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# Relation Between Niño3.4 And SOI By Wavelet-Based Multifractal Analysis

Fumio Maruyama*
*Faculty of Human Health Science, Matsumoto University, Matsumoto, 390-1295, Japan*
*Corresponding Author: Fumio Maruyama*

**-------------------------------------------------------*ABSTRACT*-------------------------------------------------------**

*In general, fractal properties may be observed in the time series of the dynamics of complex systems. The purposes of this study are to investigate the relations between the Niño3.4 index and the Southern Oscillation Index (SOI), which are both ENSO indices, from a view of multi-fractality. The Niño3.4 index and SOI are obtained from sea surface temperature and sea level pressure, respectively. To detect the changes of multifractality, we performed the multifractal analysis on the time series of the Niño3.4 index and SOI. Between 1950 and 2010, for the Niño3.4 and SOI, the matching in monofractality or multifractality was observed and the increase and decrease of multifractality were very similar especially for the 1970s, that is the change of multifractality were very similar. When the El Niño occurred the fractality of the Niño3.4 and SOI changed from multifractality to monofractality for the El Niño in 1972-1973, 1982-1983, 1991-1992, 1997-1998 and 2002-2003. When the La Niña occurred the fractality of the Niño3.4 and SOI changed from multifractality to monofractality for the La Niña in 1955-1956, 1973-1976 and 1998-2000. When the climate changed the climatic index was multifractal and when the climate became a certain state the climate index was monofractal. Hence, the unstable state shows multifractal and the stable state shows monofractal. By the wavelet coherence and phase using the Morlet wavelet and the change of fractality, the similarity of Niño3.4 index and SOI was shown. The multifractal analysis used in this study is effective as a method to analyze the complicated time series data.*

***KEYWORDS:*** *Niño3.4, SOI, global SST, wavelet, multifractal*

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## I INTRODUCTION

The Southern Oscillation Index (SOI) is a standardized index based on the observed sea level pressure differences between Tahiti and Darwin, Australia. The SOI is one measure of the large-scale fluctuations in air pressure occurring between the western and eastern tropical Pacific (i.e., the state of the Southern Oscillation) during El Niño and La Niña episodes. In general, smoothed time series of the SOI correspond very well with changes in ocean temperatures across the eastern tropical Pacific. The negative phase of the SOI represents below-normal air pressure at Tahiti and above-normal air pressure at Darwin. Prolonged periods of negative (positive) SOI values coincide with abnormally warm (cold) ocean waters across the eastern tropical Pacific typical of El Niño (La Niña) episodes. Alexander et al. (2002) showed that anomalous tropical convection induced by ENSO influences global atmospheric circulation and hence alters surface fluxes over the North Pacific. This atmospheric bridge occurs by changes in the Hadley and Walker cells and Rossby waves and surface heat fluxes are the key component.

Various objects in nature show the so-called self-similarity or fractal property. Monofractal shows an approximately similar pattern at different scales and is characterized by a fractal dimension. Multifractal is a non-uniform, more complex fractal and is decomposed into many sub-sets characterized by different fractal dimensions. Fractal property can be observed in the time series representing dynamics of complex systems as well. A change of fractality accompanies a phase transition and changes of state. The multifractal properties of daily rainfall were investigated in two contrasting climates: an East Asian monsoon climate with extreme rainfall variability and a temperate climate with moderate rainfall variability (Svensson et al., 1996). In both the climates, the frontal rain shows monofractality and the convective-type rain shows multifractality.

Hence, climate change can be interpreted from the perspective of fractals. A change of fractality may be observed when the climate changes. We attempt to explain changes in climate, referred to as regime shifts, by analyzing fractality. We use the wavelet transform to analyze the multifractal behavior of the climate index. Wavelet methods are useful for the analysis of complex non-stationary time series. The wavelet transform allows reliable multifractal analysis to be performed (Muzy et al., 1991). In terms of the multifractal analysis, we conclude that a climatic regime shift corresponds to a change from multifractality to monofractality of the Pacific Decadal Oscillation (PDO) index (Maruyama et al., 2015) and showed the influence of solar activity on climatic regime shift (Maruyama et al., 2017).

We examined the relation between the Niño3.4 index and SOI in this study. To detect the changes of multifractality, we examine the multifractal analysis on the Niño3.4 index and SOI, using the wavelet transform. Moreover we examined the wavelet coherence and phase of those indices. To investigate the relations between the Niño3.4 and SOI from a view of multi-fractality, we studied the wavelet coherence and phase between them, and the change of fractality.

## II DATA AND METHOD OF ANALYSIS

The Niño3.4 index and SOI provided by NOAA's Climate Prediction Center, USA (CPC) were used. The monthly Niño3.4 index, which is a measure of the amplitude of an ENSO event, is defined as the monthly sea surface temperature (SST) averaged over the tropical Pacific areas, (5°N–5°S, 120–170°W). The Southern Oscillation Index (SOI) is a standardized index based on the observed sea level pressure differences between Tahiti and Darwin, Australia. The SOI is one measure of the large-scale fluctuations in air pressure occurring between the western and eastern tropical Pacific (i.e., the state of the Southern Oscillation) during El Niño and La Niña episodes. Prolonged periods of negative (positive) SOI values coincide with abnormally warm (cold) ocean waters across the eastern tropical Pacific typical of El Niño (La Niña) episodes. Average global sea surface temperature (SST) obtained from Met Office Hadley Centre was used.

We used the Daubechies wavelet as the analyzing wavelet because it is widely used in solving a broad range of problems, e.g., self-similarity properties of a signal or fractal problems and signal discontinuities. The data used were a discrete signal that fitted the Daubechies Mother wavelet with the capability of precise inverse transformation. Hence, precisely optimal value of $\tau(q)$ could be calculated as explained below. We then estimated the scaling of the partition function $Z_q(a)$, which is defined as the sum of the $q$-th powers of the modulus of the wavelet transform coefficients at scale $a$. In our study, the wavelet-transform coefficients did not become zero, and therefore, for a precise calculation, the summation was considered for the entire set. Muzy et al. (1991) defined $Z_q(a)$ as the sum of the $q$-th powers of the local maxima of the modulus to avoid division by zero. We obtained the partition function $Z_q(a)$ :

$$Z_q(a) = \sum \left| W_\varphi [f](a,b) \right|^q , \qquad (1)$$

where $W_\varphi[f](a, b)$ is the wavelet coefficient of the function $f$, $a$ is a scale parameter and $b$ is a space parameter. The time window was set to six years for the following outlined reasons. We calculated the wavelets using a time window of various periods, 10, 6 and 4 years. For a time window of 10 years, a slow change of fractality was observed. Thus, this case was inappropriate to find a rapid change of regime shift because when we integrated the wavelet coefficient over a wide range, small changes were canceled. For four years, a fast change of fractality was observed. The overlap of the first and subsequent data was 3 years, which is shorter than the 9 years in the case of the 10-year calculation and thus the change of fractality was large. For six years, a moderate change of fractality was observed and hence the time window was set to this period. For small scales, we expect

$$Z_q(a) \sim a^{\tau(q)} \quad . \qquad (2)$$

First, we investigate the changes of $Z_q(a)$ in time series at a different scale $a$ for each $q$. A plot of the logarithm of $Z_q(a)$ against the logarithm of time scale $a$ was created. Here $\tau(q)$ is the slope of the linear fitted line on the log–log plot for each $q$. Next, we plot $\tau(q)$ vs $q$. The time window was then shifted forward one year and the process repeated. We define monofractal and multifractal as follows: if $\tau(q)$ is linear with respect to $q$, then the time series is said to be monofractal: if $\tau(q)$ is convex upwards with respect to $q$, then the time series is classified as multifractal (Frish and Parisi, 1985). We define that the value of $R^2$, which is the coefficient of determination, for fitting straight line, if $R^2 \geq 0.98$ the time series is monofractal and if $0.98 > R^2$ that is multifractal.

We calculated $\tau(q)$ of different moments $q$ for individual records for the Niño3.4 index. In Fig. 1, $\tau(q)$ between 1980 and 1994 is shown. The data were analyzed in six-year sets, e.g., $\tau(q)$ of n80 was calculated for the 1980–1985 period, and that of n81 was calculated for the 1981–1986 period. To examine the change of fractality, the time window was then shifted forward one year and $\tau(q)$ was calculated from n80 up to n89. A monofractal signal would correspond to a straight line for $\tau(q)$, while $\tau(q)$ would be nonlinear for a multifractal signal. Most of the multifractality observed is due to the negative value of $q$, i.e., small fluctuations are more inhomogeneous than large fluctuations. In Fig. 1, the data sets were monofractal in the cases of n80, n81, n82, n85, n86, n87, and n89 and were multifractal in the cases of n83, n84, and n88.

We plot the value of the $\tau(-6)$ in each index. The negative large values of the $\tau(-6)$ show large multifractality. For the $\tau(q)$, $q = -6$ is the appropriate number to show the change of $\tau$. The value of the $\tau(-6)$

does not always correspond to the fractality obtained from the value of $R^2$.

## III RESULTS AND DISCUSSION

### 3. 1. The relation between the Niño3.4 and SOI

The Niño3.4 index (top), and SOI (bottom) are shown in Fig. 2. The changes of Niño3.4 index, and SOI is reversed. The Niño3.4 index increase, so SOI decreased.

Wavelet power spectra of Niño3.4 index (top) and SOI (bottom) are shown in Fig. 3.

For Niño3.4, wavelet power spectra is strong in the 1980s in 4 year scale. For SOI, wavelet power spectra is strong in the 1980s in 4 year scale and for 1990–2010 in 11 year scale. The period of ENSO was about 4 years. For SOI, the influence of solar activety, the period of 11 years, was shown strongly.

We investigated the relation between the Niño3.4 and SOI. The $\tau(-6)$ of the Niño3.4 index, and SOI are shown in Fig. 4 (top). The red square shows monofractality and the green circle shows multifractality for the 6 years centered on the year plotted. For instance, the green circle for 1980 in the SSN shows multifractality between 1977 and 1982. The data was excluded from Fig. 4 (top) for cases where we could not distinguish between monofractality and multifractality. Between 1950 and 2010, especially for the 1970s, for the Niño3.4 and SOI, the matching in monofractality or multifractality was observed and the increase and decrease of multifractality were very similar, that is the change of multifractality were very similar.

We studied the relationship between the Niño3.4 and SOI by means of wavelet coherence, phase and fractality. We show the wavelet coherence and phase using the Morlet wavelet between the Niño3.4 and SOI in Fig. 4 (middle) and (bottom), respectively. The coherence between the Niño3.4 and SOI in 2–4 year scale was strong for 1960–2010, when the changes of the fractality both are very similar. The phases difference of Niño3.4 and SOI is $\pi$ and the lead of the Niño3.4 index was observed, which corresponds to the reverse change.

When the El Niño occurrsed the fractality of the Niño3.4 and SOI changed from multifractality to monofractality for El Niño in 1972-1973, 1982-1983. 1991-1992, 1997-1998 and 2002-2003. When the La Niña occurrsed the fractality of the Niño3.4 and SOI changed from multifractality to monofractality for La Niña in 1955-1956, 1973-1976 and 1998-2000. This shows that when the climate changed the climatic index was multifractal and when the climate became a certain state the climate index was monofractal. Hence, the unstable state shows multifractal and the stable state shows monofractal.

### 3.2. The relation between the SOI and SST

We investigated the relation between the global average SST and SOI. The $\tau(-6)$ of the SOI, and SST are shown in Fig. 5 (top). The data was excluded from Fig. 5 (top) for cases where we could not distinguish between monofractality and multifractality. Between 1950 and 2010 especially for the 1980s and 1990s, for the SST and SOI, the change of multifractality are very similar.

We studied the relationship between the SST and SOI by means of wavelet coherence, phase and fractality. We show the wavelet coherence and phase using the Morlet wavelet between the SST and SOI in Fig. 5 (middle) and (bottom), respectively. The coherence between the SST and SOI in 2–4 year scale was strong for 1960–1975, when the changes of the fractality both are very similar and the lead of the SST was observed. When the choherence was strong, the multifractality of SOI was strong for 1960-1975 and 1995-2000.

The multifractal analysis used in this study is effective as a method to analyze the complicated time series data.

### 3.3. The relation between the Niño3.4 and SST

We investigated the relation between the global average SST and Niño3.4. The $\tau(-6)$ of the Niño3.4, and SST are shown in Fig. 6 (top). The data was excluded from Fig. 6 (top) for cases where we could not distinguish between monofractality and multifractality. Between 1950 and 2010 especially for the 1980s and 2000s, for the SST and Niño3.4, the change of multifractality are very similar.

We studied the relationship between the SST and Niño3.4 by means of wavelet coherence, phase and fractality. We show the wavelet coherence and phase using the Morlet wavelet between the SST and Niño3.4 in Fig. 6 (middle) and (bottom), respectively. The coherence between the SST and Niño3.4 in 2–4 year scale was strong especially for 1970s and the lead of the Niño3.4 was observed. The lead of the Niño3.4 and lag of the SOI to SST was due to the reverse change of Niño3.4 and SOI. When the choherence was strong, the multifractality of Niño3.4 was strong for 1965-1975 and 1995-2000.

## IV CONCLUSIONS

In this study, we examined the relations between Niño3.4 index and SOI from a view of multi-fractality. We investigated the change of multifractal behavior of the Niño3.4 index and SOI by the multifractal analysis using the wavelet transform. We showed the change of multifractality by plotting the $\tau(-6)$. Moreover we studied the wavelet coherence and phase of these indices.

Between 1950 and 2010, especially for the 1970s, for the Niño3.4 and SOI, the matching in monofractality or multifractality was observed and the increase and decrease of multifractality were very similar, that is the change of multifractality were very similar. The changes of the Niño3.4 index and SOI is reversed, but the change of fractality is similar. When the El Niño occurrsed the fractality of the Niño3.4 and SOI changed from multifractality to monofractality for El Niño in 1972-1973, 1982-1983. 1991-1992, 1997-1998 and 2002-2003. When the La Niña occurrsed the fractality of the Niño3.4 and SOI changed from multifractality to monofractality for La Niña in 1955-1956, 1973-1976 and 1998-2000.

When the climate changed the climatic index was multifractal and when the climate became a certain state the climate index was monofractal. Hence, the unstable state shows multifractal and the stable state shows monofractal. By the wavelet coherence and phase using the Morlet wavelet and the change of fractality, the similarity of Niño3.4 index and SOI was shown. The multifractal analysis used in this study is effective as a method to analyze the complicated time series data.

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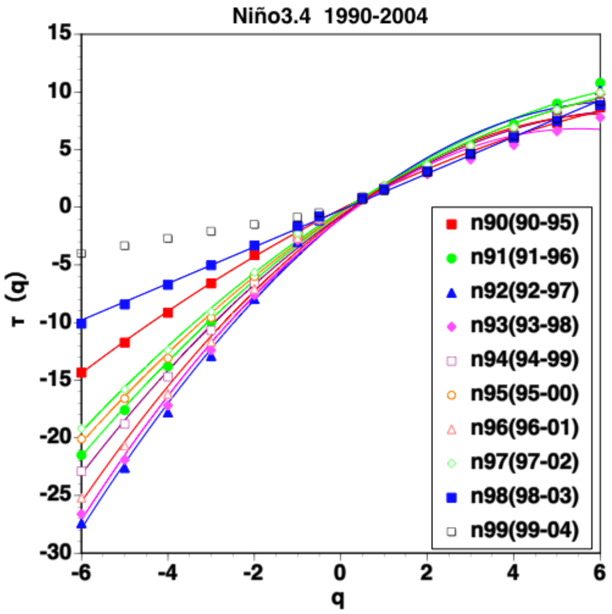



**Figure 1.** ***τ*(*q*) for individual Niño3.4 index between 1980 and 1994.**

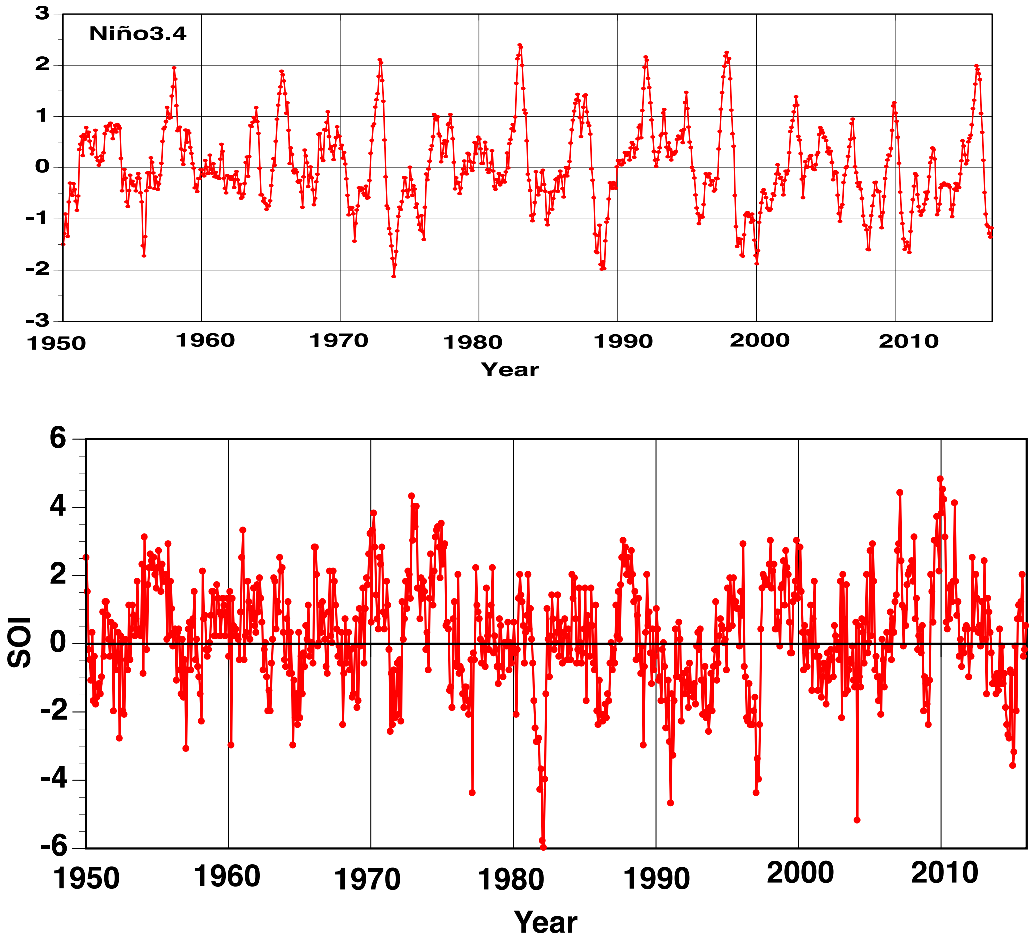
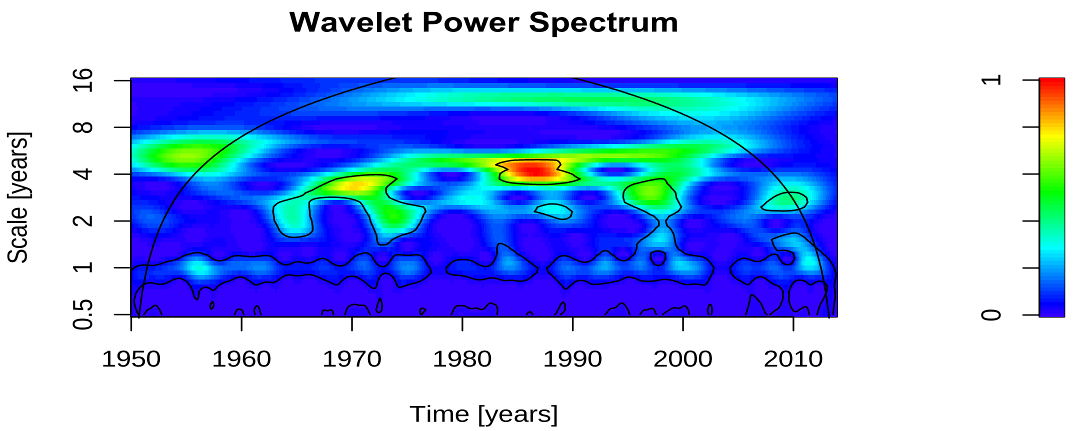



**Figure 2. Niño3.4 index (top) and SOI (bottom).**



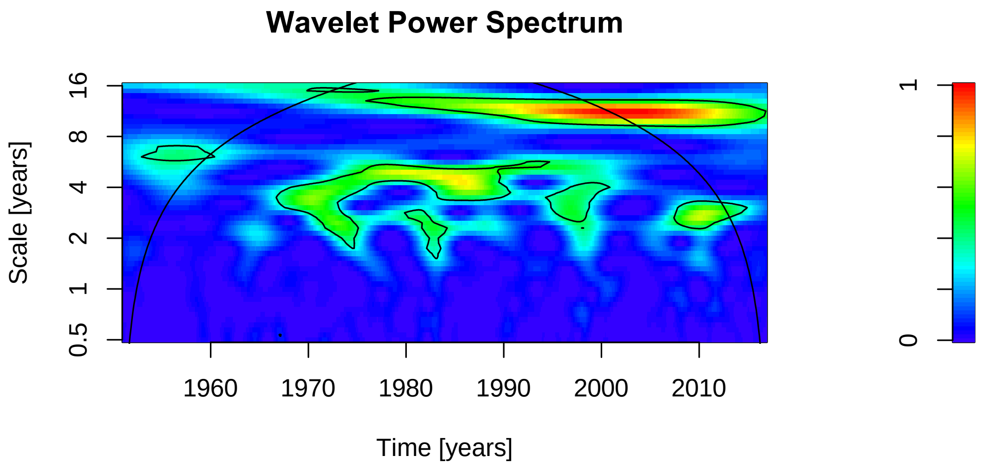



**Figure 3. Wavelet power spectra of Niño3.4 index (top) and SOI (bottom).**

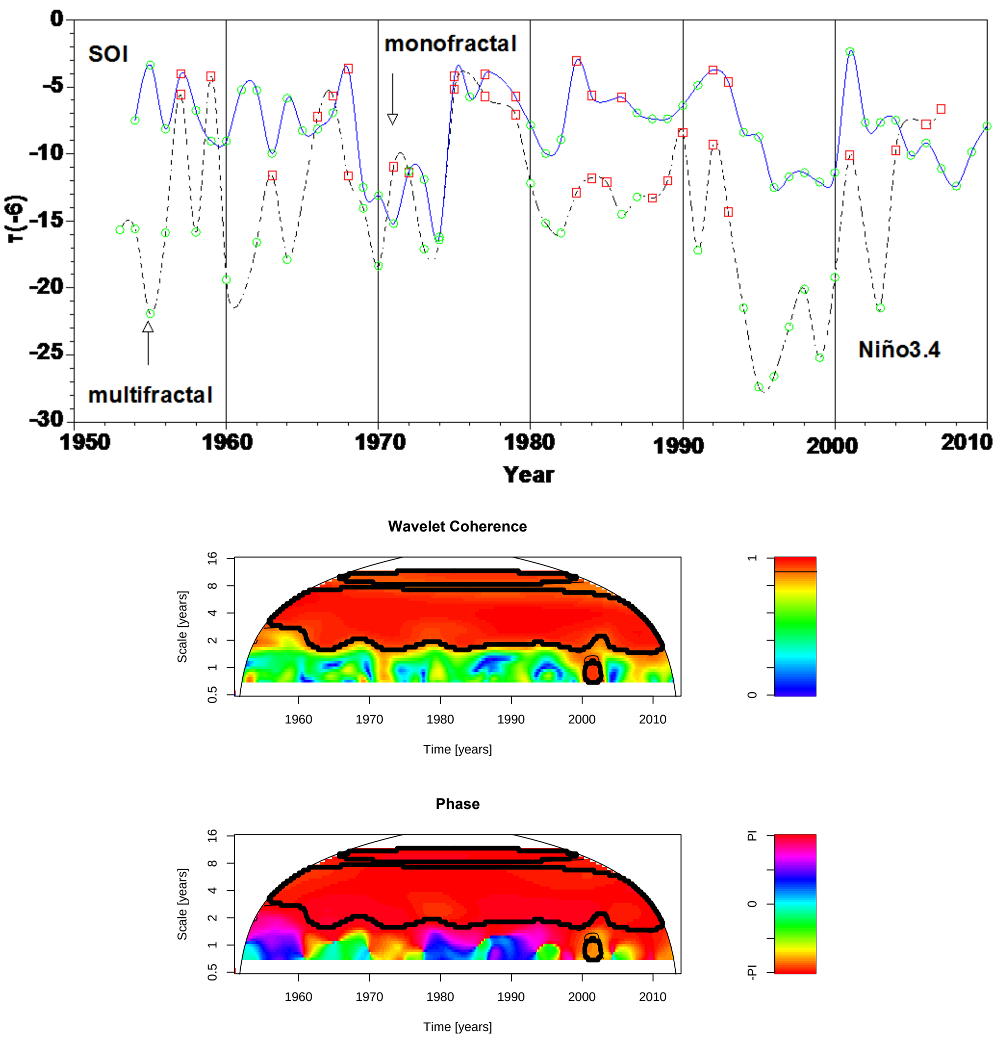



**Figure 4. The $\tau(-6)$ of Niño3.4 index and SOI (top). Wavelet coherence (middle) and phase (bottom) between the Niño3.4 index and SOI. In the wavelet phase, the positive value shown by the blue and pink shading means that the Niño3.4 index leads the SOI and the negative value shown by the green, yellow and red shading means that the SOI leads the Niño3.4 index.**

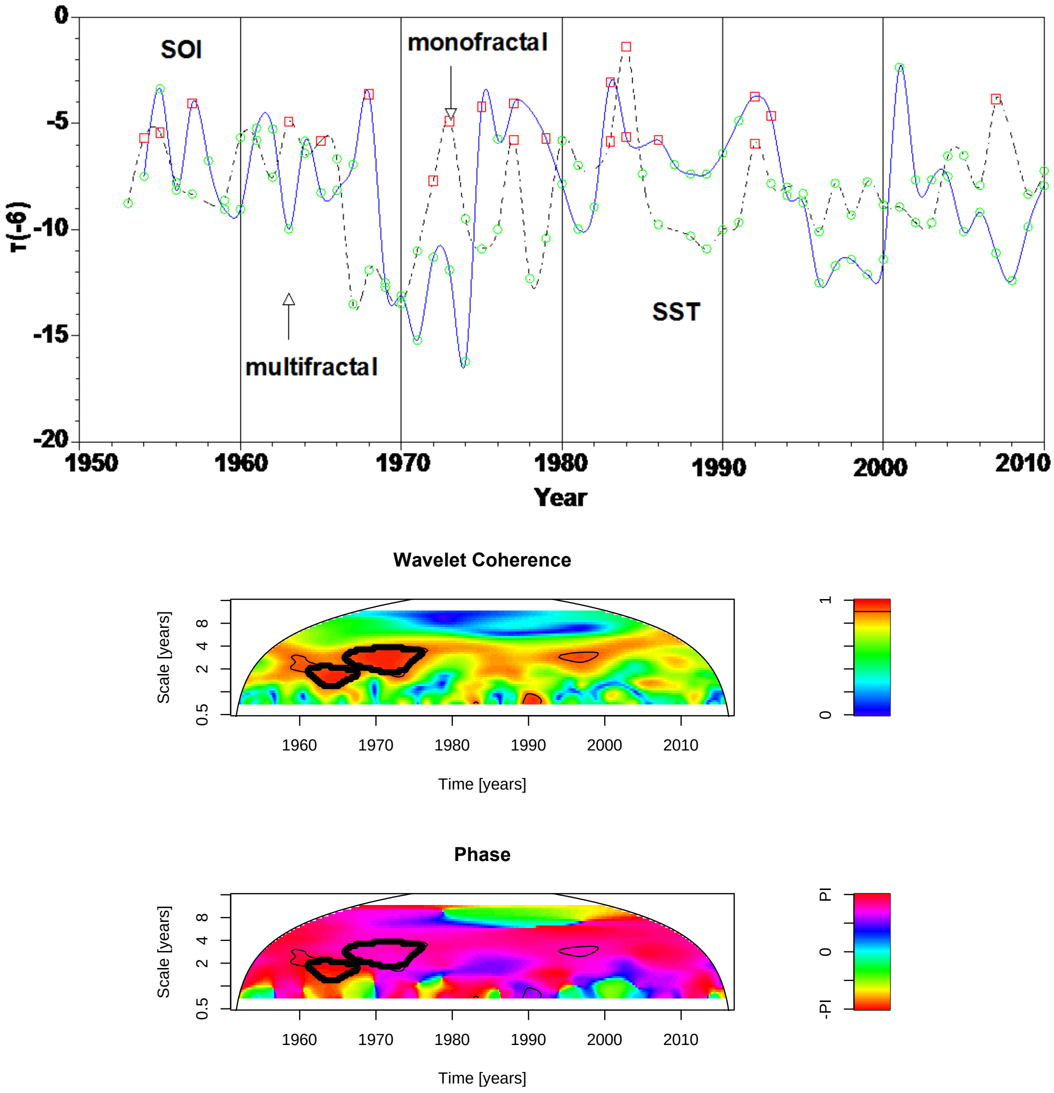



**Figure 5. The $\tau(-6)$ of the global SST, and SOI (top). Wavelet coherence (middle) and phase (bottom) between the SST and SOI. In the wavelet phase, the positive value shown by the blue and pink shading means that the SST leads the SOI and the negative value shown by the green, yellow and red shading means that the SOI leads the SST.**

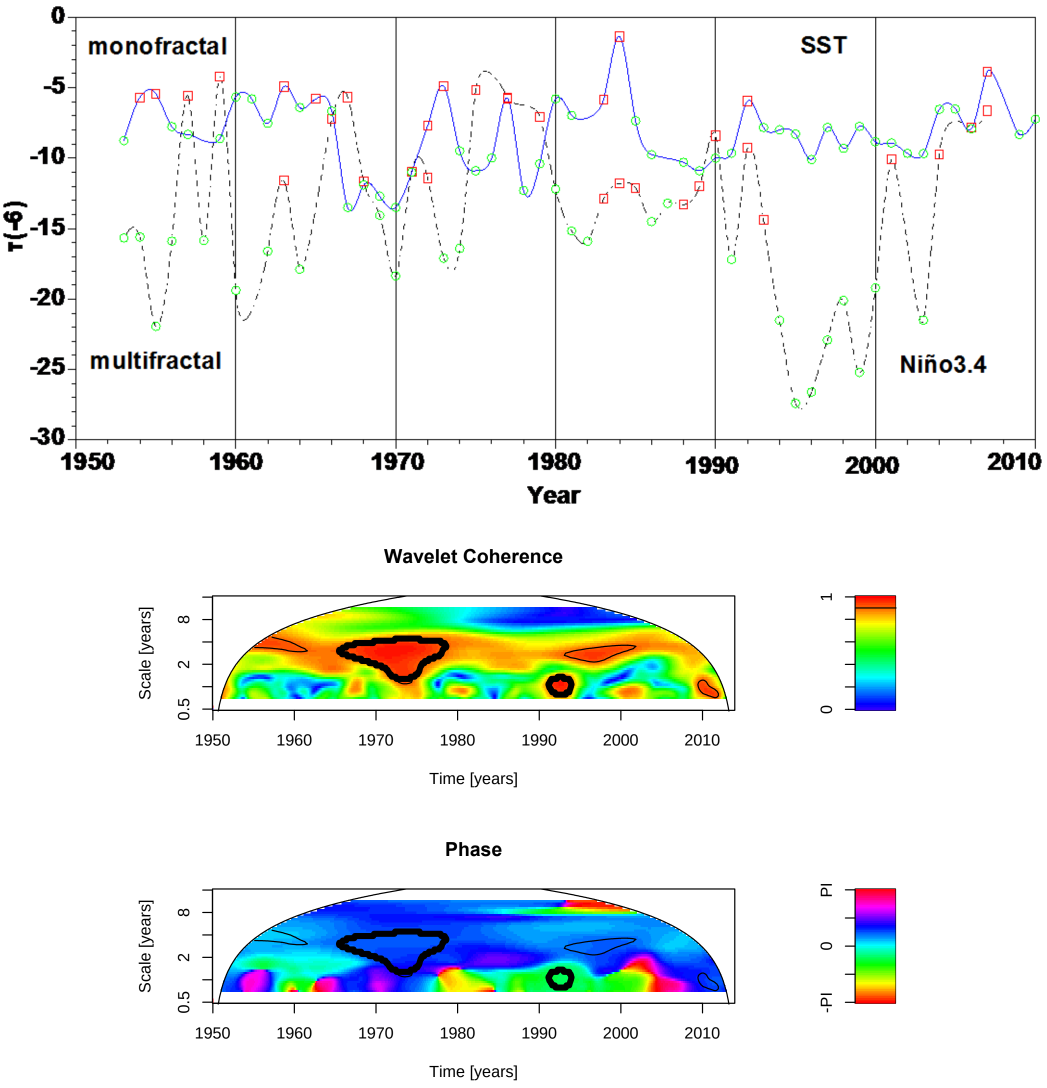



**Figure 6. The $\tau(-6)$ of the global SST, and Niño3.4 (top).Wavelet coherence (middle) and phase (bottom) between the Niño3.4 and SST. In the wavelet phase, the positive value shown by the blue and pink shading means that the Niño3.4 leads the SST and the negative value shown by the green, yellow and red shading means that the SST leads the Niño3.4.**

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