate coordinate system, it is found that the two unknowns in the governing equations can be decoupled into quartic, cubic, and/or quadratic equations which involve only oneof the two unknowns. Therefore, the analytical solutions to this problem are determined, which are essentialfor checking the soil bearing pressure when the engineering design codes migrate from the conventional Allowable Stress Design (ASD) method to the advanced Load Resistance Factor Design (LRFD) method since larger eccentricity ratios occur when factored loads are used. The solutions play acriticalrole in the optimal design of octagonal foundations for wind turbine generators since it can help significantly expedite the design processes especially when using nonlinearcontact finite element analysis.

***KEYWORDS**: Spread foundation; Octagonal foundation; Soil bearing pressure; Analytical solution.*

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I. INTRODUCTION

Pole-like structures with significant eccentric loads such as wind turbine generators, chimneys and stacks are commonly founded on a variety of types of foundations. Dependent on geotechnical conditions, the selected foundation type can be a spread foundation, pile foundation, rock anchor foundation, and Patrick & Henderson (P&H) foundation[1-4].

The spread foundation is among the most widely used foundationsdue to its simplicity, lower cost, and applicability to a broad range of sub-grade conditions.It relies on the concrete self-weight of the foundation and the overburden soil-weight above the foundation to resist the overturning moment exerted on the foundation in extreme load conditions.While circular spread foundations are generally ideal to support pole-like structures subjected tocomplex loading conditions such as wind coming from any directions, octagonal spread foundations are much more commonly built for their simplicity in constructionand ease of quality control.

So far, almost all research efforts have been focusing on the circular spread foundations. Octagonal spread foundationsare instead treated as circular cases with the same footprint[5-20].The only exception is from Czerniak[21] who directly dealt with the soil bearing pressure distribution under an octagonal spread foundation. Based on the empirical data, Czerniakplotted the curve of uplift ratio versus eccentricity ratio and the curve of maximum bearing pressureversus eccentricity ratio for loads eccentric in the flat direction and the diagonal directions of the octagonal spread foundation, respectively. These curves stop when the load eccentricity ratio reaches the value of 0.34, which may be acceptable with the allowable stress design (ASD) methodologies, since the load eccentricity ratio of the service loads is usually smaller than 0.30[22-23].However, since the last two decades, the design codes[24-28] have been migrating from the ASD methods to the load resistance factor design (LRFD) methods in checking the soil bearing capacities. It is not uncommon touse the eccentricity ratio of factored load exceeding 0.40.

While there have been plenty of research efforts investigating the spread foundations under significant eccentric loads, they are limited to simplified geometries such as circular foundations. In real engineering practices, the octagonal spread foundations are much more widely adopted due to their convenience for

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construction and quality control. Therefore, a general solution to an octagonal spread foundation with an arbitrary eccentric vertical load should be considered. This paper presents an analytical method to obtainthe foundation uplift ratio and soil bearing pressure of an octagonal spread foundation with an arbitrary eccentric vertical concentrated load, in which the uplift ratio is found to be an analytical function of the eccentricity ratio, and the soil bearing pressurecan be expressed as another analytical function of the foundation diameter, the vertical load, and its eccentricity ratio.

II. DERIVATION OF ANALYTICAL SOLUTIONS

For an octagonal spread foundation whose elevation view wasshown in **Figure 1**, the maximumbearing pressure may either occur at its tipwhen a given vertical load is eccentric inthe diagonal direction of the foundationor at its edge when the given vertical load is eccentric in the flat direction of the foundation while the rebarsareusually put in the flat direction.Therefore,twocases, in which the vertical force is eccentric in the diagonal direction and in the flat direction, respectively,are investigated in thissection. In both cases, the foundation side length, foundation diameter in the flat direction, vertical concentrated load, and eccentricity of the vertical load are denoted as $L,\ D=\left(1+\sqrt{2}\right)L,\ P$, and αD ,respectively.The area of the octagonal spread foundation can accurately be obtained as

$$S = 2(\sqrt{2}-1)D^2 = 0.8284D^2 \tag{1}$$

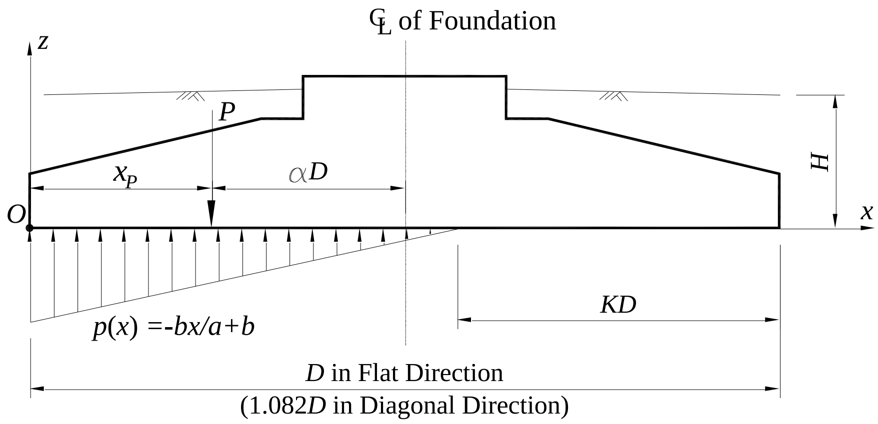

Figure 1. Illustration of an octagonal spread foundation with an eccentric vertical concentrated load and the linearly distributed soil bearing pressure.

Furthermore, withthe help of the perpendicular/parallel axis theorem,the moment of inertia of the octagonal spread foundation in both diagonal and flat directions can be determined as

$$I = \sum_{i=1}^{4}\int_{M_{i-1}}^{M_i}\left(\int_{-N_i}^{N_i}dy\right)\left(x-\frac{1.082D}{2}\right)^2 dx = \sum_{i=1}^{3}\int_{J_{i-1}}^{J_i}\left(\int_{-H_i}^{H_i}dy\right)\left(x-\frac{D}{2}\right)^2 dx = 0.05474D^4 \tag{2}$$

wherethe geometric parameters $M_0,\ M_1,\ M_2,\ M_3,\ M_4,\ N_1,\ N_2,\ N_3,$ and N_4 are shown in **Figure 2**with the definitions as:

$$\left.\begin{aligned}
&M_0 = 0\\
&M_1 = L\,cos67.5° = 0.1585D\\
&M_2 = L\left(cos67.5° + sin67.5°\right) = 0.5412D\\
&M_3 = L\left(2\,cos67.5° + sin67.5°\right) = 0.9239D\\
&M_4 = 2L\left(cos67.5° + sin67.5°\right) = 1.082D\\
&N_1(x) = x\,tan67.5° = 2.414x\\
&N_2(x) = tan22.5°\cdot\left(x - L\,cos67.5°\right) + L\,sin67.5° = 0.3170D + 0.4142x\\
&N_3(x) = -tan22.5°\cdot\left(x - L\left(cos67.5° + sin67.5°\right)\right) + L\left(cos67.5° + sin67.5°\right) = 0.7654D - 0.4142x\\
&N_4(x) = -tan67.5°\cdot\left(x - L\left(cos67.5° + 2sin67.5°\right)\right) + L\,sin67.5° = 2.613D - 2.414x
\end{aligned}\right\} \tag{3}$$

Based on the common assumption that the bearing pressure under the spread foundation is linearly distributed in the direction of the $x-$axis and uniformly distributed in the direction of the $y-$axis over the area where the non-tensional bearing stress is kept, the soil bearing pressure distribution under the octagonal spread foundation can be expressed as:

$$p(x) = -\frac{b}{a}x + b \qquad (p \geq 0) \tag{4}$$

where the coefficients of a and b represent the soil bearing lengthand maximum soil bearing pressure under the octagonal spread foundation. Although a and b have clear physical implications, they are not ideal for use in solving the governing equations since they are coupled with each other, and are dependent on not only the eccentricity ratioα, but also the foundation diameter *D*and the vertical load*P*. To make the governing equations established simple enough so that the analytical solutions can be derived, two intermediate dimensionless variables*A*and *B* are introduced, related to*a* and b as follows:

$$\begin{cases} a = AD \\ b = \dfrac{BP}{S} \end{cases} \tag{5}$$

from which, it can be seen that *A*and *B* represent the dimensionless soil bearing ratio and the dimensionless maximum soil bearing pressure under the octagonal spread foundation, respectively.

The following two subsections provide detailed solving processes for *A*and *B* for the two specific cases of the octagonal spread foundation subjected to avertical concentrated loadeccentric in the diagonal direction and in the flat direction, respectively.

2.1 Load eccentric in the diagonal direction

Figure 1and **Figure 2**show an octagonal spread foundation subjected to a vertical concentrated load with an eccentricity of $-\alpha D$ from the center of the foundation in the diagonal direction. The original point of the Cartesian coordinate system $O-x-y-z$ is set at the most left vertex of the octagonal spread foundation so that the governing equations can be manageable and be simplified to make it feasible to derive the analytical solution.

It can be observed that the maximum soil bearing pressure is $p_{max} = p(0) = b$ when $x = 0$, while the bearing pressure vanishes when $x = a$, meaning that a representsthe soil bearing lengthwith $0 < a \leq M_4 = 1.082D$,or equivalently, $0 < A \leq 1.082$.

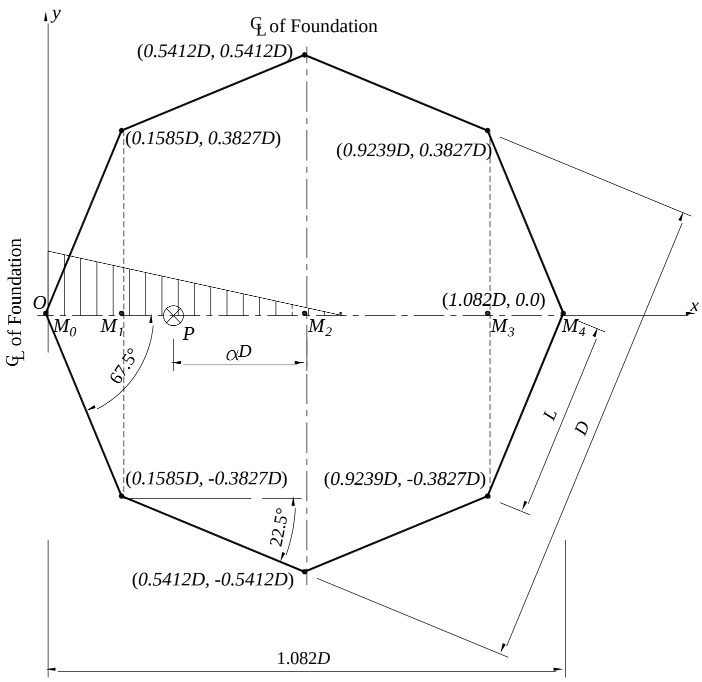

Figure 2.An octagonal spread foundation subjected to a vertical concentrated loadeccentric in the diagonal direction.

As shown later, the entire foundation base stay in contact with the underlying subgrade soil, indicating $A=1.082$, or equivalently, $a=1.082D$, when the load eccentricity ratio is smallwith $0.0 \le \alpha \le 0.1221$. From**Figure 2**, it can be seen that the critical line on which the bearing pressure under the octagonal spread foundation changes from positive to zero may occur in any of the four intervals (M_3, M_4) , $(M_2, M_3]$, $(M_1, M_2]$, and $(M_0, M_1]$,whichlater prove to correspond to the conditions in terms of A as $1.082 > A > 0.9239$, $0.9239 \ge A > 0.5412$, $0.5412 \ge A > 0.1585$, and $0.1585 \ge A > 0.0$, respectively;or conditions in terms of α as $0.1221 < \alpha < 0.1663$, $0.1663 \le \alpha < 0.3034$, $0.3034 \le \alpha < 0.4619$, and $0.4619 \le \alpha < 0.5411$,respectively. Therefore, the analytical solutions to soil bearing pressure under the octagonal spread foundation are derived separately for each of the above fivesubcases in the following five sub-subsections.

2.1.1 $0.0 \le \alpha \le 0.1221$, corresponding to $A=1.082$

When $0.0 \le \alpha \le 0.1221$, the entire octagonal foundation base stays in contact with the underlying subgrade soil without uplifting. When $a=1.082D$, it corresponds to $A=1.082$. When the minimum soil bearing pressureholds non-negative values, namely, $p_{min} \ge 0$, over the whole foundation footprint, theminimum soil bearing pressurecan be found by using of the following commonly used conventional formula[29]:

$$p_{min} = \frac{P}{S} - \frac{(\alpha D)P}{I} \cdot \frac{2(Sin67.5° + Cos67.5°)D}{2} = \frac{P}{S}(1 - 8.191\alpha) \ge 0 \tag{6}$$

from which, it can be found that the condition of $\alpha \le 0.1221$must be satisfied forthe foundation to keep in full contact with the underlying subgrade soil.When $0 \le \alpha \le 0.1221$, the bearing length $a=1.082D$, hence the corresponding dimensionless bearing length $A = a/D = 1.082$.

Correspondingly, in this case, the maximum soil bearing pressure can be found with the following equation:

$$p_{max} = \frac{P}{S} - \frac{(-\alpha D)P}{I} \cdot \frac{2(Sin67.5° + Cos67.5°)D}{2} = \frac{P}{S}(1 + 8.191\alpha) \tag{7}$$

The corresponding dimensionless maximum bearing pressure reads:

$$B = \frac{Sb}{P} = \frac{Sp_{max}}{P} = 1 + 8.191\alpha \tag{8}$$

2.1.2 $0.1221 < \alpha < 0.1663$, corresponding to $1.082 > A > 0.9239$

According to the fundamentalprinciples of force equilibrium, the total vertical force acting on the foundation and the total moment about Point *O*shall both be zero for the foundation to be in equilibrium. Therefore, when $1.082 > A > 0.9239$, the following governing equations can be established:

$$\begin{cases} P = \sum_{i=1}^{3} \int_{M_{i-1}}^{M_i} \left(\int_{-N_i}^{N_i} \left(b - \frac{b}{a}x \right) dy \right) dx + \int_{M_3}^{a} \left(\int_{-N_4}^{N_4} \left(b - \frac{b}{a}x \right) dy \right) dx \\ Px_P = \sum_{i=1}^{3} \int_{M_{i-1}}^{M_i} \left(\int_{-N_i}^{N_i} \left(b - \frac{b}{a}x \right) dy \right) xdx + \int_{M_3}^{a} \left(\int_{-N_4}^{N_4} \left(b - \frac{b}{a}x \right) dy \right) xdx \end{cases} \tag{9}$$

where x_P is the distance between the vertical concentrated loadP and the original pointO, as shown in **Figure 1**.

After integrating the right sides of the above equations andcombining with Equation(5), the above equation can be simplified as:

$$\begin{cases} 1 = \dfrac{\left(0.6906 + A\left(-2.414 + (3.154 - 0.9714A)A\right)\right)B}{A} \\ 0.5412 - \alpha = \dfrac{\left(0.3077 - 0.6906A + 1.051A^3 - 0.4857A^4\right)B}{A} \end{cases} \tag{10}$$

from which the unknown A satisfies the quartic equation:

$$A^4 - (3.247 - 2\alpha)A^3 + (3.515 - 6.494\alpha)A^2 - (1.268 - 4.971\alpha)A + (0.1361 - 1.422\alpha) = 0 \tag{11}$$

With the false roots discarded, the real root of the above equation is analytically obtained as:

$$A = \frac{1}{12}\left(\sqrt{3E_5} - 3E_3 - \sqrt{6\left(E_5 - 3E_4 - \frac{3\sqrt{3}E_6}{\sqrt{E_5}}\right)}\right) \tag{12}$$

where E_1, E_2, E_3, E_4, E_5, and E_6 are defined as:

$$\left.\begin{aligned}
E_1 &= \left(1.116 + 47.32\alpha - 906.0\alpha^2 + 3438\alpha^3 - 4958\alpha^4 + 21510\alpha^5 + 6941\alpha^6\right)^{1/2} \\
E_2 &= 4.303 - 19.34\alpha - 1.110\alpha^2 - 120.3\alpha^3 + E_1 \\
E_3 &= -3.247 + 2\alpha \\
E_4 &= \sqrt[3]{4D_2} + 2\sqrt[3]{2}\left(1.633 - 6.686\alpha + 12.35\alpha^2\right)\Big/\sqrt[3]{E_2} \\
E_5 &= -28.12 + 3D_3^2 + 51.96\alpha + \left(8.227 + 3.175\sqrt[3]{E_2^2} - 33.69\alpha + 62.26\alpha^2\right)\Big/\sqrt[3]{E_2} \\
E_6 &= 1.268 - 9.442\alpha + 12.99\alpha^2 + 8\alpha^3
\end{aligned}\right\} \tag{13}$$

Furthermore, substituting Equation (12) into the first equation of Equation Set(10), B canbe obtained.

Since the dimensionless soil bearing length A monotonicallydecreases with the increase of eccentricity α, Equation (12) can be used to back calculate α from A, and determine that $1.082 > A > 0.9239$ strictly corresponds to $0.1221 < \alpha < 0.1663$.

2.1.3 $0.1663 \le \alpha < 0.3034$, corresponding to $0.9239 \ge A > 0.5412$

When $0.9239 \ge A > 0.5412$, based on the equilibrium principles, the following governing equations can be established:

$$\left\{\begin{aligned}
P &= \sum_{i=1}^{2}\int_{M_{i-1}}^{M_i}\left(\int_{-N_i}^{N_i}\left(b - \frac{b}{a}x\right)dy\right)dx + \int_{M_2}^{a}\left(\int_{-N_3}^{N_3}\left(b - \frac{b}{a}x\right)dy\right)dx \\
Px_P &= \sum_{i=1}^{2}\int_{M_{i-1}}^{N_i}\left(\int_{-N_i}^{N_i}\left(b - \frac{b}{a}x\right)dy\right)xdx + \int_{M_2}^{a}\left(\int_{-N_3}^{N_3}\left(b - \frac{b}{a}x\right)dy\right)xdx
\end{aligned}\right. \tag{14}$$

After integrating the right sides of the above equations and combining with Equation(5), the above equation set can be simplified as:

$$\left\{\begin{aligned}
1 &= \frac{\left(0.05604 + A\left(-0.35355 + \left(0.9239 - 0.1667A\right)A\right)\right)B}{A} \\
0.5412 - \alpha &= \frac{\left(0.01455 - 0.05604A + 0.3080A^3 - 0.08333A^4\right)B}{A}
\end{aligned}\right. \tag{15}$$

from which the unknown A satisfies the quartic equation:

$$A^4 - \left(4.778 - 2\alpha\right)A^3 + \left(6 - 11.09\alpha\right)A^2 - \left(1.624 - 4.243\alpha\right)A + \left(0.1893 - 0.6725\alpha\right) = 0 \tag{16}$$

The real root of the above equation is analytically obtained as:

$$A = \frac{1}{12}\left(\sqrt{3E_5} - 3E_3 - \sqrt{6\left(E_5 - 3E_4 - \frac{3\sqrt{3}E_6}{\sqrt{E_5}}\right)}\right) \tag{17}$$

where E_1, E_2, E_3, E_4, E_5, and E_6 are defined as:

$$\left.\begin{aligned}
&E_1 = \sqrt{704.77+1419.5\alpha-68328.2\alpha^2+342697\alpha^3-684867\alpha^4+483186\alpha^5+105221\alpha^6}\\
&E_2 = 119.18-793.22\alpha+1936.78\alpha^2-1951.3\alpha^3+E_1\\
&E_3 = -4.7791+2\alpha\\
&E_4 = \sqrt[3]{4E_2}+2\sqrt[3]{2}\left(15-70.555\alpha+97.457\alpha^2\right)\Big/\sqrt[3]{E_2}\\
&E_5 = -48+3E_3^2+88.693\alpha+\left(75.595+3.1748\sqrt[3]{E_2^2}-355.58\alpha+491.15\alpha^2\right)\Big/\sqrt[3]{E_2}\\
&E_6 = -7.39104-88.9714\alpha+31.358\alpha^2+8\alpha^3
\end{aligned}\right\} \tag{18}$$

Furthermore, substitutingEquation (17) into the first equation of Equation Set(15), Bcan be found.

Since the dimensionless soil bearing length A monotonically decreases with the increase of eccentricity α, Equation (16) can be used to back calculate α from A, and determine that $0.9239 \geq A > 0.5412$ strictly corresponds to $0.1663 \leq \alpha < 0.3034$.

2.1.4 $0.3034 \leq \alpha < 0.4619$, corresponding to $0.5412 \geq A > 0.1585$

When $0.5412 \geq A > 0.1585$, from the equilibrium principles, the following governing equation can be established

$$\begin{cases} P = \int_0^{M_1}\left(\int_{-N_1}^{N_1}\left(b-\dfrac{b}{a}x\right)dy\right)dx+\int_{M_1}^{a}\left(\int_{-N_2}^{N_2}\left(b-\dfrac{b}{a}x\right)dy\right)dx \\ Px_P = \int_0^{M_1}\left(\int_{-N_1}^{N_1}\left(b-\dfrac{b}{a}x\right)dy\right)xdx+\int_{M_1}^{a}\left(\int_{-N_2}^{N_2}\left(b-\dfrac{b}{a}x\right)dy\right)xdx \end{cases} \tag{19}$$

After integrating the right sides of the above equations and combining with Equation Set (5), the above equation set can be simplified as:

$$\begin{cases} 1 = \dfrac{\left(0.003205+A\left(-0.06066+\left(0.3827-0.1667A\right)A\right)\right)B}{A} \\ 0.5412-\alpha = \dfrac{\left(0.00002540-0.003205A+0.1276A^3-0.08333A^4\right)B}{A} \end{cases} \tag{20}$$

from which the unknown A satisfies the quartic equation:

$$A^4-(0.4483-2\alpha)A^3-(2.485+4.592\alpha)A^2+(0.3556+0.7279\alpha)A-(0.01777+0.03846\alpha)=0 \tag{21}$$

The real root of the above equation is analytically obtained as:

$$A=\frac{1}{12}\left(\sqrt{3E_5}-3E_3-\sqrt{6\left(E_5-3E_4-\frac{3\sqrt{3}E_6}{\sqrt{E_5}}\right)}\right) \tag{22}$$

where E_1, E_2, E_3, E_4, E_5, and E_6 are defined as:

$$\left.\begin{aligned}
&E_1 = \left(68.83+120.4\alpha-2930.3\alpha^2-6200.4\alpha^3+6886.6\alpha^4-3604.9\alpha^5\right)^{1/2}\\
&E_2 = -27+170.3\alpha-361.4\alpha^2+258.0\alpha^3+E_1\\
&E_3 = 0.4483+2\alpha\\
&E_4 = \sqrt[3]{4E_2}+2\sqrt[3]{2}\left(5.485-23.52\alpha+25.46\alpha^2\right)\Big/\sqrt[3]{E_2}\\
&E_5 = 19.88-36.74\alpha+\left(27.64+3.175\sqrt[3]{E_2^2}-118.5\alpha+128.3\alpha^2\right)\Big/\sqrt[3]{E_2}+3E_3^2\\
&E_6 = 7.391+7.029\alpha-31.36\alpha^2+8\alpha^3
\end{aligned}\right\} \tag{23}$$

Substituting Equation(22) into the first equation of Equation(20), B can be found.

Since the dimensionless soil bearing length A monotonically decreases with the increase of eccentricity α, Equation (22) can be used to back calculate α from A, and determine that $0.5412 \geq A > 0.1585$ strictly corresponds to $0.3034 \leq \alpha < 0.4619$.

2.1.5 $0.4619 \le \alpha < 0.5411$, corresponding to $0.1585 \ge A > 0.0$

When $0.1585 \ge A > 0.0$, based on the equilibrium principles, the following governing equation can be established:

$$\begin{cases} P = \int_0^a \left(\int_{-N_1}^{N_1} \left(b - \dfrac{b}{a} x \right) dy \right) dx \\ P x_P = \int_0^a \left(\int_{-N_1}^{N_1} \left(b - \dfrac{b}{a} x \right) dy \right) x dx \end{cases} \tag{24}$$

After integrating the right sides of the above equations and combining with Equation Set (5), the above equation set can be simplified as:

$$\begin{cases} P = 0.8047 A^2 BP \\ \left(0.5412 - \alpha\right) DP = 0.4024 A^3 BDP \end{cases} \tag{25}$$

Solving the above equation set, we can obtain the solution as follows

$$\begin{cases} A = 1.082 - 2\alpha \\ B = \dfrac{1.029}{\left(1.082 - 2\alpha\right)^2} \end{cases} \tag{26}$$

The first equation in Equation Set (26) can be used to back calculate α from A, and determine that $0.1585 \ge A > 0.0$ strictly corresponds to $0.4619 \le \alpha < 0.5411$.

So far, for all the five subcases in which the concentrated vertical load is eccentric in the diagonal direction, *A*and *B* have analytically been determined.

Next, we will find the analytical solutions to *A*and *B* for the second case in which the vertical concentrated load is eccentric in the flat direction of the octagonal spread foundation.

2.2 Load eccentric in the flat direction

The second case is for the octagonal spread foundation subjected to vertical force eccentric in the flat direction. As shown in**Figure 3**, an octagonal spread foundation is subjected to a vertical concentrated load with an eccentricity of $-\alpha D$ from the center of the foundation in the flat direction. For the convenience of deriving the analytical solutions, the original point of the Cartesian coordinate system $O - x - y - z$ is placed at the midpoint of most left edge of the octagonal spread foundation.

The geometric parameters J_0, J_1, J_2, J_3, H_1, H_2, and H_3 shown in**Figure 3** are defined as follows:

$$\left. \begin{aligned} & J_0 = 0 \\ & J_1 = L\, cos 45° = 0.70711 L \\ & J_2 = L\left(1 + cos 45°\right) = 1.70711 L \\ & J_3 = L\left(1 + 2\, cos 45°\right) = 2.4141 L \\ & H_1(x) = x + L/2 \\ & H_2(x) = \left(1 + \sqrt{2}\right) L \big/ 2 \\ & H_3(x) = -x + \left(3 + 2\sqrt{2}\right) L \big/ 2 \end{aligned} \right\} \tag{27}$$

It is still assumed that the soil bearing pressure is linearly distributed under the octagonal spread foundation wherever it is non-negativeand can be expressed by Equation (4).

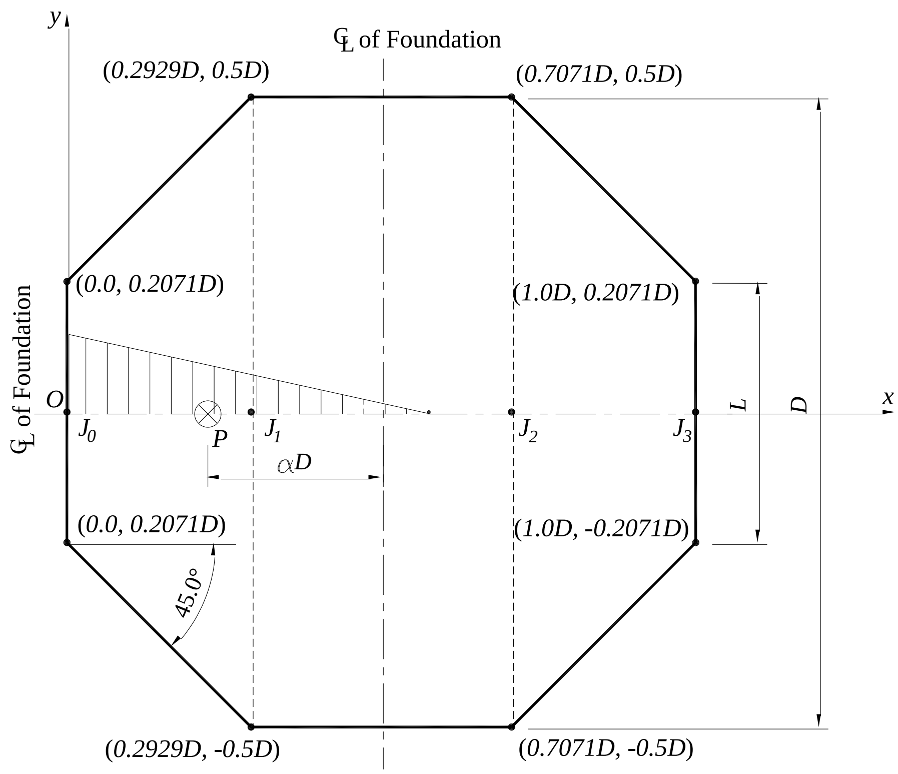

Figure 3.Octagonal spread foundation subjected to a vertical concentratedload eccentric in the flat direction.

It can be seen from **Figure 1** and **Figure 3** that the maximum soil bearing pressure will be $p_{max} = p(x=0) = b$ when $x=0$, and $x=a$ when the bearing pressure $p(x)=0$, which means that a represents the soil bearing length with $0 < a \le J_3 = D$,or equivalently, $0 < A \le 1.0$. Similarly, the foundation will keep in full touch with the underlying subgrade soil, indicating $A=1.0$, or equivalently, $a=1.0D$, when the load eccentricity ratio is small with $0.0 \le \alpha \le 0.1321$. From**Figure 3**, it can be shown that the critical line on which the bearing pressure under the octagonal spread foundation changes from positive to zero may occur in any of the four intervals (J_2, J_3), $(J_1, J_2]$, and $(J_0, J_1]$, which later prove to correspond to the conditions in terms of A as $1.0 > A > 0.7071$, $0.7071 \ge A > 0.2929$, and $0.2929 \ge A > 0.0$, respectively; or conditions in terms of α as $0.1321 < \alpha < 0.2257$, $0.2257 \le \alpha < 0.3867$, and $0.3867 \le \alpha < 0.5$, respectively. Therefore, the analytical solutions to soil bearing pressure under the octagonal spread foundation are derived separately for each of the foursubcases in the following four sub-subsections.

2.2.1 $0.0 \le \alpha \le 0.1321$, corresponding to $A=1.0$

When the vertical concentrated load is eccentric in the flat direction with $0.0 \le \alpha \le 0.1321$, the entire foundation base stays in contact with the underlying subgrade soil without uplifting. When $a=1.0D$, is corresponds to $A=1.0$. When the minimum soil bearing pressure holds non-negative values, namely, $p_{min} \ge 0$, over the entire foundation footprint, the minimum soil bearing pressurecan be found by using the following formula[29]:

$$p_{min} = \frac{P}{S} + \frac{(-\alpha D)P}{I} \times \frac{D}{2} = \frac{P}{D^2}(1.0 - 7.567\alpha) \ge 0 \tag{28}$$

from which, it can be found that the condition $\alpha \le 0.1322$ must be satisfied in order forthe entire foundation base to stay in contact with the underlying subgrade soil.When $0.0 \le \alpha \le 0.1321$, the bearing length $a=1.0D$, hence the corresponding dimensionless bearing length $A = a/D = 1.0$.

Correspondingly, in this case, the maximum soil bearing pressure can be found by using the following formula[29]:

$$p_{max} = \frac{P}{S} - \frac{(-\alpha D)P}{I} \times \frac{D}{2} = \frac{P}{D^2}(1+7.567\alpha) \tag{29}$$

Further, the corresponding dimensionless maximum bearing pressure is:

$$B = \frac{Sb}{P} = \frac{Sp_{max}}{P} = 1+7.567\alpha \tag{30}$$

2.2.2 $0.1321 < \alpha < 0.2257$, corresponding to $1.0 > A > 0.7071$

When $1.0 > A > 0.7071$, from the equilibrium principles, the following governing equation can be established:

$$\begin{cases} P = \sum_{i=1}^{2} \int_{J_{i-1}}^{J_i} \left(\int_{-H_i}^{H_i} \left(b - \frac{b}{a}x \right) dy \right) dx + \int_{J_2}^{a} \left(\int_{-H_3}^{H_3} \left(b - \frac{b}{a}x \right) dy \right) dx \\ Px_P = \sum_{i=1}^{2} \int_{J_{i-1}}^{J_i} \left(\int_{-H_i}^{H_i} \left(b - \frac{b}{a}x \right) dy \right) x dx + \int_{J_2}^{a} \left(\int_{-H_3}^{H_3} \left(b - \frac{b}{a}x \right) dy \right) x dx \end{cases} \tag{31}$$

After integrating the right sides of the above equations and combining with Equation (5), the above equation set can be simplified as:

$$\begin{cases} 1 = \dfrac{\left(0.1524 + A\left(-0.7071 + (1.457 - 0.4024A)A\right)\right)B}{A} \\ 0.5 - \alpha = \dfrac{\left(0.05178 - 0.1524A + 0.4857A^3 - 0.2012A^4\right)B}{A} \end{cases} \tag{32}$$

from which the unknown A satisfies the following quartic equation:

$$A^4 - (3.414 - 2\alpha)A^3 + (3.621 - 7.243\alpha)A^2 - (1. - 3.515\alpha)A + (0.1213 - 0.7574\alpha) = 0 \tag{33}$$

With the false roots discarded, the real root of the above equation is analytically obtained as:

$$A = \frac{1}{12}\left(-3E_3 + \sqrt{3E_5} - \sqrt{6\left(E_5 - 3E_4 - \frac{3\sqrt{3}E_6}{\sqrt{E_5}} \right)} \right) \tag{34}$$

where E_1, E_2, E_3, E_4, E_5, and E_6 are defined as:

$$\left.\begin{aligned} E_1 &= 17.26 - 103.2\alpha + 229.0\alpha^2 - 383.4\alpha^3 \\ E_2 &= -4.327 - 19.54\alpha + 31.37\alpha^2 \\ E_3 &= E_1 + \sqrt{E_1^2 - 4E_2^3} \\ E_4 &= \sqrt[3]{8E_2/E_3} + \sqrt[3]{4E_3} \\ E_5 &= 5.999 + 2E_4 + 16.97\alpha + 12\alpha^2 \\ E_6 &= 1.657 - 29.82\alpha - 16.97\alpha^2 + 8\alpha^3 \end{aligned}\right\} \tag{35}$$

Substituting Equation (34) into the first equation of Equation(32), B can be found.

Since the dimensionless soil bearing length A monotonically decreases with the increase of eccentricity α, Equation (33) can be used to back calculate α from A, and determine that $1.0 \geq A > 0.7071$ strictly corresponds to $0.1321 < \alpha < 0.2257$.

2.2.3 $0.2257 \leq \alpha < 0.3867$, corresponding to $0.7071 \geq A > 0.2929$

When $0.7071 \geq A > 0.2929$, from the equilibrium principles, the following governing equation can be established:

$$\begin{cases} P = \int_0^{J_1}\left(\int_{-H_1}^{H_1}\left(b-\frac{b}{a}x\right)dy\right)dx + \int_{J_1}^{a}\left(\int_{-H_2}^{H_2}\left(b-\frac{b}{a}x\right)dy\right)dxd \\ Px_P = \int_0^{J_1}\left(\int_{-H_1}^{H_1}\left(b-\frac{b}{a}x\right)dy\right)xdx + \int_{J_1}^{a}\left(\int_{-H_2}^{H_2}\left(b-\frac{b}{a}x\right)xdy\right)xdx \end{cases} \tag{36}$$

After integrating the right sides of the above equations and combining with Equation Set (5), the above equation set can be simplified as:

$$\begin{cases} 1 = \dfrac{\left(0.01011+\left(-0.103553+0.603553A\right)A\right)B}{A} \\ 0.5-\alpha = \dfrac{\left(0.00148058-0.01011A+0.201184A^3\right)B}{A} \end{cases} \tag{37}$$

from which the unknown A satisfies the following cubicequation:

$$A^3-(1.5-3\alpha)A^2+(0.2071-0.5147\alpha)A-(0.01778-0.05025\alpha)=0 \tag{38}$$

With the false roots discarded, the real root of the above equation can be analytically obtained as

$$A=\frac{1}{6}\left(\sqrt[3]{4}G_4-2G_2+\frac{2\sqrt[3]{2}\left(3G_1-G_2^2\right)}{G_4}\right) \tag{39}$$

where $G_0, G_1, G_2, G_3,$ and G_4 are defined as:

$$\left.\begin{aligned} G_0 &= -0.01777-0.05025\alpha \\ G_1 &= 0.2071-514719\alpha+229.05\alpha^2-383.43\alpha^3 \\ G_2 &= -1.5-3\alpha \\ G_3 &= 3\sqrt{3\left(27G_0^2+4G_1^3-18G_0G_1G_2-G_1^2G_2^2\right)+4G_0G_2^3} \\ G_4 &= \left(-27G_0+9G_1G_2-2G_2^3+G_1\right)^{1/3} \end{aligned}\right\} \tag{40}$$

Substituting Equation(39) into the first equation of Equation Set (37), B can be found.

Since the dimensionless soil bearing length A monotonically decreases with the increase of eccentricity α, Equation (33) can be used to back calculate α from A, and determine that $0.7071\geq A>0.2929)$ strictly corresponds to $0.2257\leq\alpha<0.3867$.

2.2.4 $0.3867\leq\alpha<0.5$, corresponding to $0.2929\geq A>0$

When $0.2929\geq A>0$, from the equilibrium principles, the following governing equation can be established:

$$\begin{cases} P = \int_0^{a}\left(\int_{-H_1}^{H_1}\left(b-\frac{b}{a}x\right)dy\right)dx \\ Px_P = \int_0^{a}\left(\int_{-H_1}^{H_1}\left(b-\frac{b}{a}x\right)dy\right)xdx \end{cases} \tag{41}$$

After integrating the right sides of the above equations and combining withEquation Set (5), the above equation set can be simplified as:

$$\begin{cases} 1=\left(0.25+0.4024A\right)AB \\ 0.5+\alpha=\left(0.08333+0.2012A\right)A^2B \end{cases} \tag{42}$$

Solving the above equation set, we can obtain the solution as follows

$$\begin{cases} A = 0.2929 - \alpha + 0.5\sqrt{2.8284 - 7.3137\alpha + 4\alpha^2} \\ B = \dfrac{1}{(0.25 + 0.40237A)A} \end{cases} \tag{43}$$

The first equation in Equation (43) can be used to back calculate α from A, and determine that $0.2929 \geq A > 0$ strictly corresponds to $0.3867 \leq \alpha < 0.5$.

III. RESULTS AND DISCUSSION

From the previous expressions in Section 2.1 and Subsection 2.2, Parameters a and b represent the soil bearing length along the $x-$ axis direction and maximum soil bearing pressure under the octagonal spread foundation, respectively.The solutions to the two intermediate dimensionless parameters*A*and *B*, which relate to a and b through Equation (5), have been derived for all the five subcases in which the vertical concentrated load is eccentric in the diagonal direction in Section 2.1 and for all the four subcases in which the vertical concentrated load is eccentric in the flat direction in Section 2.2.

Since $A = a/D$ represents the dimensionless soil bearing length, the uplift ratio $K(\alpha)$, as shown in **Figure 1**, can be obtained as:

$$K(\alpha) = \begin{cases} \dfrac{1.082D - a(\alpha)}{D} = 1.0824 - A(\alpha) & \text{(for a load eccentric in diagonal direction)} \\ \dfrac{1.0D - a(\alpha)}{D} = 1.0 - A(\alpha) & \text{(for a load eccentric in flat direction)} \end{cases} \tag{44}$$

Furthermore, $B(\alpha) = Sb(\alpha)/P = Sp_{max}(\alpha)/P$ exactly represents the maximum dimensionless bearing pressure under the octagonal spread foundation.

The dependence of uplift ratio $K(\alpha)$ on the eccentricity ratiofor both flat and diagonal directionsis illustrated in **Figure 4**,while the semi-log curves of the dimensionless maximum pressure *B* versus eccentricity ratio for both flat and diagonal directions are shown in **Figure 5**in which a small difference between the two semi-log curves indicates a much large change of B values.

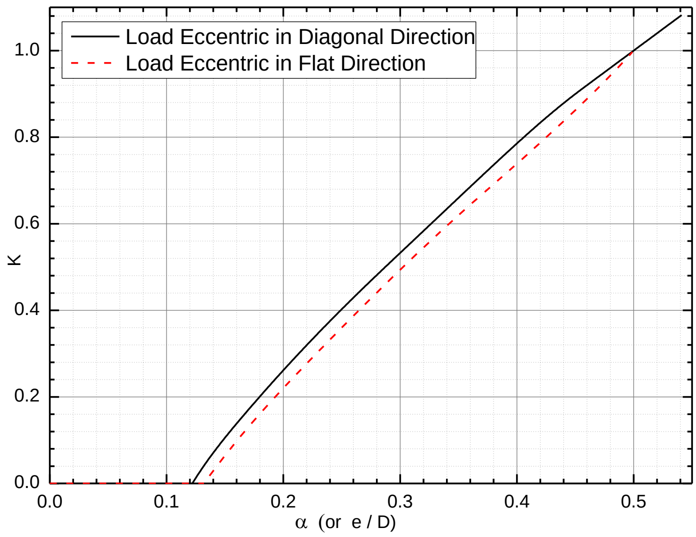

Figure 4. Curves of uplift ratio versus load eccentricity ratio

It is shown from**Figure 4** that the uplift ratio K with the same value of load eccentricity ratio α is different for the flat anddiagonal directions. It can also be seen from **Figure 5**thatthe dimensionless maximum soil bearing pressure Bwith the same value of load eccentricity ratio α is different for the flat and diagonal directions. Particularly, when the eccentricity ratio α becomes larger than 0.3, the dimensionless maximum soil bearing pressure difference can be significant.

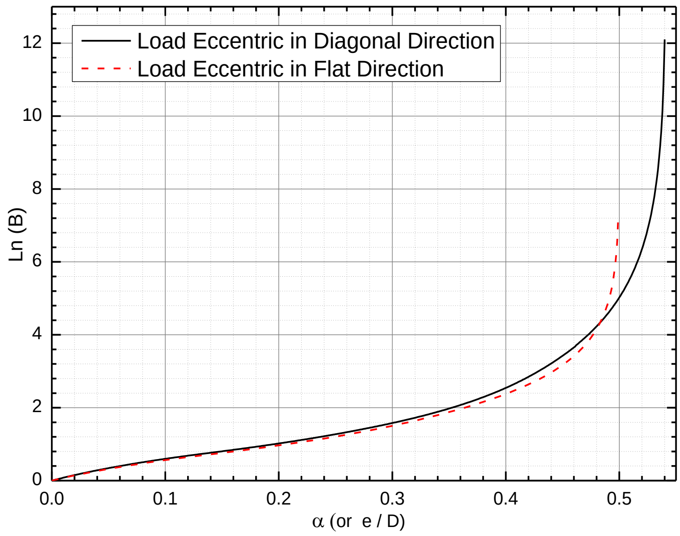

Figure 5. Semi-log curves of dimensionless maximum bearing pressure versus load eccentricity ratio

While the analytical solutions to the bearing pressure problems for an octagonal spread foundation have been derived. the expressions for the solutions are very complicated and may still not be convenient enough for daily engineering practice. In this section, highly accuratedesign curves[30] in simple forms are developed.

For the case in which the load is eccentric in the diagonal direction whose solution is derived in Section 2.1, the design curves for the uplift ratio $K(\alpha)$ and dimensionless maximum bearing pressure $B(\alpha)$ can be represented by the following two Equations, respectively:

$$K(\alpha) = 1.082 - A(\alpha) = \begin{cases} 0 & \alpha < 0.1221 \\ 2.830 - 45.67e^{-62.1504\alpha} - 3.224e^{-1.13617\alpha} & 0.1221 \le \alpha < 0.5442 \end{cases} \tag{45}$$

and

$$B(\alpha) = \frac{Sb(\alpha)}{P} = \begin{cases} 1 + 8.191\alpha & (\alpha < 0.1221) \\ 0.1837e^{1.9697859e^{1.532285\alpha}} + 0.002572e^{13.642565\alpha} & (0.1221 \le \alpha < 0.4619) \\ \dfrac{1.029}{(1.082 - 2\alpha)^2} & (0.4619 \le \alpha < 0.5442) \end{cases} \tag{46}$$

For the case in which the load is eccentric in the flat direction whose solution is derived in Section 2.2, the design curves for the uplift ratios $K(\alpha)$ and dimensionless maximum bearing pressure $B(\alpha)$ can be represented by Equations (47) and (48), respectively, as follows:

$$K(\alpha) = 1.0 - A(\alpha) = \begin{cases} 0 & (0 \le \alpha < 0.1321) \\ 2.082 - 2.576e^{-1.62056\alpha} & (0.1321 \le \alpha < 0.3867) \\ -0.4779 + 0.5604e^{1.937947\alpha} & (0.3867 \le \alpha < 0.499) \end{cases} \tag{47}$$

and

$$B(\alpha) = \frac{Sb(\alpha)}{P} = \begin{cases} 1 + 7.567\alpha & (0 \le \alpha < 0.1321) \\ 4.578 \times 10^{42} \times e^{\frac{-10000}{101.73156 + 0.44877166e^{4.363192\alpha}}} & (0.1321 \le \alpha < 0.3867) \\ \dfrac{1}{0.8232 - 2.5426\alpha + 1.792\alpha^2} & (0.3867 \le \alpha < 0.5) \end{cases} \tag{48}$$

The design curves, together with their corresponding analytical solutions and datum points from Reference [21] are plotted in**Figure** 6through **Figure 9**.It is shown that the proposed design curves match the analytical solution very well. The solutions also match the values obtained from the empirical charts in

Reference [21] well, except that the curvesin Reference [21]stop at $\alpha \approx 0.34$.Thislevel of accuracy as shown in these figures is adequate for any practical foundation engineering purpose.

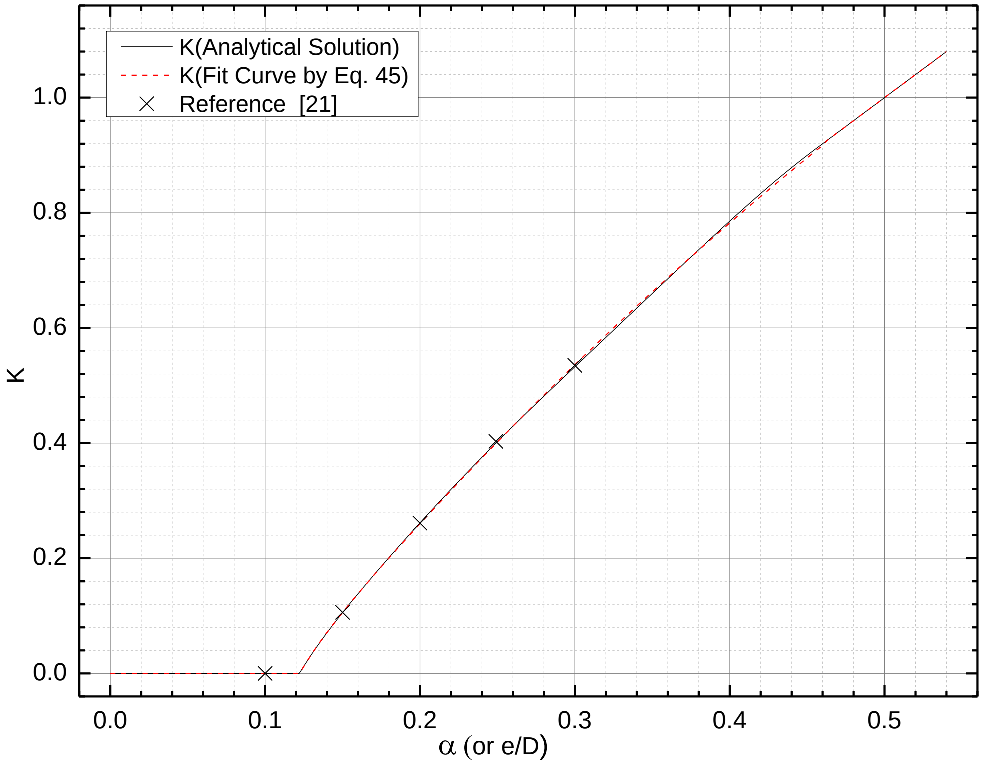

Figure 6. Curves of uplift ratio versus load eccentricity ratio (diagonal direction)

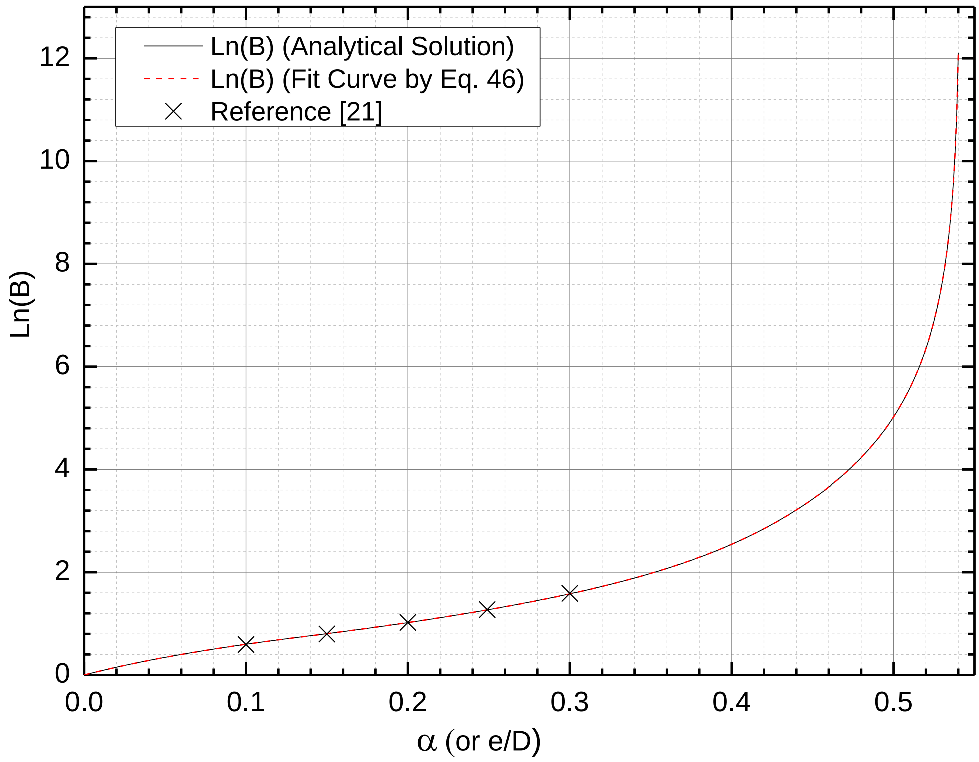

Figure 7. Semi-log curves of dimensionless maximum bearing pressure versus load eccentricity ratio (diagonal direction)

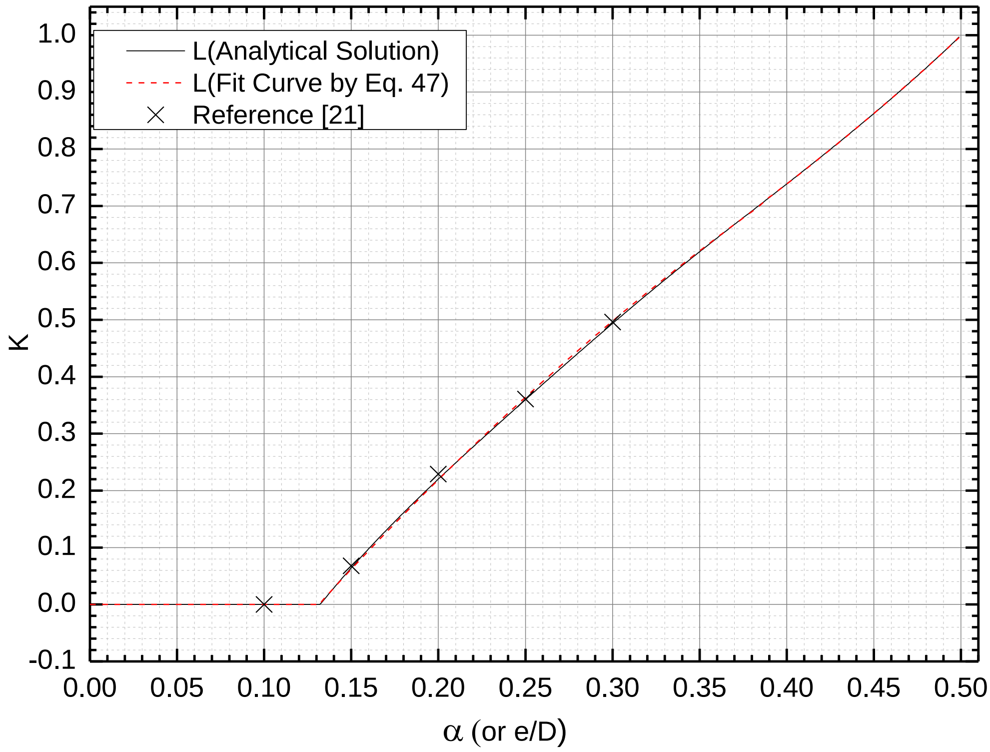

Figure 8. Curves of uplift ratio versus load eccentricity ratio (flat direction)

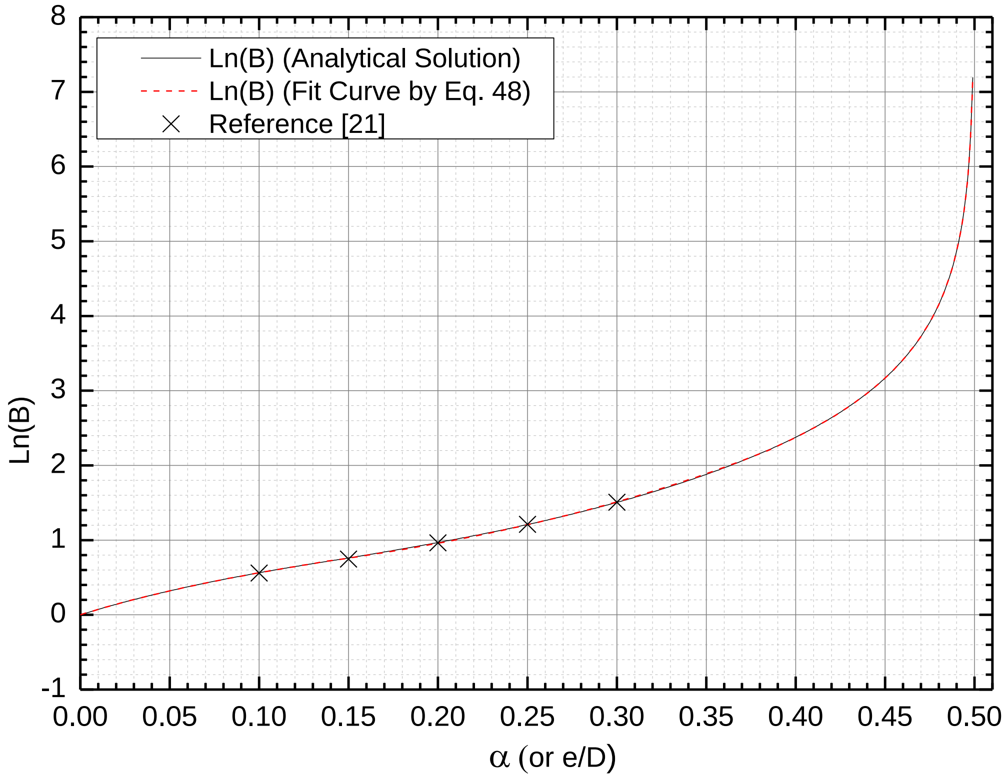

Figure 9. Semi-log curves of dimensionless maximum bearing pressure versus load eccentricity ratio (flat direction)

IV. EXAMPLE OF OCTAGONAL WIND TURBINE FOUNDATION

In the optimal engineering design of a large-scale octagonal wind turbine foundation, the following restraints must be considered:

(1) The diameter D in the flat direction shall be between 40.0ft and 72.0ft, and the embedment depth of the foundation shall be between 5.0ft and 15.0ft;

(2) The maximum factored soil bearing pressure shall not exceed 8.0ksf; and

(3) The uplift ratio shall not exceed 0.85 in both the diagonal and flat directions.

In the iteration design process,the diameter of one foundation option is $D = 56.0\ ft$. The total factored vertical load, including the weight of wind turbine, the weight of the octagonal concrete foundation, the weight of the overburden soil, and the buoyancy force, is $P_u = 3205.58\, kips$ .The total factored moment with respect to the foundation bottom is $M_u = 82839.6\, kip \cdot ft$ .Correspondingly, the eccentricity ratio is $\alpha = \frac{M_u / P_u}{D} = 0.461$.

For the case in which the load is eccentric in the diagonal direction, the uplift ratio and maximum bearing pressure can be found by substituting α into Equations(45) and (46) as follows:

$$\alpha = 0.461 \Rightarrow \begin{cases} K_{diag} = 0.9220 > 0.85 & \text{N.G.} \\ B_{diag} = 39.56 \Rightarrow p_{max} = b = B\dfrac{P_u}{S} = 48.8\, ksf > 8\, ksf & \text{N.G.} \end{cases} \tag{49}$$

The corresponding results from finite element analysis (FEA) are shown in**Figure10**, which indicates that $K_{flat} = 0.929$ and $p_{max} = 46.6\, ksf$ .The difference between the analytical solutions and the FEA results are approximately -0.75% and 4.5% for the uplift ratio and the maximum bearing pressure, which is acceptable in engineering practice.

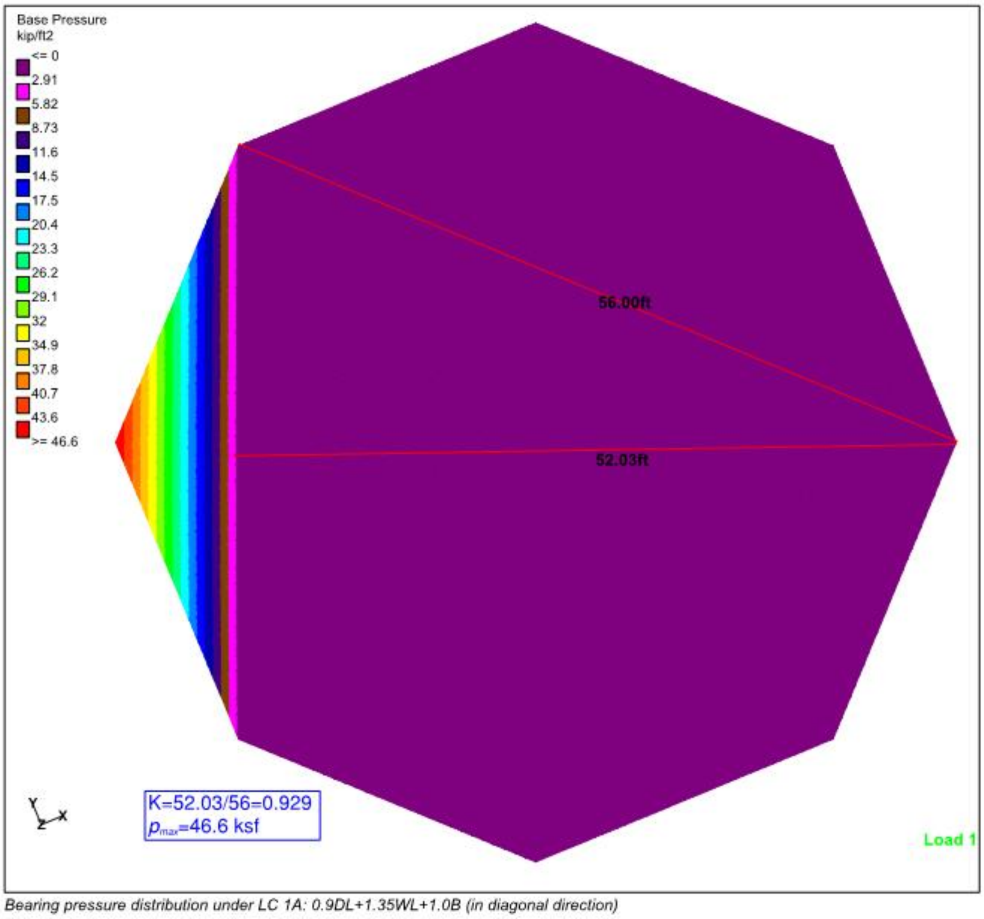

Figure 10. FEA results for the case in which the load is eccentric in the diagonal direction

For the case in which the load is eccentric in theflat direction, the uplift ratio and maximum bearing pressure can be found by substituting α into Equations(47) and (48):

$$\alpha = 0.461 \Rightarrow \begin{cases} K_{flat} = 0.8911 > 0.85 & \text{N.G.} \\ B_{flat} = 31.26 \Rightarrow p_{max} = b = B\dfrac{P_u}{S} = 38.6\,ksf > 8.0\,ksf & \text{N.G.} \end{cases} \tag{50}$$

The corresponding FEA analysis results are shown in **Figure 11**,indicating that $K_{flat} = 0.895$ and $p_{max} = 36.2\,ksf$. The difference between analytical solution and the FEA results are about -0.44% and +6.2% for the uplift ratio and the maximum bearing pressure, which is acceptable in engineering practice.

The FEA analysis results are slightlylarger in uplift ratio and slightly smaller in maximum bearing pressure than the corresponding analytical results. The small difference is expected and acceptable in engineering practice since the FEA model also considers the elasticity of the foundation itself, while the analytical study assumes that the foundation is rigid. Since the uplift ratios and the maximum soil bearing pressures fail to satisfy their respective restraints,this foundation option will be instantly excludedfrom further time-consuming FEA analysis in optimal design process, and hence much time can be saved.

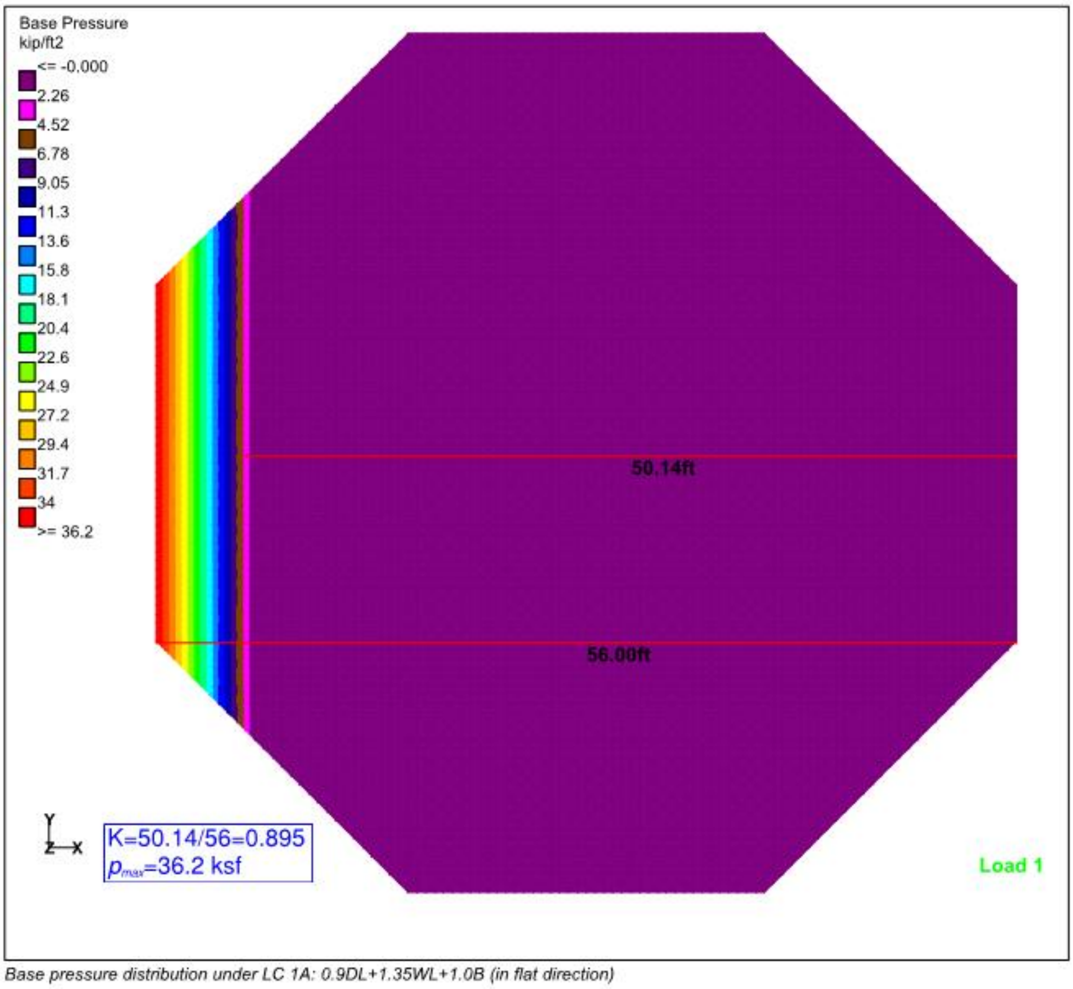

Figure 11. FEA results for the case in which the load is eccentric in the flat direction

V. CONCLUSIONS

While octagonal spread foundations are widely used for pole-like structures, easy-to-used explicit solutions to the soil bearing pressure under an octagonal spread foundation have not been found due to its intrinsic complexities. Therefore, when determining the soil bearing pressure under an octagonal spread foundation, either empirical values or equivalent circular spread foundation wasapplied to estimate the uplift ratio and maximum soil bearing pressure in the past.This approach is limited and only adequate for the ASD methodwith the small load eccentricity ratios used to check maximum soil baring pressure against the soil bearing capacity. When the design codes such as AASHTO and IBC are transitioning to use the LRFD method in which factored loads are applied and hence load eccentricity ratios are much larger, errors may occur since the maximum soil bearing pressure increases at an accelerated rate when the eccentricity ratio increases.

This paper derived an analytical solution for the soil bearing pressure under octagonal spread foundations, based on the basic principles of equilibrium andthe introduction of the two intermediate dimensionless parameters*A*and *B* that greatly reduce the complexity of the governing equations. It is found that the governing equations can be decoupled so that the quartic, cubic or quadratic equations can be obtained, leading to the direct, analytical solutions to the soil bearing pressure under an octagonal spread foundation. To facilitate the application of the solution in the daily engineering practice,the design curves for the analytical solutions are also derived. This work plays a significant role in the optimal design of octagonal spread foundations for pole-like structures, especially for those under large eccentric loads. For example, for an octagonal spread foundation for a wind turbine generator, the concrete needed can be anywhere from 180 cubic yards to 800 cubic yards and the reinforcement needed can be anywhere from 40 kips to more than 150kips. For the same wind turbine, two workable foundation layouts different in diameter, depth, thickness, etc.may frequently have up to 30% differences in total construction cost. Therefore, an optimal foundation design may save a lot of cost for the developer and owner and reduce the impact on the environment. In the process of the optimal design, many iterations of FEA analyses of the foundation are involved, making the optimal design very time consuming. The developed analytical solutions and their corresponding fitting curvescan instantly exclude many of the options that involve big uplift ratios and/or soil bearing pressure from time-consuming FEA analyses.

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