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Effects of Asymmetric Boundary Conditions on Plane Couette Flow with Limiting Nusselt Numbers and Viscous Dissipation

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ABSTRACT
Analytical investigation for the limiting value of Nusselt number, including the effect of viscous dissipation on heat transfer for a laminar plane Couette flow between two infinite parallel plates, where the bottom plate is fixed and the top plate is moving in an axial direction at a constant speed. The study concentrates on hydro-dynamically fully developed flow of a Newtonian fluid of constant properties without considering the axial conduction in the fluid. To investigate the effect of viscous dissipation on heat transfer by defining the limiting Nusselt number, plates are kept at constant unequal temperatures. Close form expressions for the limiting Nusselt numbers as a function of the Brinkman number and asymmetry parameter are evaluated. Focus is given to the viscous dissipative effect due to the shear produced by the movable top plate over and above the viscous dissipation due to internal fluid friction. The interactive effects of the Brinkman number and the degree of asymmetry on the limiting Nusselt number are investigated analytically. Specific to the cases considered for this study, the appearance of point of singularities due to the variation of Nusselt number with the Brinkman number is observed, and discussion has been made considering the energy balance, and second law analysis of thermodynamics.
Key words: *Limiting Nusselt number, heat transfer, Plane Couette flow, viscous dissipation, Brinkman number, Newtonian fluid.*

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NOMENCLATURE

Br Brinkman number
c_p Specific heat at constant pressure (J/g K)
$c_1 - c_2$ Constants
D Parameter to characterize $(1-\beta)/(1+\beta)$
h_{1c} Limiting heat transfer coefficient at lower plate (W/m^2-K)
h_{2c} Limiting heat transfer coefficient at upper plate (W/m^2-K)
H Half-channel height (m)
k Thermal conductivity (W/mK)
Nu Nusselt number
Nu_{1c} Nusselt number in the conduction limit at the lower plate
Nu_{2c} Nusselt number in the conduction limit at the upper plate
t Time (s)
$\partial p/\partial x$ Pressure gradient in the x direction (N/m^3)
$\partial p/\partial y$ Pressure gradient in the y direction (N/m^3)
q_{1c} Lower plate heat flux in the conduction limit(W/m^2)
q_{2c} Upper plate heat flux in the conduction limit (W/m^2)
$\dot{S}_{gen}$ Volumetric rate of entropy generation (W/m^{3}K)
T Temperature (K)
$\bar{T}$ Average temperature (K)

T_1 Upper plate temperature (K)

T_2 Lower plate temperature (K)

T_c Temperature in the conduction limit (K)

u Velocity (m/s)

U Dimensionless velocity (m/s)

U_p Velocity of the moving plate (m/s)

x Axial coordinate direction (m)

y Vertical coordinate direction (m)

Y Dimensionless vertical coordinate

Greek symbols

β Degree of asymmetry

θ Dimensionless temperature

θ_m Dimensionless bulk mean temperature

θ_{mc} Dimensionless mean temperature in the conduction limit

μ Dynamic viscosity (kg/m-s)

ρ Density (kg/m^3)

Subscripts

c Conduction limit

f Initial fluid

m Mean

mc Mean in the conduction limit

I. INTRODUCTION

Heat transfer in a plane Couette flow plays a major role in processes such as extrusion, metal forming, glass fibre drawing, and continuous casting, where the moving surface continuously exchanges heat with the surrounding fluid [1]. Although the thermal behaviour of the fluid is important and can affect the quality of the materials processed, viscous dissipation, which is due to work done by viscous forces acting on a fluid, can also bear great significance to processes, where large velocity gradients lead to temperature increases.

The effects of viscous dissipation on forced convective heat transfer have been widely reported in the literature [1-7]. Aydin and Avci [1] investigated the effects of viscous dissipation on fully developed convection heat transfer in pipes subjected to constant heat flux and constant wall temperature, respectively. Temperature profiles, and Nusselt numbers Nu are affected markedly when the Brinkman number Br is large. Exact solutions were obtained for a Graetz problem in studies by Ou and Cheng [2, 3] at the thermal entrance region concerning viscous dissipation effects on forced convection. Aydin [4] solved the same problem but adopted a different solution methodology, an axial conduction was assumed negligible. Aydin and Avci [6] studied heat convection in a Poiseuille flow of a Newtonian fluid and obtained exact solutions. In a study to investigate the effects of moving boundaries, Aydin and Avci [1] solved the temperature profiles analytically in a Couette-Poiseuille flow and showed that the effect of viscous dissipation is significant. Sheela-Francisca and Tso [7] furthered the study by considering asymmetric thermal boundary conditions and produced temperature solutions and Nusselt number expressions, but the results are limited to Newtonian flows with either a fixed or a moving boundary.

There are related studies with emphasis on non-Newtonian flow. Payvar [8] investigated the effects of viscous dissipation on power-law fluid, Bingham plastic fluids, and Ellis fluid for a thermally fully developed forced convection with constant wall heat flux. Davaa et al. [9,10] studied power-law and non-Newtonian fluids in Couette-Poiseuille flow between parallel plates. In [9], exact solutions for the velocity and temperature obtained for flow subjected to constant heat flux applied at the stationary and moving boundaries, respectively. In another study [10], the modified power-law model defined by Irvine and Karni [11] in the governing momentum and energy equations was used to improve the accuracy of the velocity field in the region of lower shear rates. The governing equations were solved numerically for fully developed flows subjected to constant heat flux. Hashemabadi et al. [12] included viscous dissipation effects in a Couette-Poiseuille flow between parallel plates for visco-elastic flow by adopting the simplified Phan-Thien-Tanner model. Tso et al. [13] extended the work in [7] by considering the behaviour of power-law fluids in the analysis of forced convective heat transfer between fixed parallel plates, subjected to asymmetric heating at the top and bottom plate. Sheela-

Francisca et al. [14] derived a semi-analytical solution for the temperature distribution of Couette-Poiseuille flow for pseudo plastic fluids. The temperature distribution and Nusselt number obtained for asymmetric heat flux boundary conditions are greatly affected by heat flux ratio applied to the boundaries together with the velocity of the moving plate, power-law index, modified Brinkman number, an a dimensionless parameter that is the constant of integration. In solving the momentum equation. Chan, Y.H et al. [15] improved the solution method in [14] and provide an analytical solution for heat transfer of a Couette-Poiseuille flow subjected to the asymmetric heat flux boundary condition.

All the researches mentioned above have dealt with the effect viscous dissipation on convective heat transfer in a Poiseuille flow and combined Couette-Poiseuille flow for a hydrodynamically fully developed flow between two parallel plates, considering the thermally fully developed case.

To the best knowledge of the author, Laminar forced convection in the limiting condition, giving the quantitative relation between the different performance index parameters of heat transfer including the viscous dissipation effect for a plane Couette flow between two parallel plates kept at constant unequal temperatures has not been presented in the literature.

The objective of the paper is to analytically investigate the combined effects of the Brinkman number and the degree of asymmetry on the temperature profile.

To this end, detailed analytical study is carried out to investigate the effect of viscous dissipation on the heat transfer for a plane Couette flow for varying degree of asymmetry in the wall heating. Finally, the expressions of the limiting Nusselt numbers are determined from the temperature distribution from the above mentioned condition.

II. GOVERNING EQUATIONS AND ANALYSIS OF THE PROBLEM

The fluid is flowing in the x-direction between two parallel plates where the upper plate is moving with a constant velocity U_p whereas the lower plate is fixed. The plates are $2H$ apart, and the coordinate system is attached with the center line as shown in Fig. 1.

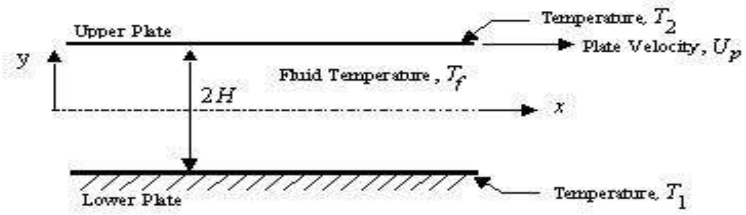

Fig. 1 Schematic Diagram

Following assumptions are made for the analysis:
i) Newtonian fluid;
ii) Incompressible fluid flow;
iii) There is no heat source and thermo-physical properties are constant;
iv) Hydro-dynamically fully developed flow;
v) Axial conduction is neglected in the fluid and through the wall;
vi) Plates are infinitely long in x and z directions.

The governing equations are continuity, momentum and energy equations. To get the velocity and temperature distributions between two plates, the governing equations, namely continuity, momentum and energy equations have been derived based on the above-mentioned assumptions.

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

From the assumptions, there is no velocity in the y-direction,

i.e., $v=0$, which gives

$$\frac{\partial u}{\partial x}=0 \tag{2}$$

Eq. (2) implies that the velocity in the x-direction is a function of y only.

X-momentum Equation:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial u}{\partial t}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

where p is the pressure. Using continuity equation and assumption (iv), one can write the x-momentum equation as follows:

$$\mu\left(\frac{d^2u}{dy^2}\right)=\frac{\partial p}{\partial x}+\rho\frac{\partial u}{\partial t} \tag{4}$$

Now, from y -momentum equation using the above assumptions, it can be shown that

$$\frac{\partial p}{\partial y}=0 \tag{5}$$

Energy Equation

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\frac{\partial T}{\partial t}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\phi \tag{6}$$

Where ϕ is the viscous dissipation term that contains only $\left(\partial u/\partial y\right)^2$. Based on the above assumptions, the energy equation reduces to

$$\rho C_p\left(u\frac{\partial T}{\partial x}+\frac{\partial T}{\partial t}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

2.1 Steady Analysis For The Movable Upper Plate With A Uniform Velocity

The fluid flow is assumed to be due to dragging of the upper plate only. Therefore, Eq. (4) reduces to

$$\mu\left(\frac{d^2u}{dy^2}\right)=C \tag{8}$$

where, C is a constant and this is equal to zero for shear driven flow. No slip condition is assumed at the plates, and thus the boundary conditions are as follows:

at $y=-H,\ u=0$ (9)

and,

at $y=H,\ u=U$ (10)

Solving Eq. (8) with above boundary conditions, the velocity profile is obtained as:

$$u=\frac{U_p}{2}\left(\frac{y}{H}+1\right) \tag{11}$$

However, defining the non-dimensional quantities $U=\frac{u}{U_p}$ and $Y=\frac{y}{H}$, the above velocity profile reduces to,

$$u=\frac{1}{2}\left(1+Y\right) \tag{12}$$

The energy Eq. (6) is written under steady condition as follows:

$$\rho C_p u\frac{\partial T}{\partial x}=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{13}$$

Now to analyze the energy equation in the conduction limit, the following case of unequal constant wall temperatures is considered while neglecting the convection term.

2.1.1 Constant Wall Temperature

For unequal constant wall temperatures, the non-dimensional quantities used as $Y = \frac{y}{H}$, $U = \frac{u}{U_p}$ and defining the dimensionless temperature $\theta = \frac{(T - \bar{T})}{(T_f - \bar{T})}$, where T_f is the initial uniform fluid temperature, the average temperature $\bar{T} = \frac{(T_1 + T_2)}{2}$, and the asymmetry of wall surface temperature $\beta = \frac{(T_2 - T_f)}{(T_1 - T_f)}$. Again the term $(\partial u/\partial y)^2$ is equal $(U_p/H)^2$. Thus the governing equation for constant wall temperatures reduces to,

$$\frac{d^2\theta}{dY^2} + \frac{1}{4}\frac{\mu U_p^2}{(T_f - \bar{T})k} = 0 \tag{14}$$

Defining the Brinkman number, $Br = \frac{\mu U_p^2}{(T_f - \bar{T})}$, Eq. (14) can be further expressed as

$$\frac{d^2\theta}{dY^2} + \frac{Br}{4} = 0 \tag{15}$$

Eq. (15) is subjected to the following boundary conditions:

$$y = H,\ \theta = (T_2 - \bar{T})/(T_f - \bar{T}) \text{ i.e. at } Y = 1,\ \theta = D \tag{16a}$$

and,

$$y = -H,\ \theta = (T_1 - \bar{T})/(T_f - \bar{T}) \text{ i.e. at } Y = -1,\ \theta = -D \tag{16b}$$

Where $$D = \frac{(1-\beta)}{(1+\beta)} \tag{17}$$

The dimensionless temperature profile is obtained by solving Eq. (15) with the above boundary conditions [Eqs. (16a,b].Therefore, Solving Eq. (15) with the above set of boundary conditions of unequal temperatures, the dimensionless temperature profile in the conduction limit is obtained as:

$$\theta_c = -Br\frac{Y^2}{8} + C_1 Y + C_2 \tag{18}$$

The constants C_1 and C_2 of Eq. (18), obtained on applying the boundary conditions given in Eq. (16a,b) are as follows:

$$\left.\begin{aligned} C_1 &= D \\ C_2 &= Br/8 \end{aligned}\right\} \tag{19}$$

However, to obtain the expression of Nusselt Number, it is usual to define the mean temperature, T_m, rather than the centerline temperature in a case of fully developed flow. The mean temperature is given by

$$T_m = \frac{\int_{y=-H}^{H} \rho C_p u T_c w dy}{\int_{y=-H}^{H} \rho C_p u w dy} \tag{20}$$

where 'w' is the width of the channel.

The non-dimensional mean temperature is given by

$$\theta_m = \frac{(T_{mc} - \bar{T})}{(T_f - \bar{T})} = \left(\frac{Br}{12} + \frac{D}{3}\right) \tag{21}$$

The local heat transfer coefficient in the conduction limit at the lower plate can be evaluated using the equation

$$q_{1c} = -k\frac{\partial T}{\partial y}\bigg|_{y=-H} = h_{1c}\left(T_1 - T_{mc}\right) \tag{22}$$

Establishing the non-dimensional quantity, Nusselt number in the conduction limit is 1

$$Nu_{1c} = \frac{h_{1c}H}{k} \tag{23}$$

However, using Eqs.(19), (22) and (23), the expression for Nusselt number at the lower plate in the conduction limit is given by,

$$Nu_{1c} = -\left[1/\left(\theta_1 - \theta_{mc}\right)\right]\left(\frac{\partial \theta_c}{\partial Y}\right)\bigg|_{Y=-1} \tag{24}$$

Similar to Eqs.(22) to (24), the Nusselt number at the upper plate in the conduction limit can be found to be

$$Nu_{2c} = \left[1/\left(\theta_2 - \theta_{mc}\right)\right]\left(\frac{\partial \theta_c}{\partial Y}\right)\bigg|_{Y=1} \tag{25}$$

Where θ_1 , θ_2 used in Eqs. (24) and (25) are the dimensionless temperatures at the lower plate and the upper plate, respectively. Finally, using Eq. (21) for θ_m and Eq. (18) for the derivative of θ_c, the expression of Nusselt numbers on both the plates in the conduction limit are obtained as.

$$Nu_{1c} = \frac{3\left(Br + 4D\right)}{\left(16D + Br\right)} = \frac{3\left[Br + 4\left\{(1-\beta)/(1+\beta)\right\}\right]}{\left[16\left\{(1-\beta)/(1+\beta)\right\} + Br\right]} \tag{26}$$

$$Nu_{2c} = \frac{3\left(4D - Br\right)}{\left(8D - Br\right)} = \frac{3\left[4\left\{(1-\beta)/(1+\beta)\right\} - Br\right]}{\left[8\left\{(1-\beta)/(1+\beta)\right\} - Br\right]} \tag{27}$$

III. RESULTS AND DISCUSSION

The Brinkman number is an important parameter governing the heat transfer and fluid flow in a channel between two parallel plates. The effects of viscous dissipation in a fluid flow and heat transfer is manifested by the representation of Brinkman number. Actually, it is a non-dimensional way of representing the effect of viscous dissipation. In this paper, the effect of Brinkman number for a hydro-dynamically fully developed flow has been analyzed. Figures (2) and (3) depict the variation of the dimensionless temperature profile for different Brinkman numbers for two different cases of asymmetric wall heating.

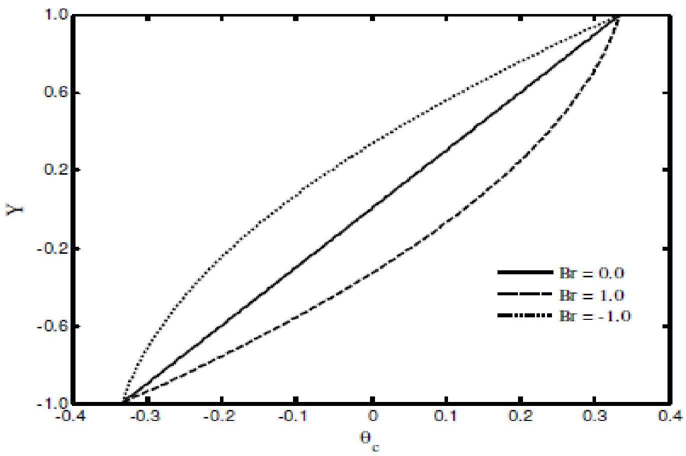

Fig. 2 Dimensionless temperature profile for $\beta = 0.5$, for different values of Br

It is observed from Fig. (2) that specific to the case of $\beta = 0.5$, the dimensionless temperature θ_c strongly depends on the Brinkman number, Br . Viscous dissipation acts as a source of energy in the flow, and this severely influences the temperature distribution in the flow field as seen from Figs. (2) and (3). In the thermal entrance region, a linear trend of developing dimensionless temperature c q is observed for both the case of wall heating for $Br = 0$, which is a pure conduction profile. Therefore, in the absence of viscous dissipation, θ_c varies with the specified values of wall surface temperature imposed on the plates. However, viscous dissipation always generates a distribution of heat source stimulating the internal energy in the fluid, and hence the temperature profile gets distorted as it is clear from the Figs. (2) and (3). A close look on the above figures also reveals that the cases with $Br \neq 0$, the profile of the dimensionless temperature gets altered in comparison to that in the case of $Br = 0$, though the imposed boundary condition on the plates remain invariant. The reason behind such a behavior of the dimensionless temperature profile obtained at different Brinkman numbers Br, is attributable to the effect of viscous dissipation coming into play due to the shear stress within the fluid layer induced by the movement of the upper plate. Positive values of Br are compatible with the wall heating case, which resembles the situation of heat transfer to the fluid across the wall. Therefore, for the cases with positive values of Br, the fluid temperature increases in comparison to the cases where Br is neglected as evident from Figs. (2) and (3). The reverse holds true for the negative values of Br. Equation (19) predicts the dimensionless temperature distribution in the conduction limit for different values of Br, which is shown in Figs. (2) and (3). The corresponding Nusselt numbers at both the plates are defined using Eqs. (24) and (25).

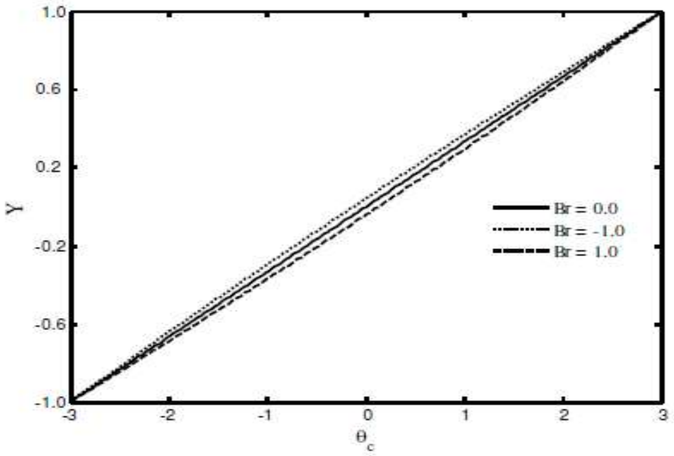

Fig. 3 Dimensionless temperature profile for $\beta = 0.5$, for different values of Br

Figures (4) and (5) are the graphical representation of Nusselt number in the conduction limit on the bottom plate versus Brinkman number Br, specific to the case of asymmetry in the wall heating for $\beta = 0.5$ and $\beta = 0.0$, respectively. Equations (26) and (27) represent the expression of Nusselt number in the conduction limit on the lower plate and upper plate, respectively. It is observed from Equations. (26) and (27) that both the limiting Nusselt numbers are functions of two independent variables, e.g. the degree of asymmetry in wall heating, β and Br. However, both the Nusselt numbers will have parametric variation with Br for $\beta \neq 1$ and with β for $Br \neq 0$. As shown, the variation of Nusselt number with Br is not continuous for the case of $\beta = 0.5$; rather a singularity is observed at $Br = -5.33$, which is very clear and expected from Eq. (26). At this point, the heat supplied by the wall balances the internal heat generation due to viscous dissipation. However, from this point of singularity with the increasing value of Br in the positive direction ($Br > 0$), the Nusselt number decreases because of the decrease in the driving potential of the heat transfer, and finally reaches at $Nu_{1c} = 3$ asymptotically (when $Br \to -\infty$). As explained the negative value of Br represents the wall cooling problem and with the increasing value of Br in the negative direction, Nusselt number decreases and an asymptote appears at $Nu_{1c} = 3$ (when $Br \to -\infty$). The result shows that the Nusselt number maintains a constant value as Br goes to infinity. The expression of Nusselt number in the conduction limit as derived in the study is given in Eq. (26). However, increasing Br will increase the temperature of the flow field, which, in turn, increases the driving temperature difference of the heat transfer, and hence the Nusselt number might alter. These changes may get reflected on the variation of Nusselt number if the convection term is included in energy equation to obtain the closed-form expression for the same. In the limiting condition the effect of increasing Br is not reflected on the variation of Nusselt number Nu_{1c} for a particular degree of asymmetry. This, however, can also be argued mathematically, from Eq. (26).

Equations (26) and (27) also yield that for any given value of b , Nusselt number depends on Br and the limiting values of Nusselt number are not equal (both in magnitude and sign) for given β . The reason behind such inequality is attributable to the movement of the top plate. The movement of the top plate induces additional shear stress, which enhances viscous heating produced by the internal friction between different fluid layers. However, it is very interesting to notice that when Br goes to infinity in either direction (i.e. the cold wall and hot wall case), the Nusselt number attains the same asymptotic
value, $Nu_{1c} = 3$.

Figure (5) also depicts the Nusselt number variation for $\beta = 0.0$. The trend observed here can be explained in the similar fashion as in the case with $\beta = 0.5$. The only difference noticed for this case is the onset of the point of singularity at $Br = -16$, which can be attributed the effect of the degree in asymmetry in the wall heating. At the point of singularity, the limiting values of Nusselt number approaches a large value for both the cases of asymmetry in wall heating at and, respectively. This is because of the equality in the bulk mean

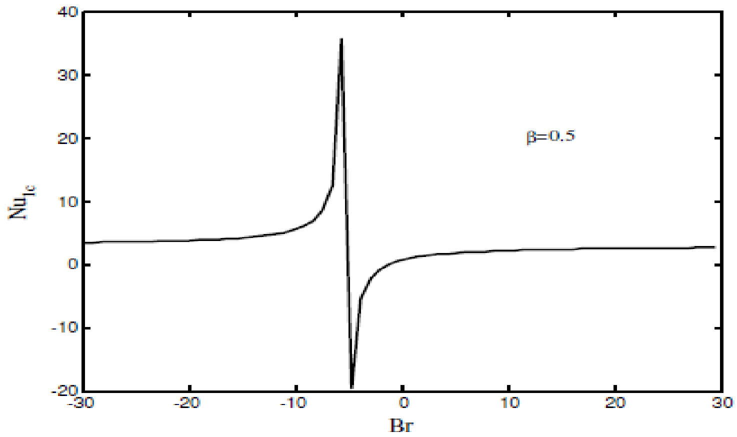

Fig. 4 The influence of Br , on the Nu_{1c} for $\beta = 0.5$

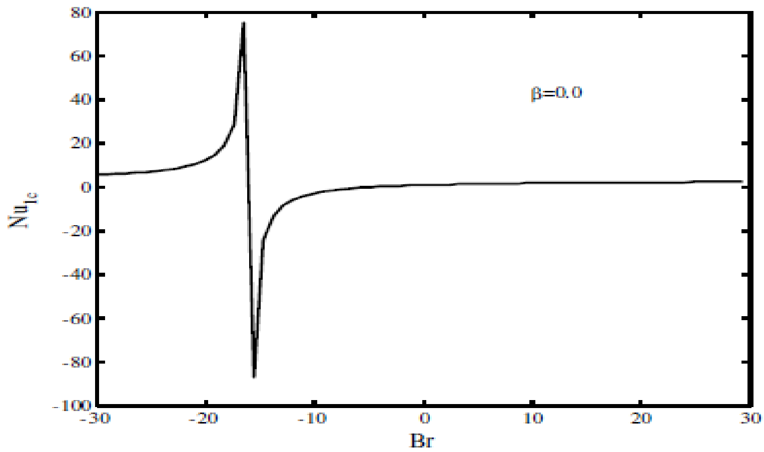

Fig. 5 The influence of Br , on the Nu_{1c} for $\beta = 0.0$

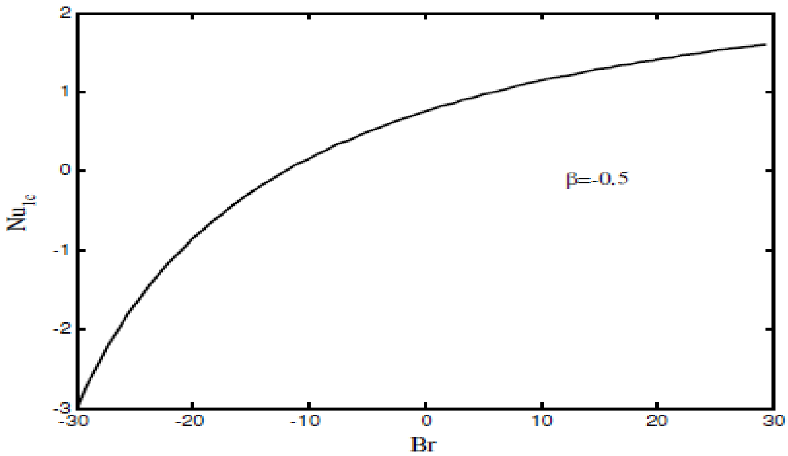

Fig. 6 The influence of Br, on the Nu_{1c} for $\beta = 0.5$

Temperature of the fluid with the average wall surface temperature in the limiting condition. Figure (6) illustrates the effect of Br on the Nusselt number at the asymmetry in wall heating for $\beta = 0.5$. In contrast to the Figs. (4) and (5), a continuous variation of Nusselt number with Br is noticed in Fig. (6). This is due to the degree of asymmetry considered in this case. However, the point of singularity is observed to appear at $Br = -48$, which is an expected result, obtained from Eq. (26) on closely looking into it.
In the present work, heat transfer characteristics in the limiting condition in a viscous dissipative environment are studied thoroughly. The volumetric rate of entropy generation can be expressed as:

$$\dot{S}_{gen} = \frac{k\left(\nabla T\right)^2}{T^2} + \frac{\mu}{T}\phi \tag{28}$$

The first term on the right side of the above equation is attributable the irreversibility due to heat transfer and the second term is the entropy generation due to viscous dissipation. Irreversible energy conversion from frictional heating of viscous dissipation into the fluid has an important bearing on the temperature field of the fluid. In the second law analysis, fluid friction irreversibility arises as a result of viscous heating is of essential importance.

Heat transfer dominates for $0 \le \phi < 1$ and fluid friction dominates when $\phi > 1$. The contributions of both heat transfer and fluid friction to entropy generation are equal when $\phi = 1$.
In the present analysis, three different degrees of asymmetry parameters of wall heating have been considered in investigating the variation of Nu_{1c} as evident from Figs. 4-6. The irreversibility associated with the heat transfer for three different values of asymmetry parameter, β, however, is related to the irreversibility due to viscous dissipation, and hence different points of onset of singularities are observed for different β.

IV. CONCLUSIONS

In the present study, the heat transfer problem for the laminar Couette flow between two plane parallel plates has been studied. The analysis has been done in the conduction limit, when the plates are kept at unequal constant temperatures. The expression for the Nusselt numbers at both the plates in the conduction limit has been obtained for a hydro-dynamically fully developed flow. After finding the velocity distribution in the flow on solving the momentum equation, it is substituted into the energy equation to obtain the expression of Nusselt numbers. In addition to the effect of viscous dissipation due to the internal fluid friction, an emphasis on viscous dissipation is given to include the effect of shear stress induced by the movement of the top plate. A strong influence of viscous dissipation is observed that is quite significant for analysis of heat transfer in the conduction limit. The interactive effects of the Brinkman number and the degree of asymmetry in the wall heating on the limiting values of Nusselt numbers have been investigated in the study. The points of singularities in the variation of Nusselt number versus Brinkman number are also observed. The onset of the points of singularities is seen to be affected by the degree of asymmetry in wall heating, and their sources of appearance have been explained in view of the energy balance, and second law of thermodynamics in the study.

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