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Different Nonlinear Control Strategies for a Rigid-Flexible Manipulator Robot

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ABSTRACT

In this paper, are studied, the development of the dynamic model, the stability analysis and the implementation of the control laws of a planar rigid-flexible robot manipulator. The model is generated using the Euler-Lagrange equations associated to the Hamilton's principle. Then a comparative study between performances of PD, fuzzy logic and gain scheduling PD fuzzy controllers applied to the rigid-flexible manipulator to ensure vibration suppression and robustness against disturbances, is presented. To improve the performance level, the stability study is accomplished using the candidate function of Lyapunov. Finally, a set of simulation results is given to compare the three strategies of control in terms of rapidity, stability and desired performances of the rigid-flexible manipulator system.

Keywords *- Rigid-flexible Manipulator, Dynamic Model, PD Control, Fuzzy Logic Control, Gain Scheduling Control.*

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I. INTRODUCTION

The interest in scientific research and technological manipulators for robots has increased considerably in recent years. It is leaded by the needs and demands in automation and industrial requirements. The use of robots is not restricted only to domains where replaced humans, but there are some areas where man can not easily intervene such as underwater environment, space environment, nuclear power plants, etc.

The manipulator robots are available in two kinds, rigid and flexible links manipulators. The recent scientific researchers are performed and focused on flexible manipulators, because they have many advantages over rigid ones: They require less material, are lighter in weight, have higher operational speed, consume lower energy, require smaller actuators, are more maneuverable and transportable, have less overall cost and higher payload to robot weight ratio. Unfortunately, the flexible manipulators have considerable vibration on the free end-point arm which causes less accuracy in the system responses, afterwards, more difficulty to establish a control law of the flexible arm. In order to develop efficient control laws and achieve the information requested of rigid-flexible manipulator, accurate mathematical dynamic models must be determined to show complexity, non-linearity and coupling terms.

Different methods are used for dynamic modeling of the rigid-flexible manipulator, among these methods we can cited the Finite Element Method (FEM) where the flexible arm is discretized into a finite number of similar items and the Hamilton's principle where model is obtained based on theory of beams of Timoshenko and Euler. To design a suitable controller for the rigid-flexible manipulator system, most researchers have tried to develop methods that do not require an accurate model of the system such as fuzzy logic control. To make a comparison between different types of control for the rigid-flexible manipulator system, a conventional PD controller, a direct fuzzy controller and a gain scheduling PD fuzzy controllers with gains normalized by knowledge rule base of a fuzzy system are chosen for study. There are many studies performed on the modeling of flexible arms using the FEM method such as Boucetta R. [1], Saad and al. [2], Baroudi M. [3], Boucetta R. and Bel Hadj Ali S. [4], [5], [6] and Spong and al. [7]. A bibliographical analysis for the dynamics of flexible manipulators can be found in Dwivedy and Eberhard [8]. De Luca and Book [9] presented in detail the modeling of robots with FEM. A study on sources of flexibilities robots series was made by Makarov M. [10]. Many other researchers use the Hamilton's principle for modeling flexible arms as Hamdi S, Boucetta R. and Bel Hadj Ali S. [11], Fung R-F. [12], Chanwikrai S. [13] and Fenili A. [14]. For the control of rigid-flexible manipulator, many regulators were used to control this kind of system, as the fuzzy logic control studied by Kalyoncu and Tinkir [15], Lianfang Tian and Curtis Collins [16], Mamdani [17], Terano T. Asai K. and Sugeno [18], Qiu, Zhi Cheng Wang and Bin Zhang [19] and Zarafshan, Payam Moosavian and Ali A. [20]. Fuzzy PD Control is proposed by Glan Devadhas G. and Lakshmi [21], Zhang, Shuai Zhang and Ya-hong Zhang [22] and Tagee J., Bingul Z. and Kizir S. [23] and classic PD control is studied by Turki Hussein M. and Najeh M. [24] and Oke G. and Istefanopulos Y. [25]. Another kind of control using fuzzy logic and neural networks was proposed by Tinkir M. and Kalyoncu M. [26], the control by neural networks was studied by Irani A.N. and Talebi H.A. [27], Sharma S. K. and Sutton R. [28]. The control by sliding mode of the manipulator is considered by Lee H.H. and Liang Y. [29] and a study on the system trajectory planning is performed by Tarvirdizadeh B. and Alipour K. [30] and Abe A. [31].

A comparison assessment of PD, fuzzy logic and a gain scheduling PD fuzzy controllers of a rigid-flexible manipulator in terms of vibration suppression and disturbance rejection presents the aim of this paper. Section 2 describes the dynamic model of the rigid flexible manipulator. Section 3, 4 and 5 present in details the PD controller, the fuzzy logic control and the gain scheduling controller for the rigid-flexible manipulator system, respectively. Section 6 show a set of simulation results to give a comparison assessment in terms of vibration suppression, stability and accuracy. The paper is ended by a conclusion.

II. DYNAMIC MODEL OF A RIGID-FLEXIBLE MANIPULATOR

Figure 1 shows the outline of the rigid-flexible manipulator robot where L_1, m_1, I_1 and E_1 represent the length, the mass, the moment of inertia of the motor giving the rotation of the rigid arm and the Young's modulus of the rigid link respectively, and L_2, ρ, I_2 and E_2 represent the length, the linear density, the moment of inertia of the motor giving the rotation of the flexible arm and the Young's modulus of the flexible link respectively. τ_1 and τ_2 represent respectively the control torques applied to the center of the joint of the rigid link and the flexible link driven by a DC gear motor. The angular displacement of the rigid link in the horizontal plane OX_0Y_0 is designated by $\theta_1(t)$ and the angular displacement of the flexible link in the horizontal plane OX_1Y_1 is designated by $\theta_2(t)$ and $\omega(r,t)$ is the deflection of the flexible link at position r from the center of articulation measured along the axis O_2X_2. OX_0Y_0, OX_1Y_1 and $O_2X_2Y_2$ represent the right-handed inertial and body frames respectively as shown in the Fig.1.

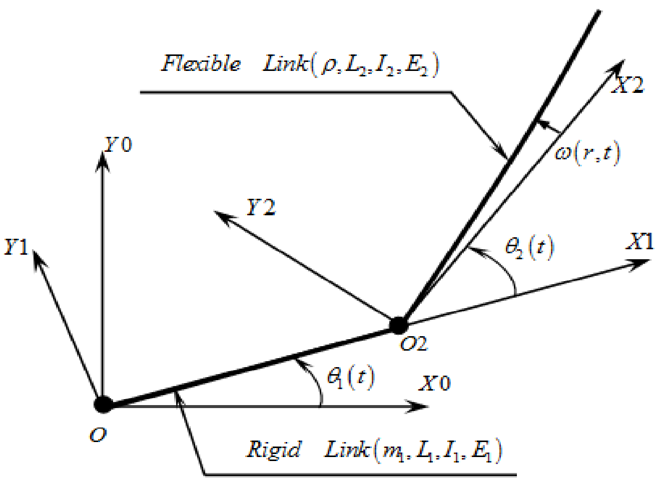


Figure1: Schematic representation of the rigid-flexible manipulator.

To obtain the dynamic equation of a rigid-flexible manipulator, the total kinetic and potential energies of the manipulator system are determined. Then the dynamic model is derived using Lagrange equations associated to the Hamilton's principle. The total kinetic energy **T** of the system is given by the following expression:

$$T=\frac{1}{2}I_1\dot{\theta}_1^2+\frac{1}{2}m_1\left(\frac{L_1}{2}\dot{\theta}_1\right)^2+\frac{1}{2}I_2\left(\dot{\theta}_1+\dot{\theta}_2\right)^2+\frac{1}{2}\rho\int_0^{L_2}\Big[\left(\omega(r,t)\left(\dot{\theta}_1+\dot{\theta}_2\right)\right)^2$$
$$+\left(r\left(\dot{\theta}_1+\dot{\theta}_2\right)\right)^2+\dot{\omega}(r,t)^2-2L_1\dot{\theta}_1\sin(\theta_2)\omega(r,t)\left(\dot{\theta}_1+\dot{\theta}_2\right)+\left(L_1\dot{\theta}_1\right)^2 \qquad (1)$$
$$+2r\left(\dot{\theta}_1+\dot{\theta}_2\right)\dot{\omega}(r,t)+2L_1\dot{\theta}_1\cos(\theta_2)\left(r\left(\dot{\theta}_1+\dot{\theta}_2\right)+\dot{\omega}(r,t)\right)\Big]dr$$

The potential energy **V** of the rigid-flexible manipulator has the following form:

$$V=\frac{1}{2}\int_0^{L_2}EI\left(\frac{\partial^2\omega(r,t)}{\partial r^2}\right)^2 dr \quad (2)$$

So, the Lagrangian **L=T-V** of the rigid-flexible manipulator is expressed as:

$$L=\frac{1}{2}I_1\dot{\theta}_1^2+\frac{1}{2}m_1\left(\frac{L_1}{2}\dot{\theta}_1\right)^2+\frac{1}{2}I_2\left(\dot{\theta}_1+\dot{\theta}_2\right)^2+\frac{1}{2}\rho\int_0^{L_2}\Big[\left(\omega(r,t)\left(\dot{\theta}_1+\dot{\theta}_2\right)\right)^2+\dot{\omega}(r,t)^2$$
$$+\left(r\left(\dot{\theta}_1+\dot{\theta}_2\right)\right)^2-2L_1\dot{\theta}_1\sin(\theta_2)\omega(r,t)\left(\dot{\theta}_1+\dot{\theta}_2\right)+\left(L_1\dot{\theta}_1\right)^2+2r\left(\dot{\theta}_1+\dot{\theta}_2\right)\dot{\omega}(r,t) \qquad (3)$$
$$+2L_1\dot{\theta}_1\cos(\theta_2)\left(r\left(\dot{\theta}_1+\dot{\theta}_2\right)+\dot{\omega}(r,t)\right)\Big]dr-\frac{1}{2}\int_0^{L_2}EI\left(\frac{\partial^2\omega(r,t)}{\partial r^2}\right)^2 dr$$

Then, the deduced Lagrangian is introduced in the Euler-Lagrange equation to obtain the dynamic equation of motion of the rigid-flexible manipulator.

$$Q=-\frac{\partial L}{\partial q}+\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{d^2}{dr^2}\left(\frac{\partial L}{\partial q''}\right) \quad (4)$$

Where $q=\left[\theta_1(t)\quad \theta_2(t)\quad \omega(r,t)\right]^T$ are the variables of the manipulator and Q is the external force vector applied to the system.

Substituting Eq.(3) in the Euler-Lagrange equation, three dynamic equations of motion of the manipulator are obtained as following:

$$\left\{m_1\frac{L_1^2}{4}+\rho L_1^2L_2+\rho L_1L_2^2\cos(\theta_2)+\int_0^{L_2}\left(\rho\omega^2(r,t)-2\rho L_1\sin(\theta_2)\omega(r,t)\right)dr+I_1+I_2\,\rho\frac{L_2^3}{3}\right\}\ddot{\theta}_1$$
$$+\left\{\int_0^{L_2}\left(\rho\omega^2(r,t)-\rho L_1\sin(\theta_2)\omega(r,t)\right)dr+I_2+\rho\frac{L_2^3}{3}+\rho L_1\frac{L_2^2}{2}\cos(\theta_2)\right\}\ddot{\theta}_2+\left(\dot{\theta}_1+\dot{\theta}_2\right)\times \qquad (5)$$
$$\left\{\int_0^{L_2}2\rho\left(\dot{\omega}(r,t)\omega(r,t)-L_1\sin(\theta_2)\dot{\omega}(r,t)\right)dr\right\}-\rho\left\{L_1\frac{L_2^2}{2}\sin(\theta_2)+\int_0^{L_2}L_1\cos(\theta_2)\omega(r,t)dr\right\}\dot{\theta}_2^2$$
$$-\rho\left\{L_1L_2^2\sin(\theta_2)+\int_0^{L_2}2L_1\cos(\theta_2)\omega(r,t)dr\right\}\dot{\theta}_1\dot{\theta}_2+\int_0^{L_2}\rho\left(r+L_1\cos(\theta_2)\right)\ddot{\omega}(r,t)dr=\tau_1$$

$$\left\{I_2+\rho L_1\frac{L_2^2}{2}\cos(\theta_2)+\rho\frac{L_2^3}{3}+\int_0^{L_2}\rho\left(\omega^2(r,t)-L_1\sin(\theta_2)\omega(r,t)\right)dr\right\}\ddot{\theta}_1$$
$$+\left\{I_2+\rho\frac{L_2^3}{3}+\int_0^{L_2}\rho\omega^2(r,t)dr\right\}\ddot{\theta}_2+\int_0^{L_2}2\rho\omega(r,t)\dot{\omega}(r,t)dr\left(\dot{\theta}_1+\dot{\theta}_2\right) \qquad (6)$$
$$+\rho\left\{L_1\frac{L_2^2}{2}\sin(\theta_2)+\int_0^{L_2}L_1\cos(\theta_2)\omega(r,t)dr\right\}\dot{\theta}_1^2+\int_0^{L_2}\rho r\ddot{\omega}(r,t)dr=\tau_2$$

$$\rho\left(r+L_1\cos\left(\theta_2\right)\right)\ddot{\theta}_1+\rho r\ddot{\theta}_2-\rho\omega\left(r,t\right)\left(\dot{\theta}_1+\dot{\theta}_2\right)^2+L_1\dot{\theta}_1^2\sin\left(\theta_2\right)+EI\omega''''\left(r,t\right)+\rho\ddot{\omega}\left(r,t\right)=0 \quad (7)$$

According to the beam theory, a separation of the variables allows to write the flexure variable in the form of modal sum as follows:

$$\omega\left(r,t\right)=\sum_{i=1}^{n}\varphi_i\left(r\right)q_i\left(t\right) \quad (8)$$

Where $q_i\left(t\right)$ are the modal coordinates and $\varphi_i\left(r\right)$ represent the shape modes of the system. To simplify the solution of the problem, the first two modes (n = 2) vibration are considered.

Substituting Eq.(8) in Eq.(5), Eq.(6) and Eq.(7), the dynamic equation of motion of the rigid-flexible manipulator robot can be written in a matrix form as:

$$M\left(q\right)\ddot{q}+D\left(q,\dot{q}\right)\dot{q}+Kq=u \quad (9)$$

Where $M\left(q\right)$, $D\left(q,\dot{q}\right)$, K , q and u have respectively the following expressions.

$$M\left(q\right)=\begin{bmatrix} m_1 & m_5 & m_6 & m_7 \\ m_5 & m_2 & m_8 & m_9 \\ m_6 & m_8 & m_3 & m_{10} \\ m_7 & m_9 & m_{10} & m_4 \end{bmatrix} \quad D\left(q,\dot{q}\right)=\begin{bmatrix} d_1 & d_2 & d_3 & d_4 \\ d_5 & d_6 & d_7 & d_8 \\ d_9 & d_{10} & d_{11} & d_{12} \\ d_{13} & d_{14} & d_{15} & d_{16} \end{bmatrix} \quad K=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix} \quad q=\begin{bmatrix} \theta_1\left(t\right) \\ \theta_2\left(t\right) \\ q_1\left(t\right) \\ q_2\left(t\right) \end{bmatrix} \quad u=\begin{bmatrix} \tau_1 \\ \tau_2 \\ 0 \\ 0 \end{bmatrix}$$

III. PD CONTROLLER OF THE MANIPULATOR

The proportional derivative controller (PD) is a control strategy that allows a closed loop control of an industrial process, where the PD controller compares the measured value of the process (the joint angle of the rigid arm and flexible arm) with set point value of the manipulator system. The error signal which is the difference between these two values is used to calculate the new input value which tends to minimize this difference. Therefore, a stability study is necessary for the rigid-flexible manipulator system with the PD controller in different reference trajectory tracking cases. The rotations of the rigid body and flexible body are assumed to follow a reference trajectory designated by a fifth order polynomial between (t=0s) and (t=T=3s). After this time, the desired output trajectory remains constant at the final value. The initial and final conditions for the desired joints trajectories are: $\theta_{1d}\left(0\right)=0;\quad \theta_{1d}\left(T\right)=\frac{\pi}{2}$ and $\theta_{2d}\left(0\right)=0;\quad \theta_{2d}\left(T\right)=\frac{\pi}{3}$.

In this case, the torques applied to the manipulator arms are described as follows:

$$\begin{cases} \tau_1=\tau_{1r}-K_{p1}\left(\theta_1-\theta_{1r}\right)-K_{d1}\left(\dot{\theta}_1-\dot{\theta}_{1r}\right) \\ \tau_2=\tau_{2r}-K_{p2}\left(\theta_2-\theta_{2r}\right)-K_{d2}\left(\dot{\theta}_2-\dot{\theta}_{2r}\right) \end{cases} \quad (10)$$

Where τ_{1r} and τ_{2r} denote feedforward control inputs determined from the inverse dynamics solution more θ_{1r} and θ_{2r} denote the paths open loop which are the results of the feedforward control, therefore:

$$\begin{aligned} &\left\{m_1\frac{L_1^2}{4}+\rho L_1^2L_2+\rho L_1L_2^2\cos\left(\theta_{2r}\right)+\int_0^{L_2}\left(\rho\omega_r^2\left(r,t\right)-2\rho L_1\sin\left(\theta_{2r}\right)\omega_r\left(r,t\right)\right)dr+I_1+I_2+\rho\frac{L_2^3}{3}\right\}\ddot{\theta}_{1r} \\ &+\left\{\int_0^{L_2}\rho\left(\omega_r^2\left(r,t\right)-L_1\sin\left(\theta_{2r}\right)\omega_r\left(r,t\right)\right)dr+\rho L_1\frac{L_2^2}{2}\cos\left(\theta_{2r}\right)+I_2+\rho\frac{L_2^3}{3}\right\}\ddot{\theta}_{2r}-\rho\left\{L_1\frac{L_2^2}{2}\sin\left(\theta_{2r}\right)\right. \\ &\left.+\int_0^{L_2}L_1\cos\left(\theta_{2r}\right)\omega_r\left(r,t\right)dr\right\}\dot{\theta}_{2r}^2-\rho\left\{L_1L_2^2\sin\left(\theta_{2r}\right)+\int_0^{L_2}2L_1\cos\left(\theta_{2r}\right)\omega_r\left(r,t\right)dr\right\}\dot{\theta}_{1r}\dot{\theta}_{2r}+\left(\dot{\theta}_{1r}+\dot{\theta}_{2r}\right)\times \\ &\left\{\int_0^{L_2}2\rho\left(\dot{\omega}_r\left(r,t\right)\omega_r\left(r,t\right)-L_1\sin\left(\theta_{2r}\right)\dot{\omega}_r\left(r,t\right)\right)dr\right\}+\int_0^{L_2}\rho\left(r+L_1\cos\left(\theta_{2r}\right)\right)\ddot{\omega}_r\left(r,t\right)dr=\tau_{1r} \end{aligned} \quad (11)$$

$$\left\{I_2+\rho L_1\frac{L_2^2}{2}\cos\left(\theta_{2r}\right)+\rho\frac{L_2^3}{3}+\int_0^{L_2}\rho\left(\omega_r^2\left(r,t\right)-L_1\sin\left(\theta_{2r}\right)\omega_r\left(r,t\right)\right)dr\right\}\ddot{\theta}_{1r}$$
$$+\left\{I_2+\rho\frac{L_2^3}{3}+\int_0^{L_2}\rho\omega_r^2\left(r,t\right)dr\right\}\ddot{\theta}_{2r}+\int_0^{L_2}\rho r\ddot{\omega}_r\left(r,t\right)dr+\rho\left\{L_1\frac{L_2^2}{2}\sin\left(\theta_{2r}\right)\right. \quad (12)$$
$$\left.+\int_0^{L_2}L_1\cos\left(\theta_{2r}\right)\omega_r\left(r,t\right)dr\right\}\dot{\theta}_{1r}^2+\int_0^{L_2}2\rho\omega_r\left(r,t\right)\dot{\omega}_r\left(r,t\right)dr\left(\dot{\theta}_{1r}+\dot{\theta}_{2r}\right)=\tau_{2r}$$

$$\rho\left(r+L_1\cos\left(\theta_{2r}\right)\right)\ddot{\theta}_{1r}+\rho r\ddot{\theta}_{2r}-\rho\omega_r\left(r,t\right)\left(\dot{\theta}_{1r}+\dot{\theta}_{2r}\right)^2$$
$$+L_1\dot{\theta}_{1r}^2\sin\left(\theta_{2r}\right)+EI\omega_r^{''''}\left(r,t\right)+\rho\ddot{\omega}_r\left(r,t\right)=0 \quad (13)$$

with $\omega_r\left(r,t\right)=\dot{\omega}_r\left(r,t\right)=\ddot{\omega}_r\left(r,t\right)=\omega_r^{''''}\left(r,t\right)=0$.

To show the stability of the closed loop system manipulator with reference trajectory, Lyapunov method is used where the candidate Lyapunov function **V** is selected as following:

$$V=\frac{1}{2}\left(I_1+M_1\frac{L_1^2}{4}\right)\left(\dot{\theta}_1-\dot{\theta}_{1r}\right)^2+\frac{1}{2}I_2\left(\dot{\theta}_1+\dot{\theta}_2-\dot{\theta}_{1r}-\dot{\theta}_{2r}\right)^2+\frac{1}{2}\rho\int_0^{L_2}\left(\dot{P}-\dot{P}_r\right)^2dr$$
$$+\frac{1}{2}EI\int_0^{L_2}\left(\omega''-\omega_r''\right)^2dr+\frac{1}{2}K_{p1}\left(\theta_1-\theta_{1r}\right)^2+\frac{1}{2}K_{p2}\left(\theta_2-\theta_{2r}\right)^2 \quad (14)$$

with $P=\begin{bmatrix}L_1\cos\left(\theta_1\right)+r\cos\left(\theta_1+\theta_2\right)-\omega\sin\left(\theta_1+\theta_2\right)\\L_1\sin\left(\theta_1\right)+r\sin\left(\theta_1+\theta_2\right)+\omega\cos\left(\theta_1+\theta_2\right)\\0\end{bmatrix}$

and $P_r=\begin{bmatrix}L_1\cos\left(\theta_{1r}\right)+r\cos\left(\theta_{1r}+\theta_{2r}\right)-\omega_r\sin\left(\theta_{1r}+\theta_{2r}\right)\\L_1\sin\left(\theta_{1r}\right)+r\sin\left(\theta_{1r}+\theta_{2r}\right)+\omega_r\cos\left(\theta_{1r}+\theta_{2r}\right)\\0\end{bmatrix}$

are respectively the real and desired position vectors for the manipulator system. K_p is the proportional gain of the PD controller. The Lyapunov function **V** is derived afterwards with respect to time to obtain the following expression:

$$
\begin{aligned}
\dot{V} = & \left(I_1 + M_1 \frac{L_1^2}{4} \right)\left(\dot{\theta}_1 - \dot{\theta}_{1r}\right)\left(\ddot{\theta}_1 - \ddot{\theta}_{1r}\right) + \frac{1}{2} I_2 \left(\dot{\theta}_1 + \dot{\theta}_2 - \dot{\theta}_{1r} - \dot{\theta}_{2r}\right)\left(\ddot{\theta}_1 + \ddot{\theta}_2 - \ddot{\theta}_{1r} - \ddot{\theta}_{2r}\right) + \rho \int_0^{L_2} \{ L_1^2 \dot{\theta}_1 \ddot{\theta}_1 \\
& + L_1^2 \dot{\theta}_{1r} \ddot{\theta}_{1r} + \omega\dot{\omega}\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 + \omega_r \dot{\omega}_r \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right)^2 + \omega^2 \left(\dot{\theta}_1 + \dot{\theta}_2\right)\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + \omega_r^2 \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right)\left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) \\
& - L_1^2 \dot{\theta}_{1r} \ddot{\theta}_1 - L_1^2 \dot{\theta}_1 \ddot{\theta}_{1r} + \left[r\left(\dot{\theta}_1 + \dot{\theta}_2\right) + \dot{\omega} \right]\left[r\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + \ddot{\omega} \right] + \left[r\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) + \dot{\omega}_r \right]\left[r\left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) + \ddot{\omega}_r \right] \\
& + \{ L_1 \ddot{\theta}_1 \omega_r \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) + L_1 \dot{\theta}_1 \dot{\omega}_r \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) + L_1 \dot{\theta}_1 \omega_r \left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) - L_1 \ddot{\theta}_1 \omega \left(\dot{\theta}_1 + \dot{\theta}_2\right) - L_1 \dot{\theta}_1 \dot{\omega} \left(\dot{\theta}_1 + \dot{\theta}_2\right) \\
& - L_1 \dot{\theta}_1 \omega \left(\ddot{\theta}_1 + \ddot{\theta}_2\right) \} \sin\left(\theta_2\right) + \{ L_1 \dot{\theta}_1 \omega_r \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) - L_1 \dot{\theta}_1 \omega \left(\dot{\theta}_1 + \dot{\theta}_2\right) \} \dot{\theta}_2 \cos\left(\theta_2\right) + \{ L_1 \dot{\theta}_{1r} \omega \left(\dot{\theta}_1 + \dot{\theta}_2\right) \\
& - L_1 \dot{\theta}_{1r} \omega_r \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) \} \dot{\theta}_{2r} \cos\left(\theta_{2r}\right) + \{ L_1 \ddot{\theta}_{1r} \omega \left(\dot{\theta}_1 + \dot{\theta}_2\right) - L_1 \ddot{\theta}_{1r} \omega_r \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) - L_1 \dot{\theta}_{1r} \omega_r \left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) \\
& - L_1 \dot{\theta}_{1r} \dot{\omega}_r \left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) + L_1 \dot{\theta}_{1r} \dot{\omega} \left(\dot{\theta}_1 + \dot{\theta}_2\right) + L_1 \dot{\theta}_{1r} \omega \left(\ddot{\theta}_1 + \ddot{\theta}_2\right) \} \sin\left(\theta_{2r}\right) + \{ L_1 \dot{\theta}_1 \left[r\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) + \dot{\omega}_r \right] \\
& - L_1 \dot{\theta}_1 \left[r\left(\dot{\theta}_1 + \dot{\theta}_2\right) + \dot{\omega} \right] \} \dot{\theta}_2 \sin\left(\theta_2\right) + \{ L_1 \ddot{\theta}_1 \left[r\left(\dot{\theta}_1 + \dot{\theta}_2\right) + \dot{\omega} \right] + L_1 \dot{\theta}_1 \left[r\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + \ddot{\omega} \right] - L_1 \ddot{\theta}_1 [\dot{\omega}_r \\
& + r\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right)] - L_1 \dot{\theta}_1 \left[r\left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) + \ddot{\omega}_r \right] \} \cos\left(\theta_2\right) + \{ L_1 \ddot{\theta}_{1r} \left[r\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) + \dot{\omega}_r \right] + L_1 \dot{\theta}_{1r} [r\left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) \\
& + \ddot{\omega}_r] - L_1 \ddot{\theta}_{1r} \left[r\left(\dot{\theta}_1 + \dot{\theta}_2\right) + \dot{\omega} \right] - L_1 \dot{\theta}_{1r} \left[r\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + \ddot{\omega} \right] \} \cos\left(\theta_{2r}\right) + \{ L_1 \dot{\theta}_{1r} \left[r\left(\dot{\theta}_1 + \dot{\theta}_2\right) + \dot{\omega} \right] - \\
& L_1 \dot{\theta}_{1r} \left[\dot{\omega}_r + r\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) \right] \} \dot{\theta}_{2r} \sin\left(\theta_{2r}\right) - \dot{\omega}\omega_r \left(\dot{\theta}_1 + \dot{\theta}_2\right)\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) - \omega\dot{\omega}_r \left(\dot{\theta}_1 + \dot{\theta}_2\right)\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) - \omega\omega_r \times \\
& \left(\dot{\theta}_1 + \dot{\theta}_2\right)\left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) - \left[r\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + \ddot{\omega} \right]\left[r\left(\dot{\theta}_{1r} + \dot{\theta}_{2r}\right) + \dot{\omega}_r \right] - \left[r\left(\dot{\theta}_1 + \dot{\theta}_2\right) + \dot{\omega} \right]\left[r\left(\ddot{\theta}_{1r} + \ddot{\theta}_{2r}\right) + \ddot{\omega}_r \right] \} dr \\
& + EI \int_0^{L_2} \left\{ \left(\omega'' - \omega_r''\right)\left(\dot{\omega}'' - \dot{\omega}_r''\right) \right\} dr + K_{p1}\left(\theta_1 - \theta_{1r}\right)\left(\dot{\theta}_1 - \dot{\theta}_{1r}\right) + K_{p2}\left(\theta_2 - \theta_{2r}\right)\left(\dot{\theta}_2 - \dot{\theta}_{2r}\right)
\end{aligned}
\qquad (15)
$$

The simplification of the Eq.(11), Eq.(12) and Eq.(13) the expression of $\dot{V}$ allows to write:

$$\dot{V} = \left(\dot{\theta}_1 - \dot{\theta}_{1r}\right)\left[\tau_1 - \tau_{1r} + K_{p1}\left(\theta_1 - \theta_{1r}\right)\right] + \left(\dot{\theta}_2 - \dot{\theta}_{2r}\right)\left[\tau_2 - \tau_{2r} + K_{p2}\left(\theta_2 - \theta_{2r}\right)\right] \quad (16)$$

Substituting Eq.(10) into Eq.(16) leads to obtain the derivative of the Lyapunov function as follows.

$$\dot{V} = -K_{d1}\left(\dot{\theta}_1 - \dot{\theta}_{1r}\right)^2 - K_{d2}\left(\dot{\theta}_2 - \dot{\theta}_{2r}\right)^2 \quad (17)$$

From Eq.(14) and Eq.(17), the manipulator system is stable if:

$$K_{p1} \succ 0, \quad K_{p2} \succ 0, \quad K_{d1} \succ 0, \quad K_{d2} \succ 0.$$

The block diagram of the rigid-flexible manipulator system controlled by the PD controller is given by the Fig.2

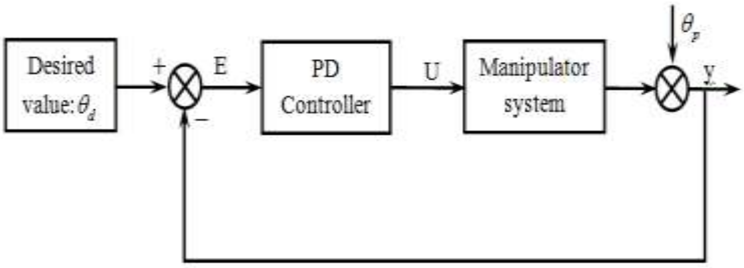


Figure 2: PD controller block diagram.

IV. FUZZY LOGIC CONTROLLER OF THE MANIPULATOR

The concept of fuzzy theory was introduced by Zadeh in 1965 and was used to describe the dynamics of systems that may be complex and sometimes ill-defined for the synthesis of controllers using conventional mathematical modeling techniques. Mamdani applied the fuzzy sets theory for controlling dynamic systems, and since then many more researchers have developed fuzzy logic controller (FLC) for various applications. Generally, a FLC consists of a set of linguistic conditional statements that are derived from human operators and represent the experts' knowledge about the controlled system. These statements define a set of control actions using If - Then rules. The FLC can be considered as a fuzzy reasoning process to mimic the control actions of human operator. The conventional structure of a fuzzy logic controller is consisted of four separate blocks.

- ✓ The fuzzification is the transformation of real variables from the outside world into fuzzy sets. An operator of fuzzification, denoted μ between 0 and 1 is used with variable measurements.

- ✓ The rule base characterized the relations between possible events classes input and the corresponding commands. The number of subsets defining the partition of the order universe of discourse is not necessarily equal to the number of rules.
- ✓ The inference mechanism calculates the fuzzy set concerning the control system from the basic rules and the fuzzy set corresponding to the fuzzification.
- ✓ Finally, defuzzification is intended to transform the fuzzy set of the universe of discourse calculated by the inference mechanism, into not fuzzy value allowing the effective control of the system.

The fuzzy controller constructed to be introduced into the forward path of the closed loop of the rigid-flexible manipulator having two inputs: the error between the joint angle and the desired angle and its derivative, and the output is the torque signal generated. The membership input and output functions are chosen triangular and symmetrical. The inputs of the universe of discourse is divided into five fuzzy sets, and that of output in seven fuzzy sets, all are between [-1, 1]. Normalizing gains are added to adjust the operation of the fuzzy controller.
The basic rules of the fuzzy controller is given by the following tables.

Table 1: Rule base of the fuzzy controller for the rigid arm

U_1		e_1				
		BN	SN	Z	SP	BP
$\dot{e}_1$	BN	BN	BN	MN	SN	Z
	SN	BN	MN	SN	Z	SP
	Z	MN	SN	Z	SP	MP
	SP	SN	Z	SP	MP	BP
	BP	Z	SP	MP	BP	BP

Table 2: Rule base of the fuzzy controller for the flexible arm

U_2		e_2				
		BN	SN	Z	SP	BP
$\dot{e}_2$	BN	BN	BN	MN	SN	Z
	SN	BN	MN	SN	Z	SP
	Z	MN	SN	Z	SP	MP
	SP	SN	Z	SP	MP	BP
	BP	Z	SP	MP	BP	BP

The following figure shows the conventional structure of a fuzzy logic control of the rigid-flexible manipulator system.

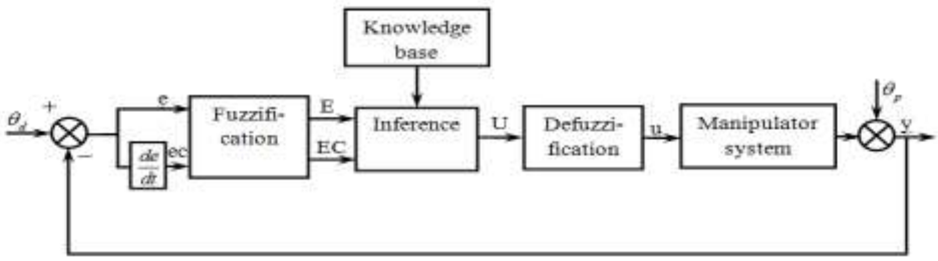


Figure 3: Diagram of fuzzy controller

V. GAIN SCHEDULING PD FUZZY CONTROLLER OF THE MANIPULATOR

The PD control system used with fuzzyfied gains of the rigid-flexible manipulator is a system which allows to make the regulation with online gain normalization according to error and the error variation of manipulator joint variables. K_p, K_d are assumed within prescribed ranges $\left[K_{p\min}, \quad K_{p\max}\right]$ and $\left[K_{d\min}, \quad K_{d\max}\right]$ respectively. So, the controller settings have the following forms:

$$\begin{cases} K_p = \left(K_{p\max} - K_{p\min}\right)\Delta K_p + K_{p\min} \\ K_d = \left(K_{d\max} - K_{d\min}\right)\Delta K_d + K_{d\min} \end{cases} \quad (18)$$

where ΔK_p and ΔK_d are expressed as:

$$\begin{cases} \Delta K_p = \left(K_p - K_{p\min}\right) / \left(K_{p\max} - K_{p\min}\right) \\ \Delta K_d = \left(K_d - K_{d\min}\right) / \left(K_{d\max} - K_{d\min}\right) \end{cases}$$

These parameters are obtained by a set of if-then fuzzy rules. The controller inputs (errors and their derivatives) have five triangular membership functions, while the outputs (ΔK_p and ΔK_d) have Gaussian membership functions. An example of a knowledge rule base for the proportional gain adjustment is presented by the next table. It is the same for the others parameters. The block diagram of the rigid-flexible manipulator system with the Gain Scheduling PD fuzzy controller is shown in the Fig.4.

Table 3: The fuzzy rule base of proportional gain for the rigid arm

K_{p1}		e_1				
		BN	SN	Z	SP	BP
$\dot{e}_1$	BN	Big	Small	Small	Small	Big
	SN	Big	Big	Small	Big	Big
	Z	Big	Big	Big	Big	Big
	SP	Big	Big	Small	Big	Big
	BP	Big	Small	Small	Small	Big

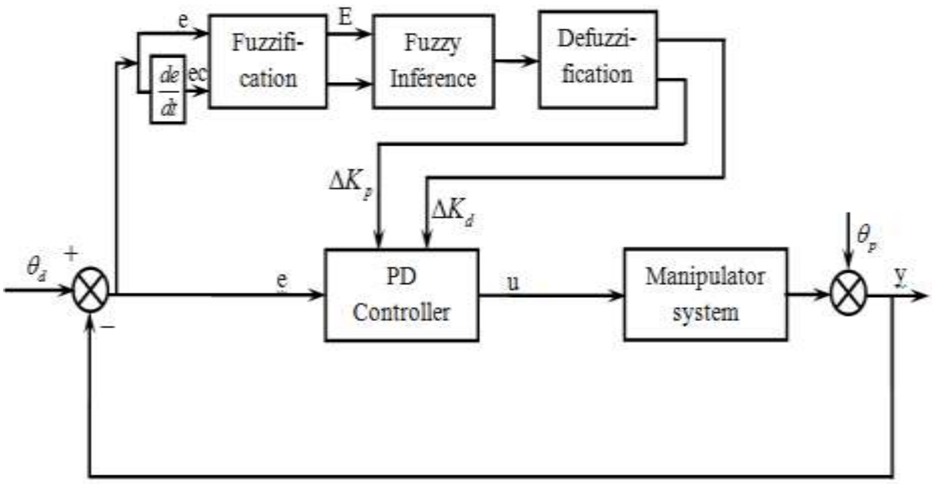


Figure 4: Gain Scheduling PD fuzzy controller block diagram.

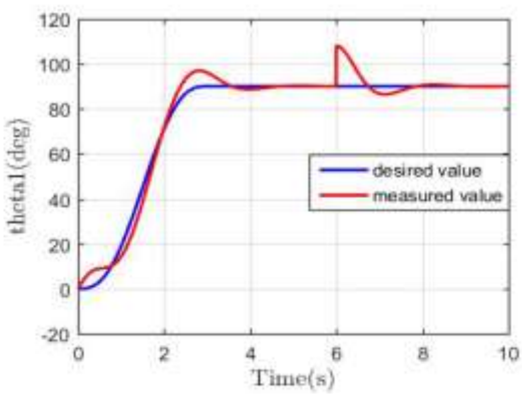
VI. SIMULATION RESULTS

The physical parameters of the rigid-flexible manipulator system used in this work are illustrated in Table.4.

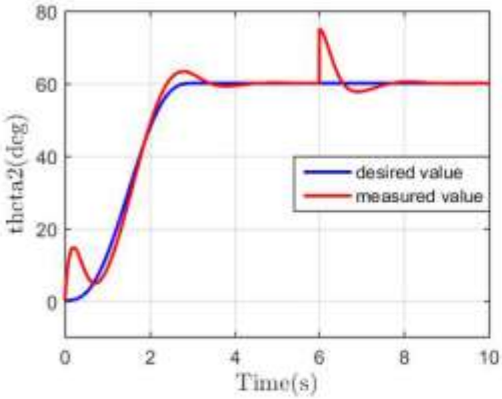
Table 4: Parameters of the rigid-flexible manipulator

Parameter	Symbol	Value
Rigid link length	L_1	0.21 m
Flexible link length	L_2	0.22 m
Mass of rigid link	m_1	0.08 kg
Moment of inertia of rigid link	I_1	0.0082 kg/m^2
Moment of inertia of flexible link hub and motor	I_2	5.0536×10^{-4} kg/m^2
Flexural rigidity	EI	0.1143 N/m^2
density of flexible link	ρ	0.0182 kg/m

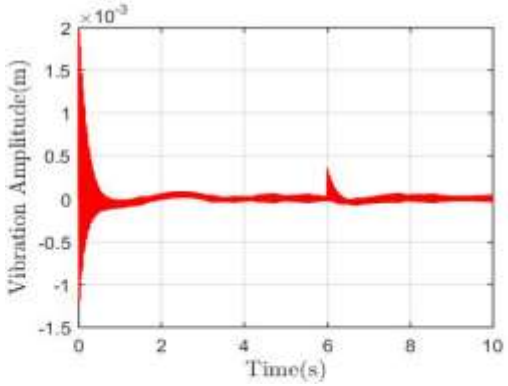
To simulate the closed loop system, Particle Swarm Optimization (PSO) has been used to optimize the gains of the PD controller that allows us to obtain the following parameters $K_p = [0.1 \quad 0.02]$ and $K_d = [0.03 \quad 0.005]$, the parameters of the gain scheduling PD fuzzy controller are varied in the following intervals $K_{p1} \in [0.05 \quad 0.15]$, $K_{p2} \in [0.005 \quad 0.035]$, $K_{d1} \in [0.01 \quad 0.05]$, $K_{d2} \in [0.001 \quad 0.009]$ and for the fuzzy controller we use for each joint two inputs and one output. The two inputs have 5 triangular membership functions, the output has 7 membership functions. The knowledge base was illustrated in Table.1 and Table.2 and the responses of the rigid-flexible manipulator system with different controllers are illustrated by the following figures.



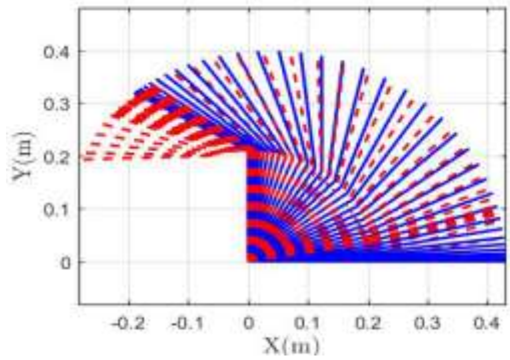
(a) Evolution of the first joint $\theta_1(t)$



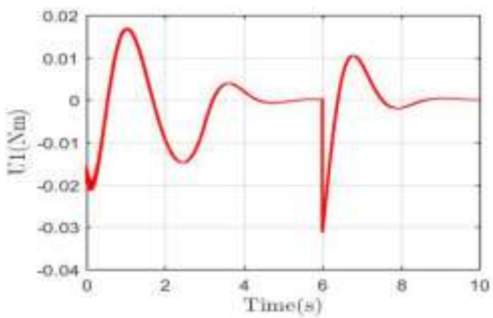
(b) Evolution of the second joint $\theta_2(t)$



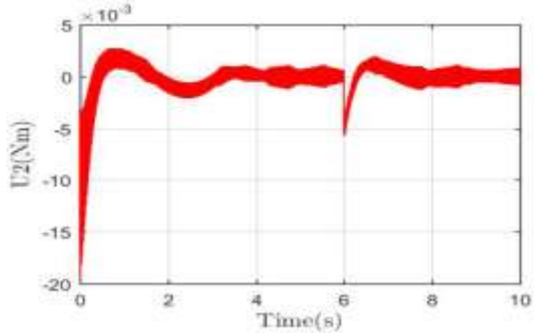
(c) Evolution of the flexibility $\omega(r,t)$



(d) Animation of the rigid-flexible manipulator

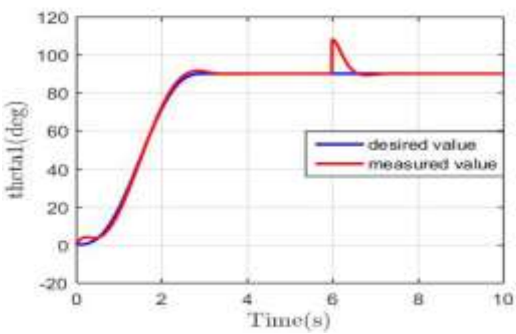


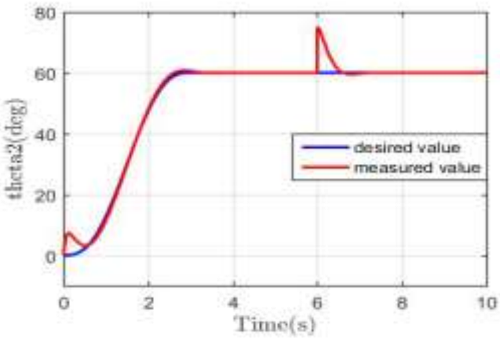
(e) Control signal of the rigid arm



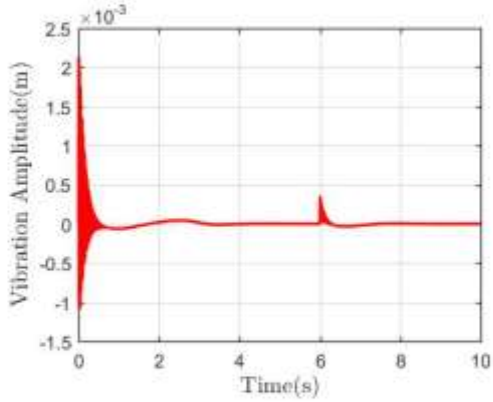
(f) Control signal of the flexible arm

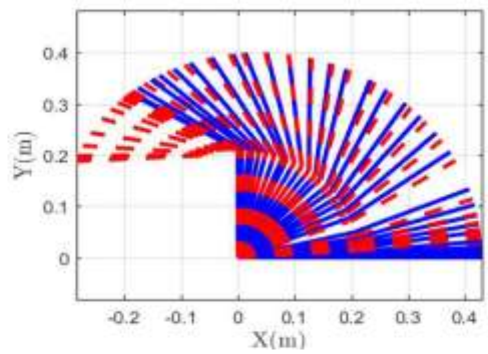
Figure 5: Rigid-flexible manipulator responses with PD controller.



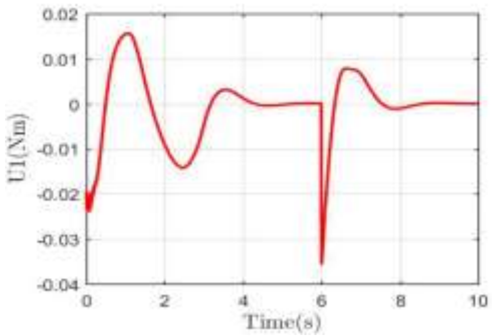


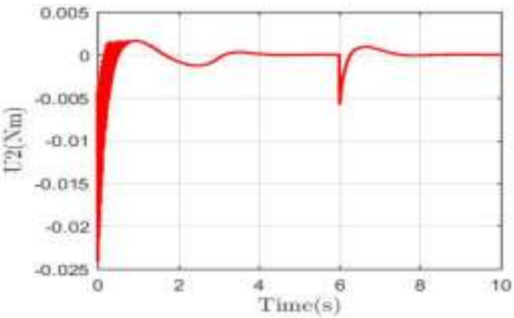
(a) Evolution of the first joint $\theta_1(t)$ (b) Evolution of the second joint $\theta_2(t)$



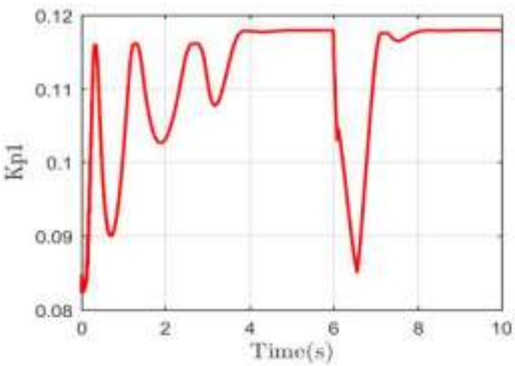


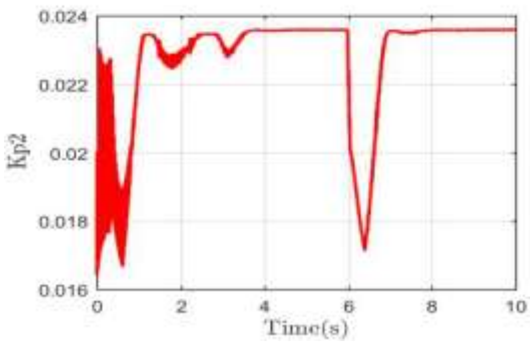
(c) Evolution of the flexibility $\omega(r,t)$ (d) Animation of the rigid-flexible manipulator



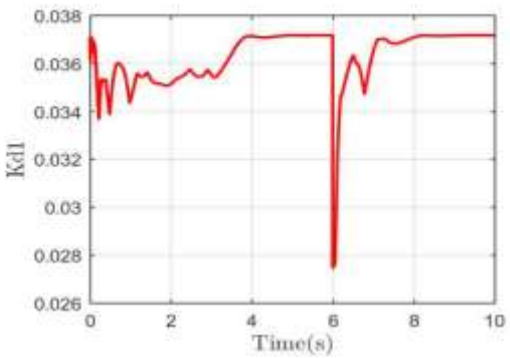


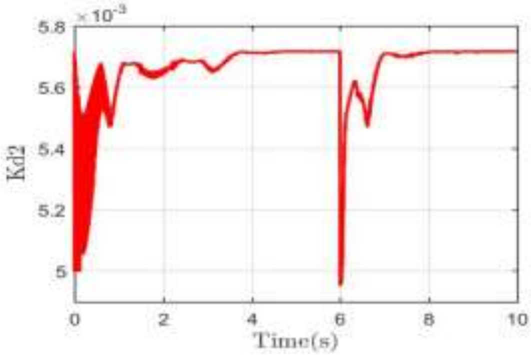
(e) Control signal of the rigid arm (f) Control signal of the flexible arm





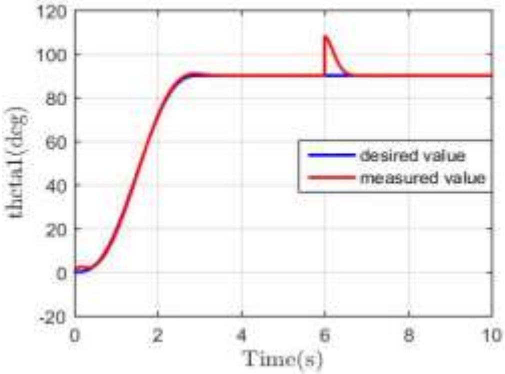
(g) Evolution of K_{p1} (h) Evolution of K_{p2}



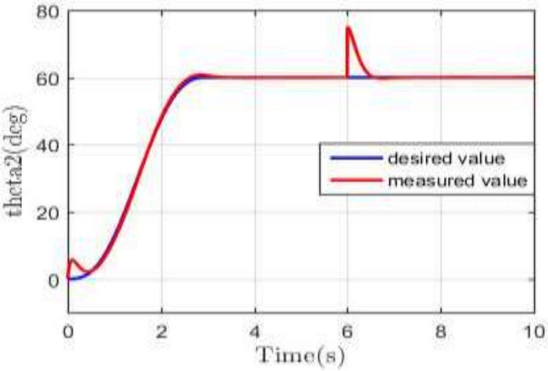


(i) Evolution of K_{d1} (j) Evolution of K_{d2}

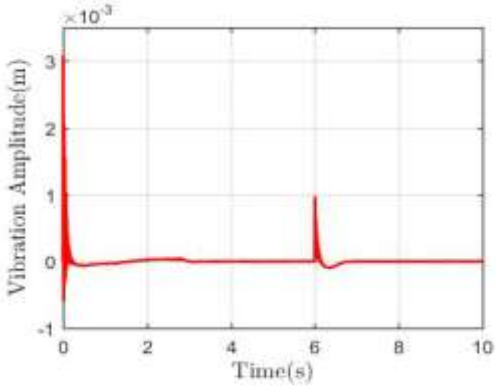
Figure 6: Rigid-flexible manipulator responses with gain scheduling PD fuzzy controller.



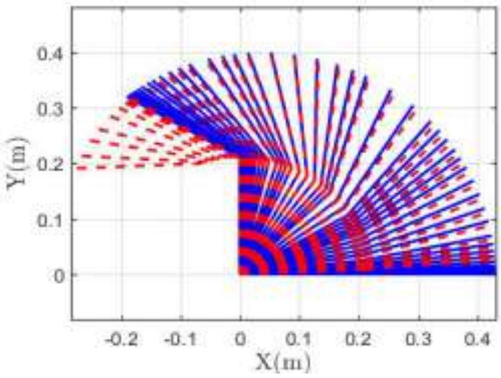
(a) Evolution of the first joint $\theta_1(t)$



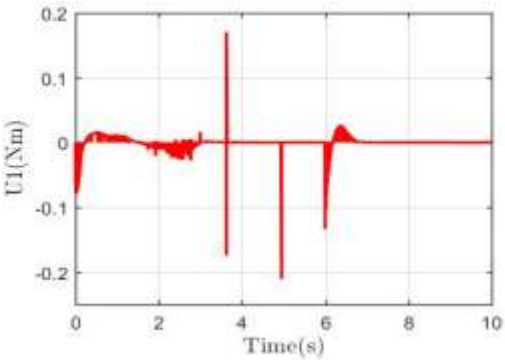
(b) Evolution of the second joint $\theta_2(t)$



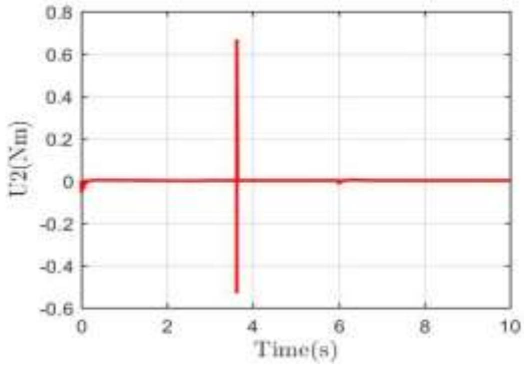
(c) Evolution of the flexibility $\omega(r,t)$



(d) Animation of the rigid-flexible manipulator



(e) Control signal of the rigid arm



(f) Control signal of the flexible arm

Figure 7: Rigid-flexible manipulator responses with fuzzy controller.

The assessment comparison between PD, fuzzy logic and gain scheduling PD fuzzy controllers in terms of performances shows that the FLC and gain scheduling is more efficient than PD controllers regarding path track and disturbance rejection. Moreover, the FLC satisfied vibration suppression in the end-point flexible arm. The command signal applied via PD and gain scheduling controllers has a lower magnitude compared to the FLC, which means an outstanding reduced energy consumption in this case.

VII. CONCLUSION

In this work, a novel kind of controller is developed to control a rigid-flexible manipulator wherein PD parameters are performed by a fuzzy system. The controller is implemented and compared with a simple PD controller and a direct fuzzy logic control.

A comparison assessment has been elaborated in terms of trajectory tracking performance and low control signal reached to minimize energy consumption, the gain scheduling PD fuzzy controller is more efficient compared to the others controllers.

Finally, a set of simulation results are given to show the performance at the level of the two joint angles of the rigid-flexible manipulator system in terms of the vibration suppression and robustness against disturbances .

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