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Dynamic Analysis Of A Mdof System Subjected To Inclined Force Components.

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--***ABSTRACT***--
This paper presents a multi degree of freedom (MDOF) Model that analyses a system subjected to dynamic inclined forces. The source of these forces could be from earthquake, wind, etc. In analysis of structures with possibilities of dynamic occurrences, only the normal vertical and lateral force components have been extensively dealt with. However, there are other possibilities of having inclined components of the dynamic force on structures which are neither vertical nor lateral. This paper, therefore, presents an analysis of inclined components of dynamic force. Flexibility influence coefficient method was used in this analysis considering a system with two degrees of freedom. The solution of the formulation is for any MDOF system with 'n' degrees of freedom. From the solution, the natural frequencies, inertia forces were evaluated based on angles of inclination of the force to the system. Dynamic effects of the inclined forces were compared to the vertical force components. From the results obtained, presented in table and graphs, it was observed that there is significant effects from these inclined forces on the natural frequency and the inertia force proving its importance in dynamic analysis.
KEYWORDS: *Multi-degrees of Freedom (MDOF), Natural Frequencies, Inertia Force, Dynamic motion, Resonance.*

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I. INTRODUCTION

When vibration occurs be it from wind or earthquake forces, buildings undergo dynamic motion. Designing buildings to resist dynamic motion requires that they be translated into forces acting upon the buildings [4]. Buildings are always designed to handle normal vertical force components and lateral force components. Earthquake forces are called lateral forces because their predominant effect is to apply horizontal loads to the building and also the vertical force components are considered in special cases.

Some researchers argued that the inclusion of vertical force components is very significant in dynamic analysis especially from earthquake motions as damage to structures was predominantly by vertical motions [5,8,1]. The IS 1893 (part 1) 2002 code specifies that earthquake generated vertical forces effects should be given special attention and considered in designs unless checked and proven by calculation to be not significant.

Beside the dynamic lateral and vertical force components, there also exist force components that are neither lateral nor vertical. They are inclined and these inclined motion components are mostly silent in the dynamic analysis. Analysis by [2,8] showed the value of the vertical ground motion to be one-third to two-third of the horizontal components and argued for their inclusion in the Code. In like manner, there is great need to properly investigate these inclined forces as their behavioural effects and characteristics might be significant to dynamic analysis of the structure as the vertical components.

Common features of all dynamic excitations are that they all generate vibrations in structures upon which they act and these vibrations can be severe and undesirable [7]. The proneness of a structure to dynamic excitation is assessed by the knowledge of the natural frequency of the structure and the associated mode of vibration which enables the structural engineer in evaluating dynamic design parameters and also to predict the likelihood of resonance [3]. The closer the natural frequency is to the frequency of forcing, the more the chances of resonance. Therefore it is desirable to use higher natural frequencies in design.

It is also necessary to apply the principles of dynamics of a structure rather than those of statics in determining structural responses when set in motion. This helps in analyzing inertia forces that are completely absent when static forces are acting on the structure [7].

This paper models an MDOF system of two degrees of freedom for analysing the influence of inclined force components from dynamic motion and this analysis will greatly outline the significance of this force on dynamic motion of structures through the determination of their natural frequencies and forces of inertia at some inclined angles.

II. DESCRIPTION OF THE SYSTEM

The system is modelled as a MDOF beam element with two degrees of freedom as in Fig.1.a. It is acted upon by excitation forces hitting the system at an inclined angle 'ϕ'.

$F_1(t)$, $F_2(t)$, $F_3(t)$ are the amplitude of the external forces acting on the system at an inclined angle 'ϕ' and they are taken as one force '$Q(t)$' = $q_0$sin□t. The deflection effect 'F_{iq}' caused by the external loadings is in of Fig.1.d.

Dynamic displacements of the masses M_1 and M_2 on the system at any arbitrary time are $x_1(t)$ and $x_2(t)$ and since the system is in motion, the forces of inertia developed are shown in Fig.1b. The deflection effect 'δ_{ij}' from the application of unit force at the point of masses M_1 and M_2 in Fig.1 e – f.

Equations of motion for this MDOF system will be derived using the flexibility influence coefficient method from whose solution, will be used to determine natural frequencies and inertia forces of the MDOF system resulting from the inclined dynamic force components.

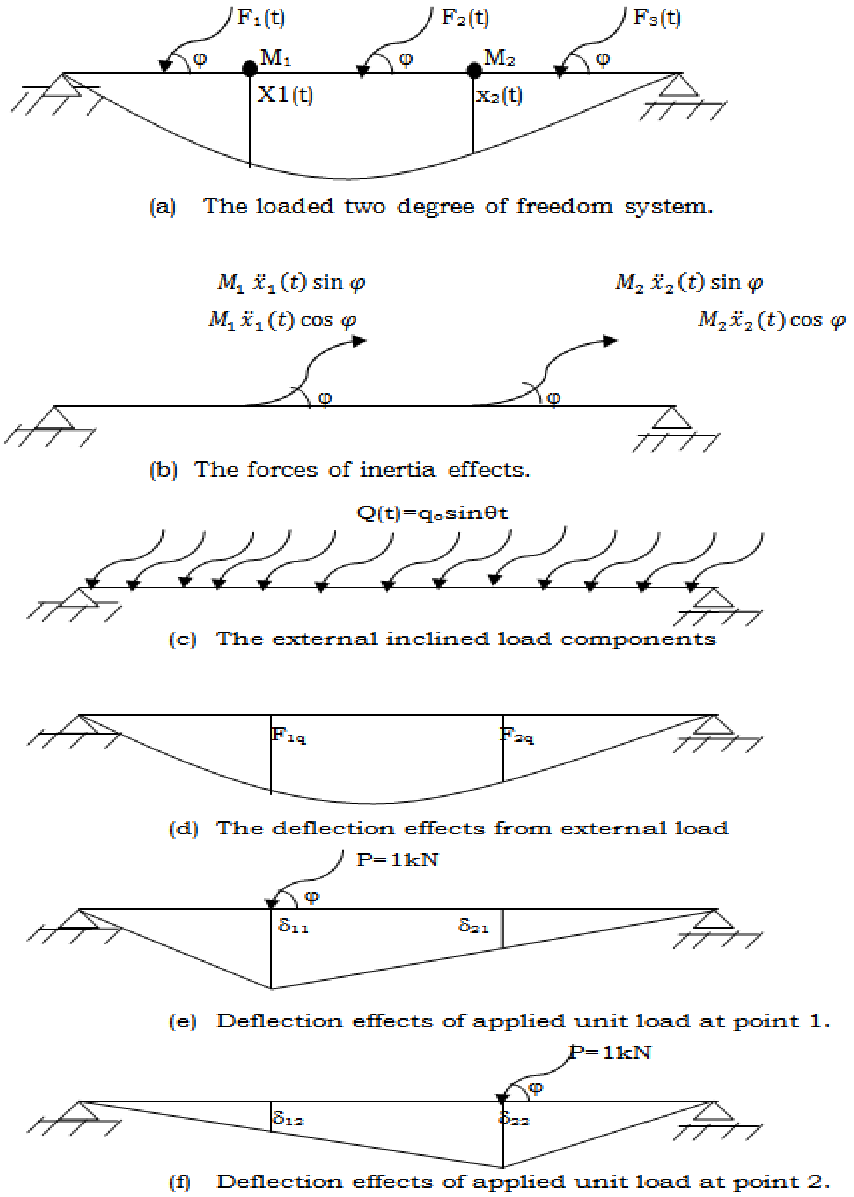

Fig.1: Load effects of the MDOF system.

III. DERIVATION OF THE EQUATIONS OF MOTION

The formulation is to calculate the behaviour of the loaded beam system against inclined force components from dynamic loads, using the superposition principle applied to split forces acting on the system. Equations of motion resulting from the inclined force components are derived using Newton's second law of motion and applying D' Alembert principle, which states that a dynamic problem can be treated as a static problem if the forces of inertia are considered in addition to the imposed load. Applying these descriptions to Fig.1a-f, resulted to the equations below;

$$x_1(t) = (M_1\ddot{x}_1(t)\sin\varphi + M_1\ddot{x}_1(t)\cos\varphi)\delta_{11} - (M_2\ddot{x}_2(t)\sin\varphi + M_2\ddot{x}_2(t)\cos\varphi)\delta_{12} + F_{1q}\sin\theta t \quad (1)$$

$$x_2(t) = (M_1\ddot{x}_1(t)\sin\varphi + M_1\ddot{x}_1(t)\cos\varphi)\delta_{21} - (M_2\ddot{x}_2(t)\sin\varphi + M_2\ddot{x}_2(t)\cos\varphi)\delta_{22} + F_{2q}\sin\theta t \quad (2)$$

In generalised form, equation (1) and (2) gives;

$$x_i(t) = -(M_i\ddot{x}_i(t)\sin\varphi + M_i\ddot{x}_i(t)\cos\varphi)\delta_{ij} + \sum_{j=1}^{n} F_{iq}\sin\theta t \quad (3)$$

Where;

δ_{ij} is the deflection effects on the system at the ith nodal point due to a unit load applied at the jth nodal point.

Equations (1) and (2) are arranged as;

$$x_1(t) + (M_1\ddot{x}_1(t)\sin\varphi + M_1\ddot{x}_1(t)\cos\varphi)\delta_{11} + (M_2\ddot{x}_2(t)\sin\varphi + M_2\ddot{x}_2(t)\cos\varphi)\delta_{12} = F_{1q}\sin\theta t \quad (4)$$

$$x_2(t) + (M_1\ddot{x}_1(t)\sin\varphi + M_1\ddot{x}_1(t)\cos\varphi)\delta_{21} + (M_2\ddot{x}_2(t)\sin\varphi + M_2\ddot{x}_2(t)\cos\varphi)\delta_{22} = F_{2q}\sin\theta t \quad (5)$$

Equations (4) and (5) are equations of motion of an MDOF system with two degrees of freedom from dynamic effects of inclined force components on the system.

As the natural vibration dies off with time, only contribution from the forcing is left. In solution of equations (4) and (5), displacement in steady-state regime is in the form;

$$x_i(t) = C_i\sin\theta t \quad (6)$$

Where;

C_i = amplitude of motion

$\sin\theta t$ = frequency of forcing.

$$\ddot{x}_i(t) = C_i\theta^2\sin\theta t \quad (7)$$

Substituting equations (6) and (7) into (4) and (5) results in;

$$C_1\sin\theta t(M_1C_1\theta^2\sin\theta t\sin\varphi + M_1C_1\theta^2\sin\theta t\cos\varphi)\delta_{11}(M_2C_2\theta^2\sin\theta t\sin\varphi + M_2C_2\theta^2\sin\theta t\cos\varphi)\delta_{12} = F_{1q}\sin\theta t \quad (8)$$

$$C_2\sin\theta\, t(M_1C_1\theta^2\sin\theta t\sin\varphi + M_1C_1\theta^2\sin\theta\, t\cos\varphi)\delta_{21}(M_2C_2\theta^2\sin\theta t\sin\varphi + C_2\theta^2\sin\theta\, t\cos\varphi)\delta_{22} = F_{2q}\sin\theta t \quad (9)$$

Let amplitude of the force of inertia be Z_i

Then;

$$Z_i = M_iC_i\theta^2 \quad or \quad C_I = \frac{Z_i}{M_i\theta^2} \quad (10)$$

But for this system, i = 1, 2. Dividing equations (8) and (9) with 'sinθt', multiplying by -1 and applying equation (10) gives;

$$-\frac{Z_1}{M_1\theta^2} + (Z_1\sin\varphi + Z_1\cos\varphi)\delta_{11} + (Z_2\sin\varphi + Z_2\cos\varphi)\delta_{12} + F_{1q} = 0 \quad (11)$$

$$-\frac{Z_2}{M_2\theta^2} + (Z_1\sin\varphi + Z_1\cos\varphi)\delta_{21} + (Z_2\sin\varphi + Z_2\cos\varphi)\delta_{22} + F_{2q} = 0 \quad (12)$$

But

$$T = \sin\varphi + \cos\varphi \quad (13)$$

$$\delta f_{ii} = [\delta_{ii}(\sin\varphi + \cos\varphi) - \frac{1}{M_i\theta^2}] \quad (14)$$

Substituting equations (13) and (14) into (11) and (12) gives solution of equations (4) and (5) of this system under inclined load component for a two degree of freedom system as;

$$\delta f_{11}Z_1 + T\delta_{12}Z_2 + F_{1q} = 0 \quad (15)$$

$$T\delta_{21}Z_1 + \delta f_{22}Z_2 + F_{2q} = 0 \quad (16)$$

IV. DETERMINATION OF NATURAL FREQUENCIES

Since the system is vibrating under self excitation, there is absence of external forces 'F_{iq}' and frequency of forcing 'θ' will now be 'ω'. The frequency equation is determined from equation (15) and (16) from flexibility formulation;

Thus in generalised form for the two degree of freedom;

$$\delta f_{ii} Z_1 + T\delta_{ij} Z_2 = 0 \tag{17}$$

$$T\delta_{ji} Z_1 + \delta f_{jj} Z_2 = 0 \tag{18}$$

Also equation (14) becomes;

$$\delta f_{ii} = [\delta_{ii}(\sin\varphi + \cos\varphi) - \frac{1}{M_i\omega^2}] \tag{19}$$

Substitute (19) into equation (17) and (18) and putting into frequency matrix where;

$$N = \frac{1}{M_i\omega^2} \tag{20}$$

If flexibility influence coefficients 'δ_{ij}' used in equations (17) and (18) are multiples of the rigidity then;

$$N = \frac{\mathrm{EI}}{\mathrm{M_i}\omega^2} \tag{21}$$

The frequency matrix is;

$$\begin{bmatrix} (\mathrm{T}\delta_{ii} - \mathrm{N}) & \mathrm{T}\delta_{ij} \\ \mathrm{T}\delta_{ji} & (\mathrm{T}\delta_{jj} - \mathrm{N}) \end{bmatrix} \begin{matrix} Z_i \\ Z_j \end{matrix} = 0 \tag{22}$$

Formulation of equation (22) is an eigen-value problem leading to determination of modal characteristic values of 'N' where

$N_1 > N_2 > \ldots\ldots\ldots\ldots N_n.$

For non-trivial solution of equation (22), requires that determinant of matrix factor of the amplitude vector be equal to zero, that is;

$$\text{Det A} = 0 \tag{23}$$

$$\begin{bmatrix} (\mathrm{T}\delta_{ii} - \mathrm{N}) & \mathrm{T}\delta_{ij} \\ \mathrm{T}\delta_{ji} & (\mathrm{T}\delta_{jj} - \mathrm{N}) \end{bmatrix} = 0 \tag{24}$$

From equation (20) and (21), the modal frequencies are;

$$\omega_i = \sqrt{\frac{1}{M_i N_i}} \text{ or } \sqrt{\frac{EI}{M_i N_i}} \quad (i = 1,2) \tag{25}$$

Where: $\omega_1 < \omega_2 \ldots\ldots\ldots\ldots \omega_n$, are natural frequencies of the system.

V. NUMERICAL ANALYSIS

The model system of Fig.2 below is used. It has three forces acting on it to be considered. They include; two lumped masses (M_1 and M_2) of equal mass and the external load.

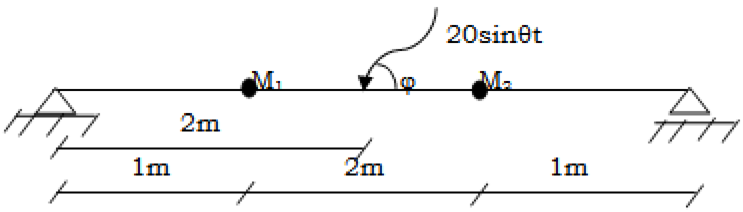

Fig.2: the model system.

Degrees of freedom of the model are two. Rigidity of the system is constant and the inclined force components are at 15°, 30°, 45°, 60°, 75°, 90° respectively to the structure. The superposition principle was applied for effect of the various forces. Forcing frequency 'θ' is half of first and second natural frequency of the inclined angle.

Drawing bending moment diagrams (M_1 and M_2 diagrams) for the two points of lumped masses on the beam by applying inclined load P = 1kN. The bending moment (M_E diagram) due to amplitude of the external load at point of its application also drawn. See Fig.3.

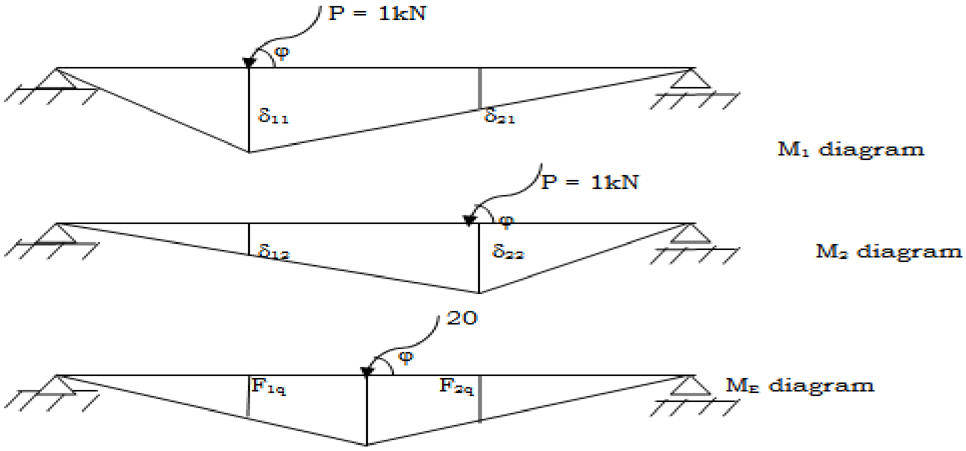

Fig.3: the load effects at inclined angles.

The natural frequencies 'ω' results from this analysis using eqn.(17) and (18) and the inertia forces 'Z' from eqn.(15) and (16), resulting from the dynamic force are presented in table 1 below;

Table: 1.

φ^o	$\omega_1 \sqrt{\frac{EI}{M}}$ (Hz)	$\omega_2 \sqrt{\frac{EI}{M}}$ (Hz)	Z_1 (KN)	Z_2 (KN)	N_1	N_2
15	2.99	9.13	14.88	14.88	0.112	0.012
30	1.47	4.23	13.19	13.19	0.464	0.056
45	1.03	2.89	13.64	13.64	0.950	0.120
60	0.86	2.5	13.77	13.77	1.360	0.160
90	0.86	2.43	19.04	19.04	1.330	0.170

The natural frequency effects against the forces of inclination gave the graphical Fig.4 below where W1 and W2 are the first and the second natural frequencies;

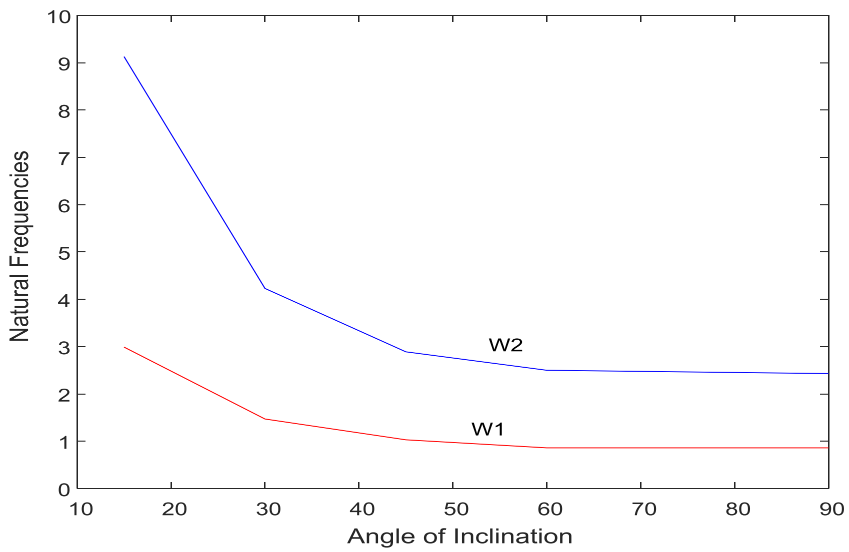

Fig.4: Natural Frequencies Vs Angles of Inclination

Presenting analysis effect of inertia forces against inclination angles graphically in Fig.5 gives;

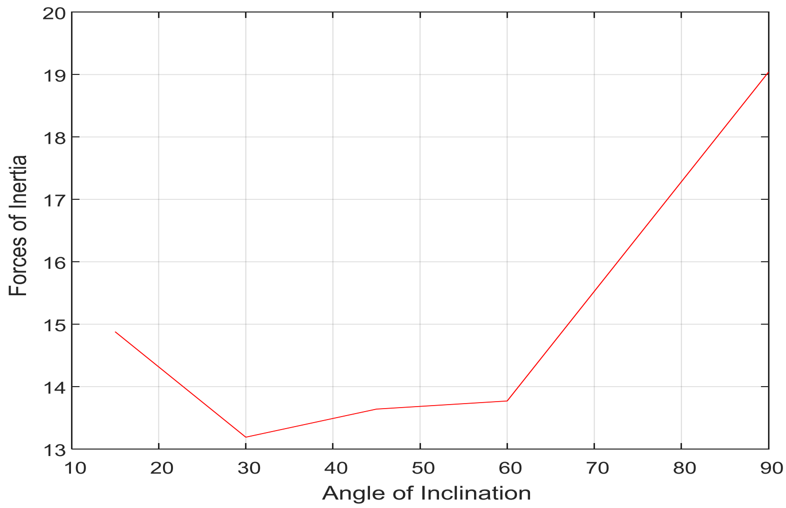

Fig.5: Inertia Forces Vs Angles of Inclination

VI. DISCUSSION OF RESULT AND CONCLUSION

Results obtained from the analysis as presented in table 1, the natural frequencies of the analysed two-degree of freedom system shows that as the angle of inclination of the vibrating force on the system increases from 0^o to 90^o, the natural frequencies of the system decreases (Fig.4). It also shows that inertia force increases as inclination increases Fig.5. As forces goes from inclined components to pure vertical components, the natural frequency decreases showing that inclined force components have higher natural frequencies than pure vertical force acting on the system at 90^o. A modal characteristic (N) of the system which is a function of natural frequencies shows high impact at the pure vertical force components (90^o).

It is better to design using natural frequency that is far higher than the forcing frequency. Buildings with high natural frequencies are safer as high forcing frequency would easily cause buildings to resonate.

It was observed that the effects of inertia was more pronounced when the inclined force is near to the lateral (15^o) or the vertical force (60^o) components (Fig.5) showing that the inclined forces can cause significant change to the structure if not properly guided as overall stability during vibration is the major objective in dynamic analysis.

In conclusion, from the results of this analysis, the inclined force components are capable of changing the overall stability of the system when considering the effects of the natural frequencies. Although the vertical force components have greater effects on force of inertia developed, as some researchers have validated, the inclined force components showed some significant effects and when improved on, its characteristics and structural behaviour could be included the design codes.

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