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Proposal of an algorithm to transform the seismic signal into linear elastic response spectrum

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ABSTRACT

This work deals with the global characterization of the seismic signal from the accelerations of the ground. It consists in solving the differential equation of the movement and construct of the graph according to the respective periods by using the maximums of the response. We use the method which combine the Duhamel integral and the linear interpolation of the seismic signal. Linear behavior of an oscillator has been studied. The results obtained by our approach are less than those obtained by PS_Duhamel's Software except the first. The developed algorithm allows automatic transformation of all seismic signals to response spectrum.

KEY WORDS: *Damping, Simple oscillator, Seismic signal, Spectrum of response, linear behavior.*

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I. INTRODUCTION

Earthquake-resistant building measures have found wide application in the field of civil engineering and have made it possible to obtain resistant structures. Nevertheless, some earthquakes remain destructive and bruised. Hence the need of improving seismic measures. Thanks to the development of digital tools, the characterization of the seismic signal is no longer a problem today, but the transformation of these signals into a spectrum response is still relevant today. It is known that to predict a dynamic response, we need a response spectrum. The prediction of the dynamic response is a complicated calculation because it leads to solving the differential equation generated by the motion of the mass. This kind of problem is solved either by the Duhamel integral, or by the linear interpolation method of the seismic signal [1], [2], [3], [4]. In this paper, we propose a quasi-analytical resolution of this differential equation by a method combining the Duhamel integral and the linear interpolation of the seismic signal; the construction of a response spectrum network of simple oscillators with linear elastic behavior under the action of the earthquake, for different damping ratio.

II. METHODOLOGY

When designing structures, it is necessary to predict the dynamic response of the structure. To arrive at this estimate, the structure is reduced to a very simple model called skewer, combining elements of mass and stiffness. Then Newton's motion equation is applied to the mass and we obtain a heterogeneous differential equation of second order with constant coefficients, with second member [5]. The seismic signal is not available in analytical form. The seismic response of the structure is thus determined by making a good dose of approximation. Several approaches are used for its resolution, **Chopra and al** suggests using one of the two approaches because they are quasi-analytic [3].

- The convolution integral or integral of Duhamel;
- The linear interpolation of the seismic signal.

Erick Ringot proposed a resolution using the first approach, but in this article, we propose another resolution method by combining the two approaches evoked. The development of the linear elastic response spectrum requires the following chronological steps [6]: from modeling to equation; from solving the equation to developing an algorithm to plot response spectrum.

II.1 Modeling and Equation

To do this modeling, the structure is reduced to a system combining elements of mass and stiffness. Generally, the structure of the structure is modeled by the skewer model with a mass, a single degree of freedom of translation and linked to the ground with a vertical rod of negligible mass.

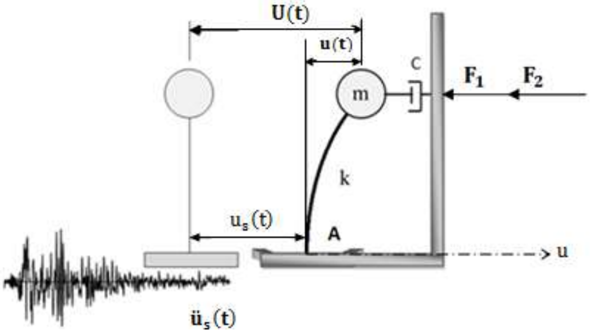

Figure1 : Single oscillator subjected to the seismic signal$\ddot{u}_s(t)$

When a seismic signal $\ddot{u}_s(t)$ is applied to the base of this oscillator, it oscillates and generates anelastic restoring force ($F_1 = k.u(t)$) and a damping force ($F_2 = c.\dot{u}(t)$). By applying the fundamental principle of dynamics to mass, we obtain the equation of the motion of damped forced oscillations following [1], [2].

$$m.\ddot{U}(t) + c.\dot{u}(t) + k.u(t) = 0 \qquad (1)$$

With $\ddot{U}(t) = \ddot{u}_s(t) + \ddot{u}(t)$, we will have:

$$m.\ddot{u}(t) + c.\dot{u}(t) + k.u(t) = -m\,\ddot{u}_s(t) \qquad (2)$$

Where m, c, k are the mass, the damping constant and the rigidity of the rod, respectively.

$\ddot{U}(t), \ddot{u}(t), \dot{u}(t), u(t)$ denote respectively the total acceleration, the acceleration, the velocity and the relative displacement of the mass of the oscillator.

II.2 Resolution of the equation of motion

To solve this equation, divide by m, and put $\xi = \frac{c}{2m\omega}$, $\omega^2 = \frac{k}{m}$,we obtain the refined expression:

$$\ddot{u}(t) + 2\xi\omega\dot{u}(t) + \omega^2 u(t) = -\ddot{u}_s(t) \qquad (3)$$

Where ξ is the damping ratio and ω the oscillator's pulsation.

Alain PECKER and Anil K. Chopra propose the expression below as the ideal method for solving equation (3). The general solution this linear differential equation for the under-damped oscillator is of the form:

$$u(t) = A\cos(\omega_D t - \varphi)e^{-\xi\omega t} - \frac{1}{\omega_D}\int_{\circ}^{t} \ddot{u}_s(t)e^{-\xi\omega\,(t-\tau)} \sin\omega_D(t-\tau)d\tau \qquad (4)$$

Where ω_D is the pulsation of damped oscillations and is expressed by $\omega_D = \omega\sqrt{1-\xi^2}$.

The first member to the right of equation (4) describes the free oscillations with damping while the second member describes the forced oscillations with damping. In the general case, the free oscillations disappear in a few fractions of a second to let the forced oscillations established. This general solution therefore takes the following form:

$$u(t) = -\frac{1}{\omega_D}\int_{\circ}^{t} \ddot{u}_s(t)e^{-\xi\omega\,(t-\tau)} \sin\omega_D(t-\tau)d\tau \qquad (5)$$

Equation (5) is called the Duhamel integral, which we solve by a quasi-analytical method by considering that the ground acceleration $\ddot{u}_s(t)$ varies linearly within each increment of time Δt. Thanks to the acquired experience [7], [8], we can then express it by:

$$\ddot{u}_s(t) = \frac{\Delta\ddot{u}_s}{\Delta t}t + \ddot{u}_s(t_i) \qquad (6)$$

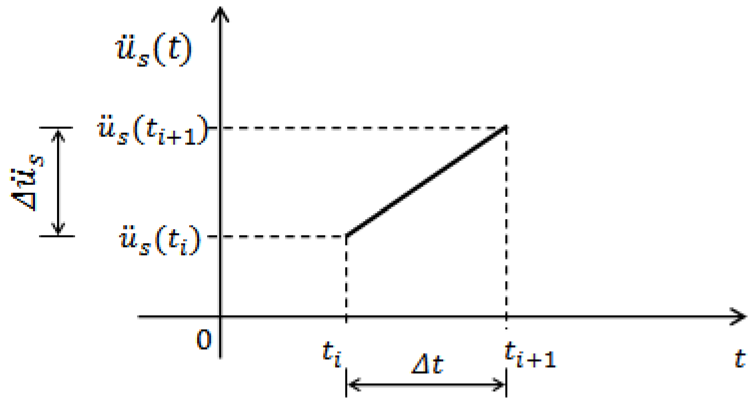

Figure 2 : Linear approximation of ground acceleration $\ddot{u}_s(t)$

With the expression of the acceleration of the ground $\ddot{u}_s(t)$ known, we can thus solve this equation by making several integrations by part.

By putting $h(t-\tau) = \frac{1}{\omega_D} e^{-\xi\omega(t-\tau)} \sin\omega_D(t-\tau)$, the integral (5) becomes

$$u(t) = -\int_\circ^t \ddot{u}_s(\tau)\, h(t-\tau)\, d\tau \qquad (7)$$

Otherwise: $\sin\omega_D(t-\tau) = \sin\omega_D t\cos\omega_D\tau - \cos\omega_D t\sin\omega_D\tau$

$$e^{-\xi\omega(t-\tau)} = e^{-\xi\omega t} . e^{\xi\omega\tau} = \frac{e^{\xi\omega\tau}}{e^{\xi\omega t}}$$

And the expression (7) becomes:

$$h(t-\tau) = \frac{1}{\omega_D}\frac{\sin\omega_D t}{e^{\xi\omega t}} e^{\xi\omega\tau}\cos\omega_D\tau - \frac{1}{\omega_D}\frac{\cos\omega_D t}{e^{\xi\omega t}} e^{\xi\omega\tau}\sin\omega_D\tau \qquad (8)$$

By introducing the expression (8) in (7), we have:

$$u(t) = -\left[\frac{1}{\omega_D}\frac{\sin\omega_D t}{e^{\xi\omega t}}\int_\circ^t \ddot{u}_s(t) e^{\xi\omega\tau}\cos\omega_D\tau\, d\tau - \frac{1}{\omega_D}\frac{\cos\omega_D t}{e^{\xi\omega t}}\int_\circ^t \ddot{u}_s(t)\, e^{\xi\omega\tau}\sin\omega_D\tau\, d\tau\right] \qquad (9)$$

By asking

$$A(t) = \int_\circ^t \ddot{u}_s(t) e^{\xi\omega\tau}\cos\omega_D\tau\, d\tau \quad and \quad B(t) = \int_\circ^t \ddot{u}_s(t) e^{\xi\omega\tau}\sin\omega_D\tau\, d\tau \qquad (10)$$

The expression (9) is still written in the form

$$u(t) = -\left[\frac{1}{\omega_D}\frac{\sin\omega_D t}{e^{\xi\omega t}} A(t) - \frac{1}{\omega_D}\frac{\cos\omega_D t}{e^{\xi\omega t}} B(t)\right] \qquad (11)$$

Let us study the functions A (t) and B (t) in the interval [0, t], for that, subdivide this interval in small intervals of time $[t_i, t_{i+1}]$

Just now $t = t_i$, we have $A(t_i) = \int_\circ^{ti} \ddot{u}_s(t) e^{\xi\omega\tau}\cos\omega_D\tau\, d\tau$ *and* $B(t_i) = \int_\circ^{ti} \ddot{u}_s(t) e^{\xi\omega\tau}\sin\omega_D\tau\, d\tau$ (12)

And at the moment $t = t_{i+1}$,

$$A(t_{i+1}) = \int_\circ^{ti+1} \ddot{u}_s(t) e^{\xi\omega\tau}\cos\omega_D\tau\, d\tau \quad and \quad B(t_{i+1}) = \int_\circ^{ti+1} \ddot{u}_s(t) e^{\xi\omega\tau}\sin\omega_D\tau\, d\tau \qquad (13)$$

Let's find the relation between A (t_i) and A (t_{i+1})

$$A(t_{i+1}) = A(t_i) + \int_{ti}^{ti+1} \ddot{u}_s(t) e^{\xi\omega\tau}\cos\omega_D\tau\, d\tau \qquad (14)$$

In the same way as before, we find

$$B(t_{i+1}) = B(t_i) + \int_{ti}^{ti+1} \ddot{u}_s(t) e^{\xi\omega\tau}\sin\omega_D\tau\, d\tau \qquad (15)$$

Introduce the analytic expression $\ddot{u}_s(t)$in the expressions below and after several integrations per part of equation (5), we obtain the following general discretized solution:

$$\left.\begin{aligned} u(t_i) &= -\left[\frac{1}{\omega_D}\frac{\sin\omega_D t_i}{e^{\xi\omega t_i}} A(t_i) - \frac{1}{\omega_D}\frac{\cos\omega_D t_i}{e^{\xi\omega t_i}} B(t_i)\right] \\ u(t_{i+1}) &= -\left[\frac{1}{\omega_D}\frac{\sin\omega_D t_{i+1}}{e^{\xi\omega t_{i+1}}} A(t_{i+1}) - \frac{1}{\omega_D}\frac{\cos\omega_D t_{i+1}}{e^{\xi\omega t_{i+1}}} B(t_{i+1})\right] \end{aligned}\right\} \qquad (16)$$

II.3 Plotting the linear elastic response spectrum

It consists in finding the maximum amplitude (u_{max}) of the response of the simple oscillator. This result is then plotted on the ordinate of the graph as a function of the natural period T of the oscillator on the abscissa, corresponding to a value of the damping ratio ξ. This operation is then applied to several simple oscillators, by varying their eigen values and invariant damping ratio. The curve connecting all its points constitutes the linear elastic response spectrum. Thus, we construct several other spectra by varying the value of the relative damping ratio.

III. NUMERICALS VALIDATION OF THE THEORY

To implement the developed algoritm, we used data from the North – South component of the accelerogram of the EL CENTRO earthquake of May 18, 1940 [9]. Its duration is 31.18 sec and the ground accelerations are recorded every 0.02 sec. These accelerations have for unit the acceleration of the gravity and is worth $9{,}81\mathrm{m/s^2}$ for this case. Oscillators have periods ranging from 0.2 sec to 20 sec with a pitch of 0.2 sec. For the damping ratio, they vary from 0 % to 10 % with a pitch of 2%.

1- Limit conditions

When the oscillators are in equilibrium, that is to say no displacement; velocity and acceleration are zero.

$$u(t=0)=\dot{u}(t=0)=\ddot{u}(t=0)=0$$

Which also implies that $A\,(t=0)=\ B\,(t=0)=\ 0.$

2- PS_Duhamel Software

Figure 3 presents the linear elastic seismic response spectrum obtained by the PS_Duhamel's Software, for the damping ratio varying from 0% to 10% with a pitch of 2%. The first curve from the top corresponds to the one with the lowest damping ratio. Let's remake that when its value increase, the response drops.

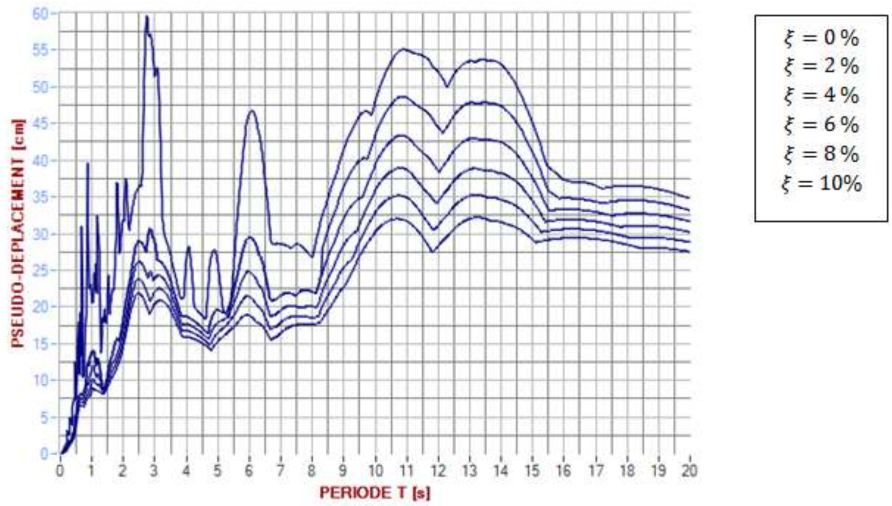

Figure 3 : Linear Elastic Seismic Response Spectrum Obtained by the PS_Duhamel's Software

Analysis of the response spectrum obtained allowed us to determine the oscillation periods giving the maximum responses as a function of the damping ratio. This analysis is summarized in Table 1 below. Where we have in the first column the damping ratio, the second the periods corresponding to the values of the maximum responses and the third the values of the maximum responses. The largest value of the response (57,44cm) is obtained when the damping ratio is 0 %.

Table 1. Oscillations Periods giving the maximum responses by the PS_Duhamel Software

Damping ratio ξ in %	Oscillations Periods T in (s)	Maximum Responses u_{maxPSD} in (cm)
0	02,70	57,44
2	10,90	48,64
4	10,80	43,33
6	10,80	38,88
8	13,25	35,21
10	13,25	32,16

3- Combination of the Integral of Duhamel with linear interpolation of the seismic signal

The implementation of function (16) in the developed algorithm, gives us the curves below. These curves describe the history of oscillator displacement. With the maximum amplitude in absolute value and the period of each oscillator, we obtain the coordinates of the response spectrum. Figure 5 presents the linear elastic seismic response spectrum obtained by combining of the integral of Duhamel with linear interpolation of the seismic signal, for the damping ratio varying from 0% to 10% with a pitch of 2%. As well as for the first approach, the first curve from the top corresponds to the one with the lowest damping ratio. Let's remake that when its value increase, the response drops.

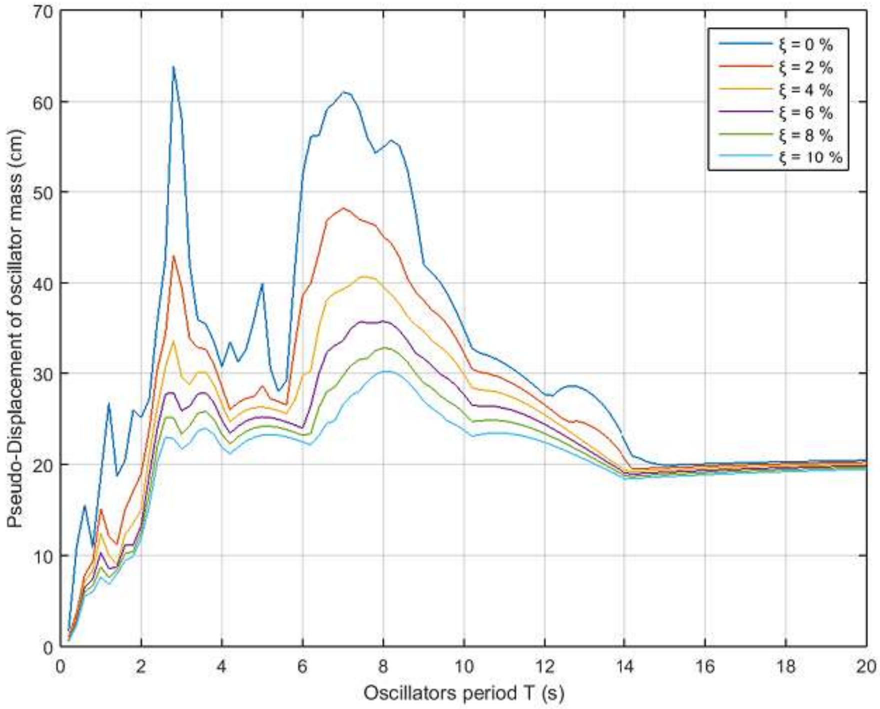

Figure 5 : Linear elastic seismic response spectrum obtained by combining the Duhamel integral with the linear interpolation of the seismic signal.

Analysis of the response spectrum obtained allowed us to determine the oscillation periods giving the maximum responses as a function of the damping ratio. This analysis summarized in Table 1 below. As well as for the first approach, we have in the first column the damping ratio, the second the periods corresponding to the values of the maximum responses and the third the values of the maximum responses. The largest value of the response (63,86cm) is obtained when the damping ratio is 0 %.

Table 2. Oscillations Periods giving the maximum responses by combining the Duhamel integral with the linear interpolation of the seismic signal

Damping ratio ξ in %	Oscillations Periods T in (s)	Maximum Responses u_{maxPSD} in (cm)
0	2,80	63,86
2	7,00	48,16
4	7,60	40,61
6	8,00	35,75
8	8,00	32,83
10	8,00	30,26

4- Comparison of the results obtained

The purpose of this analysis is not to make a detailed study, but to compare the results obtained by the different approaches. The analysis of the results obtained will make it possible to identify the differences between the spectral responses taken in the same configurations, and to highlight the relationship between them.

Table 3. Analysis of the responses of simple oscillators subjected to the El Centro earthquake

Damping ratio	PS-Duhamel Software (PSD)		Integral Duhamel with linear interpolation seismic signal (DIL)		Percentage difference
ξ in %	Oscillations Periods T in (s)	Maximum responses u_{maxPSD} in (cm)	Oscillations Periods T in (s)	Maximum responses u_{maxDIL} in (cm)	$E = \left(\frac{u_{maxPSD} - u_{maxDIL}}{u_{maxPSD}}\right) \times 100$
0	02,70	57,44	2,80	63,86	-11,18
2	10,90	48,64	7,00	48,16	0,99
4	10,80	43,33	7,60	40,61	6,28
6	10,80	38,88	8,00	35,75	8,05
8	13,25	35,21	8,00	32,83	6,76
10	13,25	32,16	8,00	30,26	5,91
				Medium difference	2,80

At the end of this analysis, we find an average decrease of 2.80% of the seismic responses of our approach compared to PS-Duhamel software responses.

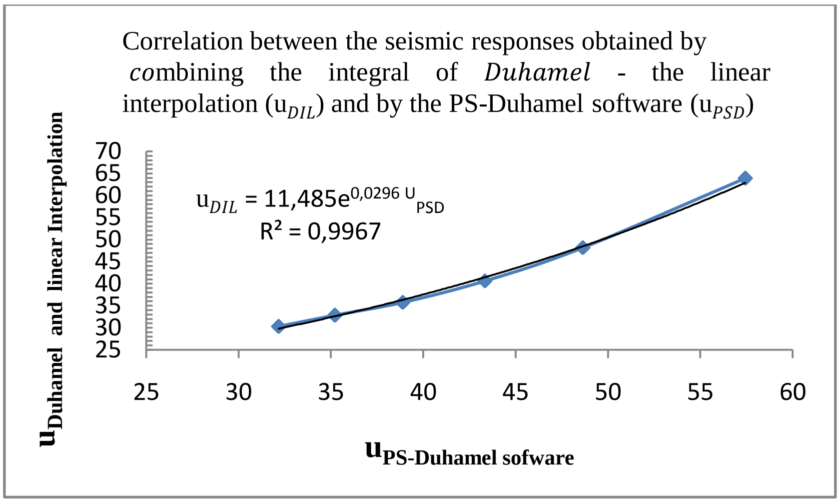

Figure 6 : U_{DIL} seismic response according to the same size U_{PSD}

Figure 6 shows the relationship between the seismic responses of the two approaches studied. Note here that the exponential smoothing function is very close to the real curvature and has a correlation coefficient R^2 close to unity.

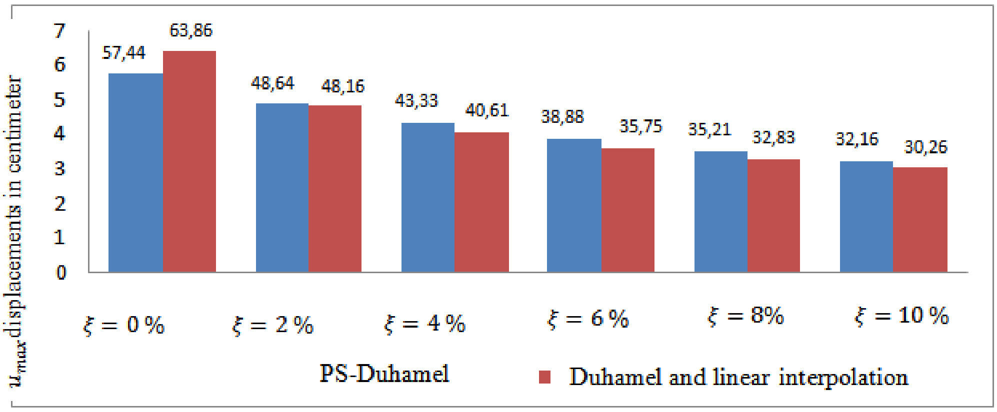

Figure 7 : Comparison of the seismic responses obtained

Figure 7 illustrates the difference between the seismic responses from the two approaches for the same damping ratio.

IV. CONCLUSION

We solve the differential equation of simple oscillator by using the combination of Duhamel integral and the linear interpolation of the seismic signals. We developed an algorithm which allows automatic transformation of all seismic signals to response spectrum. We examined the linear behavior of a simple oscillator using our approach. The results show that when $\xi = 0\ \%$ our result is greater than those obtained. In other cases our results are less than PS_Duhamel's software.

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