

Simulation of Fluid Flow in Subsurface Porous Media: An Introductory Overview

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-----ABSTRACT-----

Materials and systems of applications in engineering science and technology are majorly porous media. Investigating the fundamentals of fluid flow during serviceability of such media are analytical and quit involving, hence numerical simulations. Overview of common subsurface porous media was carried out with a view to presenting some common numerical schemes already in use for the analysis and investigation of fluid flow in them. Common shortcomings of those numerical schemes were exposed and few recommendations were made to further enhance better computation analysis and the investigation of subsurface porous media.

KEYWORDS: Porous, subsurface, numerical, simulation

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I. INTRODUCTION

A porous medium / material is a solid (often called frame or matrix) permeated by an interconnected network of pores (voids) filled with fluid(s) (either liquid, gas or both). Usually both the solid matrix and the pore network (also known as the pore space) are assumed to be continuous, so as to form two or more interpenetrating continua (as in a sponge). Many natural substances such as rocks, soils, biological tissues (e.g. bones), kidney, human skin and man-made materials such as (cements, foams, sand wood and ceramics) can be considered as porous media. A poro-elastic medium is characterized by its porosity, permeability as well as the properties of its constituents (i.e. Solid matrix and fluid) (Saman, 1992). Most materials in nature can be considered as porous, meaning they are a composition of voids and solid parts. The non-penetrable part of the material is hereafter called the matrix, and the voids are called pores, with respective volumes, V_M and, V_P . The total volume, $V_B = V_P + V_M$, of the medium is called the bulk volume (Dyrdahl, 2014). The concept of porous media is used in many areas of applied sciences, engineering and engineering – technology; mechanics, acoustics, thermal engineering, geo-mechanics, soil-mechanics and rock-mechanics, petroleum engineering, construction engineering, geosciences (hydro-geology, petroleum geology, geophysics), biology and biophysics, material sciences etc. (Luilan *et al.*, 2005). It is also used in MPP (Micro Perforated Panels) absorbers, nanotechnology and Aquifer management.

A subsurface porous media is a one that exist underneath the earth crust deeper enough to trap fluid at considerable high pressures. Two common examples in engineering and technology are the Oil reservoirs and Aquifers. Subsurface porous media which culmates to Aquifers or Oil reservoirs are basically and naturally rocks that are sedimentary in nature housing and allowing the passage and flow of water, gas or crude oil. Porous media can be classified as homogeneous or heterogeneous. Theoretically, fluid flow in porous medium is understood as the flow of liquid or gas or both in a medium filled with small solid grains packed in homogeneous manner. The concept of heterogeneous porous medium then introduced to indicate properties change (mainly porosity and permeability) within that same solid grains packed system. An average estimation of properties in that system is an obvious solution, and the case is still simple (Islam *et al.*, 2010). Fluid flow through porous media is a subject of common interest and has emerged a separate field of study. The study of more general behaviour of porous media involving deformation of the solid frame is called poro-mechanics. The porous media are so deeply located with fluids and the flows under very high pressures which makes investigations and analysis pretty difficult to be carried out. All sedimentary rocks consist of a solid matrix with an interconnected void. The void pore space allows the rocks to store and transmit fluids. The ability to store fluids is determined by the volume fraction of pores (the rock porosity), and the ability to transmit fluids (the rock permeability) is given by the

interconnection of the pores. Rock formations found in natural petroleum reservoirs are typically heterogeneous at all length scales, from the micrometer scale of pore channels between the solid particles making up the rock to the kilometer scale of a full reservoir formation. On the scale of individual grains, there can be large variation in grain sizes, giving a broad distribution of void volumes and interconnections. Moving up a scale, laminae may exhibit large contrasts on the mm-cm scale in the ability to store and transmit fluids because of alternating layers of coarse and fine-grained material (Lie, 2014). The numerical equations and expressions governing the flows of scientifically separable but physically immiscible fluids in subsurface porous stratum (earth) flow are inherently nonlinear and the geometries and material properties characterizing many empirical problems in petroleum / oil and groundwater engineering can be quite irregular and involving. Numerical simulation offers the only viable approach to the modelling of fluid flows generally (e.g. single phase or multiphase / multispecies fluid flows).

The analysis of fluid flow in porous media is a very complex phenomenon and as such cannot be described as explicitly as flow through pipes or conduits in which it is rather easy to measure the length and diameter of a pipe and compute its flow capacity as a function of pressure; however, in porous media, fluids flow are different in that there are no clear-cut flow paths which lend themselves to measurement (Chen *et al.*, 2007). Many engineering, and natural materials can be regarded as porous media within which fluid flows are considered to be in their pore naturally or artificially. And since the pores are of varying sizes and in a complex interconnectivity, the mechanism of fluid in them is quite involving and of greater intense. Sometimes there may be more than one fluid flow in the pores of such media. This, of course readily occurs in the Oil reservoirs where there are basically three or more scientifically separable but immiscible kinds / types of fluids namely; Oil (Black / Crude), gas and water or their constituents flowing through their different pores (Cottet *et al.* 2002). Various methods are available to study fluid flow in surface passages, conduit flows, pipes, oil reservoir, and groundwater, viz-`a- viz porous media. Figs.1. 1 and 1.2 respectively shows block diagram and cross-section of a typical single phase and multiphase porous media system consisting of only water and oil, water gas, in porous and non-porous rocks is shown:

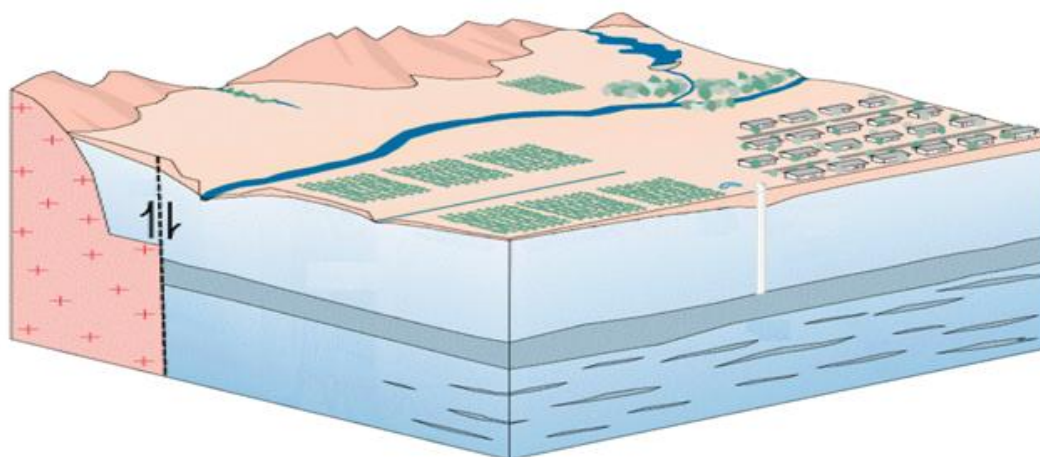


Fig.1.1. Block diagram of a part of a hypothetical basin-fill groundwater system.

The blue segments show the direction of ground-water flow. Among the features shown are an unconfined aquifer overlying a confining unit and confined aquifer, a gaining stream, infiltration from irrigated agriculture, and mountain-front recharge. Retrieved Sept. 20, 2015 from Reilly and McAdam, 2015.

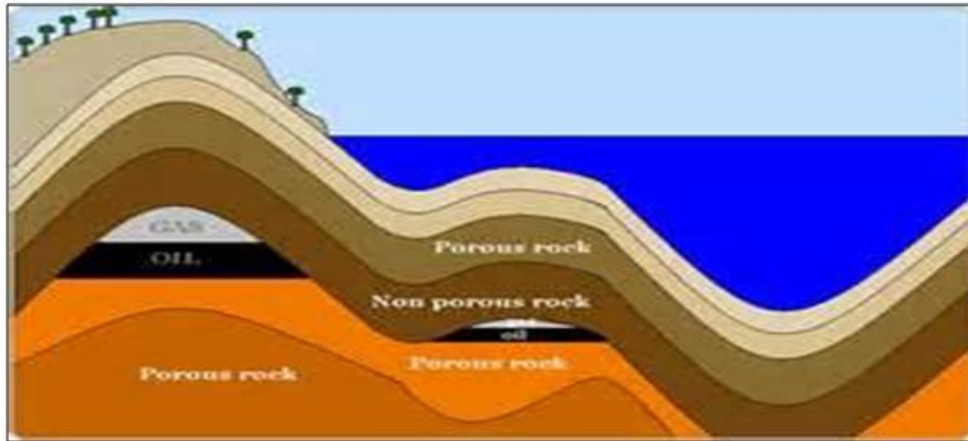


Fig. 1.2. Crossection of a typical multiphase porous media system

Retrieved Oct. 10, 2014 from Abu-Khamsin, 2004.

Actual fluid flows are governed by unsteadiness (turbulent flows) but sensible assumptions can be employed to make a worthwhile study and investigations as does this. The empirical quantitative method required for the study of fluids flow through any media is traditionally called fluid mechanics while the modern way of doing the study and investigation is called Computational Fluid Dynamics (CFD); a branch of fluid mechanics that uses numerical methods and algorithms to solve and analyze problems that involve fluid flows. (Gavalas *et al.*, 1976). Computers are used to perform the calculations required to simulate the interaction of liquids and gases with surfaces defined by appropriate boundary conditions. Single and multiphase flow through porous media are shown below in Figs. 1.3 and 1.4 with γ and κ denoting the fluid and solid control volume respectively. Thus, γ and k phase shows the liquid-solid phase interaction description.

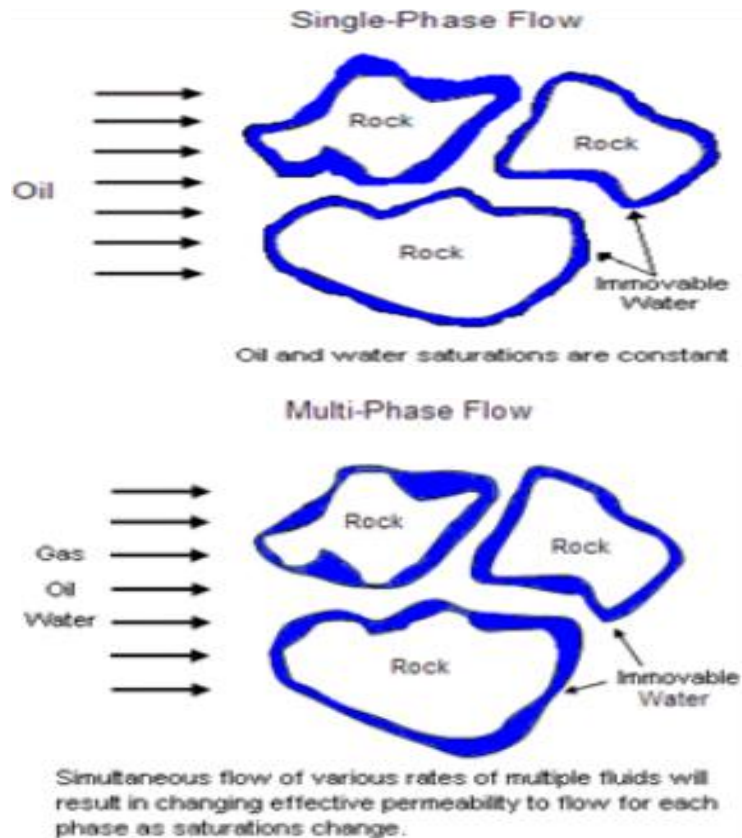


Fig. 1.3 Comparison of Single-phase and Multi-phase porous media

Retrieved Sept. 24, 2014 from Chen, *et al.*, 2007.

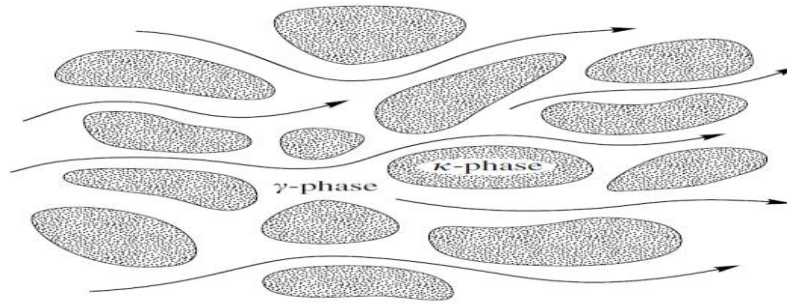


Fig. 1.4 Enlarged view of porous media system

Retrieved Sept. 24, 2014 from Chen, *et al.*, 2007.

As defined by groundwater / reservoir engineering dictionary, 2015 aquifers are formations having structures that permit appreciable water to move through them under ordinary field conditions; an example is sand. In contrast, an aquiclude is an impermeable formation which may contain water but is incapable of transmitting significant water quantities examples are clay and shale. An aquifer age is an impermeable formation which neither contains nor transmits water e.g. granite. Aquifers may be classified as unconfined or confined, depending upon the presence or absence of water table. An unconfined Aquifer is one in which a water table serves as the upper surface of the zone of saturation. It is also known as free, phreatic or non-artesian Aquifer. The water table varies depending upon areas of recharge and discharge, pump age from wells and permeability. Rises and falls in the water table correspond to changes in the volume of water in storage within an Aquifer. Lowering the water table may decrease and even stop natural discharge, but increase natural recharge, especially in certain types of soil. Among other factors affecting groundwater levels are, seasonal variations in evaporation and evapotranspiration, changes in barometric pressure, passage of moving loads like trains over artesian formations, land and ocean tides, and earthquakes. Confined Aquifer which is also known as pressure aquifer, occurs where groundwater is confined under pressure greater than atmospheric by the overlying impermeable strata. If a well is sunk such that it penetrate such an aquifer, the water level will rise and falls off water in wells penetrating. This type of aquifer is as a result of changes in pressure, rather than changes in storage volumes. The water level in these wells defines the elevation of the piezo-metric surface at that point. If the piezo-metric surface lies above ground surface, a flowing well results but if it falls below the bottom of the upper confining bed, then the aquifer becomes an unconfined one (Ground water and wells, 2011). Simulation of flow in fractured aquifers is a complex issue. The problem of mathematical modeling of highly heterogeneous porous media, typical of natural systems, couples with the needs making proper simplifying assumptions and approximations (Carloni *et al.*, 2011).

II. NUMERICAL METHODS OF INVESTIGATION

Numerical schemes such as finite difference, finite element and finite volume methods have been explored in fluid flow analysis of such media. Implementation of these techniques require many mathematical operations that place high demand on computer system resource and human expertise. Literature is sparse on the use of cellular vortex element scheme with low demand on computer system resource and human expertise in modeling fluid flow in subsurface porous media. This study was designed to investigate the use of cellular vortex element technique for the modeling of fluid flow in subsurface porous media in which confined aquifers and heterogeneous oil reservoirs are typical examples. While fluid flow is a ubiquitous phenomenon on both Earth's surface and elsewhere in the cosmos, its existence, as a mathematical field quantity without discrete form, color, or shape, defies representation in the visual arts. Both physical biology and computational physics are, at their roots, very large systems of interacting agents. The field of computational fluid dynamics deals with solving the essential formulas of fluid dynamics over large numbers of interacting elements (Stock, 2009). Computational fluid dynamics (CFD) is commonly used to visualize and understand complicated fluid flow and heat transfer in many industries. It is imperative to validate CFD computer models in order to avoid costly design choices where experimentation cannot be used to ratify the predictions of computer models (Chen *et al.*, 2007). Computational fluid dynamics (CFD) came into apparent limelight around the year 1965 when scientists like Jaquard, Jain, Eringen and Ingram worked on various aspects of CFD ranging from numerically computing sensitivity coefficient for history matching purposes and theory of fluid mixtures. However the works of Eringen and Ingram were periodically reviewed by Atkin and Crain. Large numbers of Computational Fluid Dynamics (CFD) calculations and analysis employs Eulariandescription of the fluid flow where the governing equations are written as finite difference equations relating properties at two or more spatially fixed adjacent points. As reported by Cakmak, 2011, Rajagopalan and Tien, Tufenkji and Elimelech, Yao *et al.*, submitted that the Lagrangian approach focuses on the movement of distinct particles and tracks particle position in a moving fluid. In contrast,

the Eulerian approach considers the concentration distribution of particles in a porous media. The Eulerian approach has advantages over the Lagrangian approach, in that it does not require high computational performance, and it is easy to incorporate Brownian motion.

The numerical simulation of multidimensional problems in fluid dynamics and nonlinear solid mechanics often requires coping with strong distortions of the continuum under consideration while allowing for a clear delineation of free surfaces and fluid–fluid, solid–solid, or fluid–structure interfaces. A fundamentally important consideration when developing a computer code for simulating problems in this class is the choice of an appropriate *kinematical description* of the continuum. In fact, such a choice determines the relationship between the deforming continuum and the finite grid or mesh of computing zones, and thus conditions the ability of the numerical method to deal with large distortions and provide an accurate resolution of material interfaces and mobile boundaries (Donea *et al.*, 2014). In the last few years, much attention has been dedicated to depleted petroleum reservoirs in the oil industry all over the world. The interdependency of economy on oil and gas reserves, price fluctuation, increasing demands and politics are major factors affecting interest in developing alternative sources of energy, and enhanced oil recovery from abandoned fields, heavy oil reservoirs, tar sands, bitumen shales, etc. Among the potential targets for enhanced oil recovery projects are mainly many depleted oil reservoirs, some with such a low pressure that the gas cap is at atmospheric pressure. This is the case when gas solubility is low, or for water flooded shallow reservoirs which have produced much of the oil, water and associated gas as reported by Cakmak, 2011. Numerical simulation of fluid mechanics problems is one of the most challenging scientific computing’s field of research of the last decades. With the increasing power of computers in terms of memory available, speed and number of processors, more and more complex problems arising in industrial applications become accessible to numerical simulations. In complex geometries, the discretization of the Navier Stokes Equations (NSE) by finite element or finite volume methods on body-fitted grids allows to simulate flows at low to moderate Reynolds numbers. However, generating an efficient conformal mesh is a challenging problem when the geometry gets complex and this pre-processing step is very CPU-time consuming (James *et al.*, 2013).

Most traditional solution methods use finite element and finite difference methods for solution technique. Some are quantitative while some are qualitative. Some are largely deterministic, probabilistic while some are both, as reported by Ogundare (2006). The Finite Element method is a numerical methodology that solves differential equations by using a ‘piecewise approximation’. The domain of interest is subdivided into a large number of interconnected sub-regions called finite elements. The finite element analysis of physical problems can be summarized as follows:

1. Discretization of the domain: The physical system is discretized into a series of finite elements that are connected at a discrete number of nodal points.
2. A matrix expression is developed to relate the nodal variables of each element creating an element matrix.
3. The element matrices are combined to form a set of algebraic equations that describes the entire global system.
4. The imposed boundary conditions are incorporated into the assembled matrix equation.
5. The resulting set of simultaneous algebraic equations is solved.

The Finite Element method assumes that there is an infinite sum of functions that will exactly represent the solution to the groundwater flow equation. The series approximation provides an exact representation as N approaches infinity. The Princeton Transport Code (PTC) is a numerical simulator that solves a system of partial differential equations in order to represent the subsurface flow, the velocities and the contaminant mass transport in the simulated aquifer. PTC employs a unique splitting algorithm for solving the fully three-dimensional equations, which significantly reduces the computational burden. The algorithm discretizes the domain into approximately parallel horizontal layers and within each layer a finite element discretization is used allowing for the accurate representation of irregular domains. In the Finite Element method the discrete equations are applicable over an element, but in the Finite Difference method the discrete equation normally used on rectangular grids. Two kinds of Finite Difference discretizations are available: one that has nodes at the corners of the element and the other that has nodes in the centre (Papadopoulou *et al.*, 2009). In a simulation model, a set of mathematical functions is used to accurately describe the physical system. Numerical methods such as Finite Element (FE) and Finite Difference (FD) are used for the solution of the governing set of partial differential equations (James *et al.*, 2013). The purpose of simulation is the estimation of reservoir performance (e.g., oil recovery) under one or more producing schemes. Whereas a reservoir, through its entire lifetime, can produce only once at a considerable expense, a model can “produce” or run many times at low expense over a short period of time. Observation of a model performance under different producing conditions aids selection of an optimal set of producing conditions for the reservoir.

In 1870, Adolph Theim of Germany, modified Dupuit’s formula so that he could actually compute the hydraulic characteristics of an Aquifer by pumping a well and observing the effects in other wells in the vicinity (Theim, 1963). He continued perfecting his method by applying it to various field situations. Many works have

been done on Aquifers and Black Oil reservoirs simulation of one-dimensional to three-dimensional geometry, and of any phase ranging from one-phase to three-phase. Some are theoretical, numerical, statistical and mathematical while others applied different techniques in solving the black oil equations so generated. Advances in groundwater hydraulics can be more easily identified with individual people because, specific formulae are commonly published rather than applying generalized concepts which are so important in classical geology. Jules Dupuit of France was the first scientist to develop a formula for the flow of water into a well. The work was published only seven years after Darcy's monograph, yet successfully utilized Darcy's law (Papadopoulou *et al.*, 2009).

In the seventh decade of this century, various automatic and semi-automatic history matching techniques have been introduced. In 1965 Jacquard and Jain, presented a technique based on a version of the method of steepest descent. They did not consider their method to be fully operational, however, due to the lack of experience with convergence. In 1965, Jahns presented a method based on the Gauss-Newton equation with step - wise solution for speeding the convergence, but his procedure still required a large number of reservoir simulation to lead to a solution. Coats *et al.*, 2003 presented a workable automatic history matching procedure based on least-squares and linear programming. Slater and Durrer, 1971 presented a method based on a gradient method and linear programming. In their study they highlighted the difficulty of choosing a step size for their gradient method, especially for problems involving low values of porosity and permeability. They also pointed out the need for a fairly small range in their reservoir description parameters for highly non-linear problems.

Zhan, 2007 reported that Jacquard and Jain, presented the first procedure for numerically computing sensitivity coefficient for history matching purposes. They applied their method to the estimation of permeability in a two-dimensional reservoir from pressure data. Working on this premise, Carter *et al.*, 1974 presented an elegant derivation of a method to compute sensitivity coefficient for two-dimensional single-phase flow problems using electric circuit analogue, but for non-linear problem like multiphase flows, where the derivation do not apply. Thus other alternatives like efforts on ad-joint or optimal control method from Chen *et al.*, 2007 and Chavent *et al.*, 1975 and also Lee and Seinfeld, 1987, Yang *et al.*, 1988 and Makhlof *et al.*, 1993 who worked on multi - phase flow problems with the same methods were eminent. However, these method proved to be computationally inefficient.

Rezael and Donaldson, 1998 described a single phase, two-dimensional compressible front tracking model for micro-computer which can be used to simulate water floods. The model can be used for any type of heterogeneous reservoir with variable thickness, permeability, porosity and flow boundaries. A complete discussion on the mathematical model, operating logic of the program, and sensitivity to input data was also provided. Odeh, 1990 presented the results of the comparison of the solutions to a 3-Dimensional black oil reservoir simulation study. Seven companies participated in the study using different simulators. Two runs were made in the study. Case one was run with the bubble point (Saturation) pressure constant and equal to the original value. Case two varied the saturation pressure with gas saturation. The reservoir was subdivided into a 2 or 3-D network of rigid blocks and he later showed that the simulation model equations are basically the familiar volumetric material balance equation written for each phase for each grid block with an excellent description of a simulation model. He illustrated the subdivision of a reservoir into 2-D or 3-D network grid blocks and later showed that the simulation model equation are basically familiar volumetric balance equation written for each block. Mattax and Dalton, 1990 described the modeling concept and how to test the model validity. They agreed that the precision with which variation in reservoir properties can be modeled is determined by the number of blocks in the model. Ajiroba, 2006 reported that McCarty and Barfield, illustrated a method for carrying out some 2-Dimensional analysis so as to obtain a more precise control of petroleum reservoir performance with using a high speed digital computer. They approached the problem by developing a representation numerical model of the reservoir and then employed a suitable numerical technique to solve the basic equation of flow. They also defined the required engineering information for the particular case represented by the model. They carried out their analysis by calculating the pressure distribution in the reservoir and tracking the progress of the interface between the displaced and displacing fluids in a step wise manner to provide a depletion history of the operation. The list could go on and on but the main observation is that current industrial researches are rooted in the past works of these people. As reported by Ogundare, 2006 an important aspect is the regularized automatic history matching which are deeply rooted in the works of Gavals *et al.* and Shah *et al.*, although only one-dimensional single-phase flow was considered, the work of these authors laid the ground work for much of the research work being done today on automatic pressure and saturation history matching. Different solution techniques have been employed over the years from the one that is deterministic to the probabilistic, all in effort to provide suitable, economical solution technique to Navier-Stokes equation. Amongst these methods are, Random Walk method, Core Expansion Method, Vortex element method in elliptic Gaussian Blob amongst others. Not left out are the Numerical methods of solution, these include: Finite Difference Method, Finite Element Method, Monte Carlo Method and the Hybrid Monte Carlo Method (Ogundare, 2006). Zheng *et al.*, 1998 successfully applied the Finite

Element Method using Biot Poro-elastic approach to simulate 2D axisymmetric 3-D and full 3-D reservoir in the subsistence of Venice in Italy.

III. DISCUSSION

This overview has shown that most common numerical methods like finite element method, finite difference method, and finite volume method and combination of one or more of the mentioned methods are prone to various constrains. These constrains are expected to be relaxed by developing or adopting other numerical methods suitable for the systems in context. Most existing solution techniques for Aquifer and Reservoir simulation are rigidly grid based (i.e. solutions falls on grid points otherwise no solution exist). These solutions that are iterative in nature does not always capture the dynamic nature of the fluids in relation to fluid particle location and phase behavior..

IV. CONCLUSION AND RECOMMENDATION

For the fact that those shortcomings with the existing numerical methods in subsurface fluid flow simulations has been identified, there is need for other better methods that can sufficiently replace the existing ones with better efficiency and accuracy. The simulation should also be such a one that will be able to handle various attributes of parameters of properties, geometries and boundaries subsurface porous media

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