The International Journal of Engineering and Science (IJES)
|| Volume || 7 || Issue || 4 Ver. II || Pages || PP 31-40 || 2018 ||
ISSN (e): 2319 – 1813 ISSN (p): 23-19 – 1805

Effects of Residuals of Autocorrelation Function and Partial Autocorrelation Function in Long-Range Dependence Market Analysis

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ABSTRACT

The univariate data of the Nigerian All-Share Index (NASI) obtained from the Nigerian Stock Market (NSM), covering the period of 18 years, was studied and analysed through time series. The NASI considered exhibited long-range dependence with $H > 0.5$.The result of the analysis in a long-range dependence showed that the autocorrelation function (ACF), partial autocorrelation function (PACF) and their respective residuals agreed with their theoretical concepts. The multiplicative trend type (MULT) of the seasonal decomposition of NASI MULT is simple seasonal. The residual of ACF and PACF of NASI MULT for simple seasonal is shown with its model fit having zero predictors, zero outliers, mean zero and unit variance for 24 lags of NASI MULT. Further result of its trend showed model fit statistic with stationary R-squared value as 0.069, R-squared value as 0.938, with Ljung-Box $Q(18)$for 18 years having 16 degrees of freedom, 0.832 significance level, various fit statistic with respective mean, minimum, maximum and percentile value ranging from 5 to 95 with no standard error. Hence, the result of the analysis in a long-range dependence phenomenon showed that the residual functions in financial data could be of help to further describe the nature of the trading activities of the financial market trends in a given economy.

Index Terms: *Autocorrelation Function, Nigerian All-Share Index, Partial Autocorrelation Function, Residual Function, Seasonal Multiplicative Trend.*

Date of Submission: 27-03-2018 Date of Publication: Date: 11-04-2018

I. INTRODUCTION

This paper considered the residual functions as some of the essential dynamics of financial markets with respect to the behaviour of the autocorrelation function and partial autocorrelation function [2], [3]. The Nigerian Stock Exchange (NSE) maintains an All-Share Index formulated in January 3,1984 with all listings included in the Nigerian Stock Exchange All-share index while data on listed companies performances are published daily, weekly, monthly, quarterly and annually [26.] The theory of correlations and autocorrelations with some introductory concepts to Statistics were shown in the works by [30],[31]and [32], and [6]. Some time series models were considered by [29] and [28] as well as [4] who emphasised on the theoretical time series autocorrelated sampling properties. [8]extensively studied the variance of the regression residual method for estimating the Hurst coefficient of times series. [9J presented the approximate distribution of serial correlation coefficients while [12] and [13] respectively showed some tests of hypothesis in certain linear autoregressive model. Moreover, some approximate tests of correlation in time series could be found in [15] and [22]. [I] and [27] respectively worked on estimation and information in stationary time series. [25]emphasized on fractional Brownian motion and long-range dependence while [10J highlighted on certain theory and application of long-range dependence. Expressions for covariances having a bilinear representation of time series with applications while assuming that the random variables $e_t$are Gaussian with $E(e_t) = 0$was given by [11]. [23]measured forecast performance of ARMA and ARFIMA models with application to US dollar and UK pound. In[17], [18] and [19], computer experiments with fractional noises were respectively presented in various parts while [20] discussed the robustness of rescaled range Statistical measurement. [7]presented a good work on time series analysis forecasting and control. Also, [21] gave an elaborate forecasting details with univariate Box-Jenkins Model. Ljung-Box test was recorded in [14]. [16]gave an insight into operational hydrology.

II. METHODOLOGY

We consider three processes associated with ARIMA (p,d,q) models. These include:

$$z_t = c + \phi_1 z_{t-1} + a_t \qquad (2.1)$$

$$z_t = c - \theta_1 a_{t-1} - \theta_2 a_{t-2} + a_t \qquad (2.2)$$

$$z_t = c + \phi_1 z_{t-1} - \theta_1 a_{t-1} + a_t \qquad (2.3)$$

$$c = \mu\left(1 - \sum_{i=1}^{p} \phi_i\right) \qquad (2.4)$$

Equation (2.1) is called an AR(l) process because it contains only one AR term (including the constant term and the current random shock) where the maximum time lag on the AR terms is two. Equation (2.2) is called an MA(2) since it has only MA terms with a maximum time lag on the MA terms of two. Equation (2.3) is an example of a mixed process because it contains both AR and MA terms. It is an ARIMA(l,l) with AR(l) and MA(l). Equation (2.4) gives the constant term of an ARIMA process. If no AR terms are present, then $c = \mu$. This is true for all MA processes.

ARIMA processes are characterized by the values of p, d, q in this manner:

ARIMA(p,d,q), where $p, d, q \in \mathbb{Z}^+$ such that p is the AR order of the process, q is the MA order of the process and d is the number of times a realization must be differenced to achieve a stationary mean. The letter "I"in the acronym ARIMA refers to the integration step which corresponds to the number of times, *d*, the original series has been differenced. If a series has been differenced *d* times, it must subsequently be integrated *d* times to return to its original overall level [21:p.95].

2.1 ARIMA Models in Back shift Notation

ARIMA models are often written in back shift notation. The back shift operator, *B*, alters the time subscript on the variable by which it is multiplied, that is

$$B^k z_t = z_{t-k} \qquad k = 1, 2, \ldots, n \qquad (2.5)$$

Also,

$$B^k C = C \forall k < \infty \qquad (2.6)$$

Multiplying z_t by the differencing operator, $(1-B)^d$, produces the d^{th} differences of z_t:

$$(1-B)^d z_t = (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) z_t \qquad (2.7)$$

The procedure for non-seasonal processes in back shift form has six steps as seen in [19]. Using the six steps, a non-seasonal process in back shift notation has the general form

$$(1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p)(1-B)^d \tilde{z}_t = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q) a_t \qquad (2.8)$$

where $(1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p)$ is the general form of the AR operator of order $(1 - \theta_1 B - \theta B^2 - \cdots - \theta_p B^p)$ is the general form of the MA operator of order q, z_t written in deviations from its mean, $\tilde{z}_t$, and a_t is the random shock. Equation (2.8) can be written in compact notation by substituting the following symbols. Let

$$\nabla = 1 - B \qquad (2.9)$$

$$\emptyset(B) = 1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^q \qquad (2.10)$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q \qquad (2.11)$$

Thus, a compact way of saying that the random variable z_t evolves according to an ARIMA (p,d,q) process is

$$\emptyset(B) \nabla^d \tilde{z}_t = \theta(B) a_t \qquad (2.12)$$

It is more difficult to show that MA terms represent past $z's$. By demonstrating this for MA(1) rather than proving it for the general case, we will find out that the MA(1) process can be interpreted as an AR process of infinitely high order. MA(1) process in back shift form is:

$$\tilde{z}_t = (1 - \theta_1 B) a_t \qquad (2.13)$$

$$\Longrightarrow \quad a_t = (1 - \theta_1 B^{-1}) \tilde{z}_t \qquad (2.14)$$

For a geometric series with $|\theta_1| < 1, (1 - \theta_1 B)^{-1}$ is the sum of a convergent infinite series, that is,

$$(1-\theta_1 B)^{-1} = 1 + \theta_1 B + \theta_1^2 B^2 + \theta_1^3 B^3 + \cdots, \qquad (2.15)$$

$$\Rightarrow a_t = (1 + \theta_1 B + \theta_1^2 B^2 + \theta_1^3 B^3 + \cdots)\tilde{z}_t \qquad (2.16)$$

Equation (2.16) is an AR process of infinitely high order with the $\emptyset$ coefficients given by

$$\left.\begin{aligned} \phi_1 &= -\theta_1 \\ \phi_2 &= -\theta_2 \\ \phi_3 &= -\theta_3 \\ &\vdots \end{aligned}\right\} \qquad (2.17)$$

2.2 The Estimated ACF and PACF.

The estimated *ACF* and the estimated PACF are very important tools at the identification stage of the UBJ method [7].

2.2.1 Estimated ACF

The idea in an autocorrelation analysis is to calculate a correlation coefficient for each set of ordered pairs $(\tilde{z}_t, \tilde{z}_{t+k})$. Since we need to find the correlation between sets of numbers that are part of the same series, the resulting statistic is called an autocorrelation coefficient.

Let r_k be the estimated autocorrelation coefficient of observations separated by k time periods within a given series. The standard formula for calculating autocorrelation coefficients is:

$$r_k = \frac{\sum_{t=1}^{n-k}(z_t - \tilde{z})(z_{t+k} - \tilde{z})}{\sum_{t=1}^{n}(z_t - \tilde{z})^2} \qquad (2.18)$$

or more compactly as

$$r_k = \frac{\sum_{t=1}^{n-k}\tilde{z}_t \tilde{z}_{t+k}}{\sum_{t=1}^{n}\tilde{z}_t^{\,2}} \qquad (2.19)$$

where $\tilde{z}_t$ and $\tilde{z}_{t+k}$ have their usual meaning as shown.

2.2.2 Estimated PACF

An estimated PACF is broadly similar to an estimated ACF. An estimated PACF is also a graphical representation of the statistical relationship between sets of ordered pairs $(\tilde{z}_t, \tilde{z}_{t+k})$ drawn from a single time series. The estimated PACF is used as a guide, together with the estimated ACF, in choosing one or more ARIMA models that might fit the available data.

The estimated partial autocorrelation coefficient measuring this relationship between $\tilde{z}_t$ and $\tilde{z}_{t+k}$ is designated by $\hat{\phi}_{kk}$. (Recall that $\hat{\phi}_{kk}$ is a statistic because it is calculated from sample information and provides an estimate of the true partial autocorrelation coefficient ϕ_{kk} *.)* The steps to obtain $\hat{\phi}_{kk}$ are as follows. Initially, we estimate the following regression

$$\tilde{z}_{t+1} = \phi_{11}\tilde{z}_t + U_{t+1} \qquad (2.20)$$

for $\hat{\phi}_{11}$ where ϕ_{11} is the true partial autocorrelation to be estimated by regression for $k = 1$, where U_{t+1} is the error term representing all things affecting $\tilde{z}_{t+1}$ that do not appear elsewhere in the regression equation. Using least squares regression computer program, we obtain $\hat{\phi}_{11}$ for $k = 1$. To obtain, $\hat{\phi}_{22}$ we have to estimate the multiple regression:

$$\tilde{z}_{t+2} = \phi_{21}\tilde{z}_{t+1} + \phi_{22}\tilde{z}_t + U_{t+2} \qquad (2.21)$$

where ϕ_{22} is the true partial autocorrelation coefficient to be estimated for $k = 2$.Therefore, $\hat{\phi}_{22}$ estimates the relationship between $\tilde{z}_t$ and $\tilde{z}_{t+1}$ with $\tilde{z}_{t+1}$ accounted for. Next, we estimate the following regression

$$\tilde{z}_{t+3} = \phi_{31}\tilde{z}_{t+2} + \phi_{32}\tilde{z}_{t+1} + \phi_{33}\tilde{z}_t + U_{t+3} \tag{2.22}$$

to find $\hat{\phi}_{33}$ where $\hat{\phi}_{33}$ is the partial autocorrelation coefficient to be estimated for $k = 3$. Thus, $\hat{\phi}_{33}$ estimates the relationship between $\tilde{z}_t$ and $\tilde{z}_{t+3}$ with $\tilde{z}_{t+1}$ and $\tilde{z}_{t+2}$ accounted for. There is a slightly less accurate though computationally easier way to estimate the ϕ_{kk} coefficients. It involves using the previously calculated autocorrelation coefficients,r_k. If the data series is stationary, then the following set of recursive equations gives fairly good estimates of the partial autocorrelations.

$$\hat{\phi}_{11} = r_1$$

and

$$\hat{\phi}_{kk} = \frac{r_k - \sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_{k-j}}{1 - \sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_j} \qquad (k = 2,3,....) \tag{2.23}$$

where $\quad \hat{\phi}_{kj} = \hat{\phi}_{k-1,j} - \hat{\phi}_{kk}\hat{\phi}_{k-1,k-j} (k = 3,4,...; \quad j = 1,2,...,k-1$

2.3 Long-Range Dependence

Definition 2.1A stationary sequence $(X_n)_{n\in\mathbb{N}}$ exhibits long-range dependence if the autocovariance function $\rho(n) := cov(X_k, X_{k+n})$ satisfy

$$\lim_{n\to\infty} \frac{\rho(n)}{cn^{-\alpha}} = 1 \tag{2.24a}$$

for some constant c and $\alpha \in (0,1)$. In this case, the dependence between X_k and X_{k+n} decays slowly $as\ n \to \infty$ and

$$\sum_{n=1}^{\infty} \rho(n) = \infty \tag{2.24b}$$

Hence, according to [4], we obtain immediately that the increments

$$X_k := B_k^H - B_{k-1}^H \tag{2.25}$$

and

$$X_{k+n} := B_{k+n}^H - B_{k-1}^H \tag{2.26}$$

of B^H have the long-range dependence property for the Hurst parameter, $H > \frac{1}{2}$ since

$$\rho_H(n) = \frac{1}{2}[(n+1)^{2H} + (N-1)^{2H} - 2n^{2H}] \sim H(2H-1)n^{2H} asn \to \infty \tag{2.27}$$

In particular,

$$\lim_{n\to\infty} \frac{\rho_H(n)}{H(2H-1)n} = 1 \tag{2.28a}$$

Summarizing, we obtain:

$$\sum_{n=1}^{\infty} \rho_H(n) = \infty, \qquad H > \frac{1}{2} \tag{2.28b}$$

$$\sum_{n=1}^{\infty} |\rho_H(n)| < \infty, \qquad H < \frac{1}{2} \tag{2.28c}$$

There are alternative definitions of long-range dependence. We recall that a function L, is slowly varying at zero (respectively, at infinity) if it is bounded on a finite interval and if for all $\alpha > 0 \; \frac{L(ax)}{L(x)} \to 1 \; as \; n \to \infty$ respectively. The spectral density of the autocovariance $\rho(k)$ is given by

$$f(\lambda) := \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-i\lambda k} \rho(k) \quad \forall \lambda \in [-\pi, \pi] \tag{2.29}$$

Definition 2.2 For stationary sequences, $(X_n) n \epsilon \mathbb{N}$, with finite variance, we say that $(X_n)_{n \in \mathbb{N}}$ exhibits long-range dependence if one of the followings holds:
For some constants $c \; and \; \beta \in (0,1)$,

$$\lim_{n \to \infty} \sum_{k=-n}^{\infty} \rho(k) / cn^{\beta} L_1(n) = 1 \tag{2.30a}$$

For some constant $c \; and \; \gamma \in (0,1)$,

$$\lim_{k \to \infty} \rho(k) / ck^{-\gamma} L_2(k) = 1 \tag{2.30b}$$

For some constant $c \; and \; \delta \in (0,1)$,

$$\lim_{\lambda \to 0} f(\lambda) / c|\lambda|^{-\delta} L_3 |\lambda| = 1 \tag{2.30c}$$

where L_1, L_2 are slowly varying functions at infinity, while L_3 is slowly varying at zero.

Lemma 2.1. For fractional Brownian motion, (fBm), B_t^H of Hurst index, $H \in (\frac{1}{2}, 1)$, the three definitions of long-range dependence of Definition (2.2) are equivalent. They hold with the following choice of parameters and slowly varying functions [**3**]:

$$\beta = 2H - 1, L_1(x) = 2H \tag{2.31a}$$

$$\gamma = 2 - 2H, \; L_2(x) = H(2H-1) \tag{2.31b}$$

$$\delta = 2H - 1, \; L_3(x) = \pi^{-1} H\Gamma(2H) sin \pi H \tag{2.31c}$$

Proof. See [4], [23] and [8].

Definition 2.3. We say that an $\mathbb{R}^d$ $-$*valued* random process, $X = (X_t)_{t \geq 0}$, is self-similar or satisfies the property of self-similarity if for every *a* >0 there exists *b* > 0 such that

$$Law(X_{at,} \, t \geq 0) = Law(bX_t, t \geq 0) \tag{2.32}$$

Hence the two process $X_{at}$and bX_thave the same finite-dimensional distribution functions, $i.e. \forall t_0, \ldots, t_n \in \mathbb{R}$,

$$P(X_{at0} \ldots, X_{at_n} \leq x_n) = P(bX_{t_0} \leq x_{0,} \ldots, bX_{t_n} \leq x_n) \quad \forall x_{i,} i, \ldots, n \in \mathbb{R} \tag{2.33}$$

Definition2.4If $b = a^{-H}$in Definition (2.3), then we say that $(X_t)_{t \geq 0}$is a self-similar process with Hurst index H or that it satisfies the property of (statistical) self-similarity with Hurst index H. The NASI considered exhibited long-range dependence with $H > 0.5$ [2].

III. ANALYSIS ON NASI DATA

The seasonal decomposition on NASI data to observe some trends is presented in this section. We find two trends in the computer simulation of the seasonal decomposition of NASI namely:- the multiplicative type with MULT as trend name and NASI MULT as serial name; and the additive type with ADDT as trend name and NASI ADDT as serial name. The choice of NASI MULT was due to the fact that it had a higher average periodicity percentage growth rate than that of NASI ADDT. See [2], [3] for more explanation on Additive Trend: NASI ADDT.

3.1 Multiplicative Trend: NASI MULT

The seasonal multiplicative trend type, MULT, with serial name, NASI MULT, has length of seasonal period 4. In computing the method of moving averages, the observations span is equal to the periodicity and all points are weighted equally. Applying the model specifications from MULT, the seasonal factors (%) for four periods are: Period 1, 99.2%; period 2,100.5%; period 3,100.6% and period 4,99.7%.

3.2 NASI MULT of ARIMA(1,1,1)

The time series modeler trend type of NASI MULT analysed was ARIMA (1,1,1) with the model statistic summary chart given in Table 3 while Table 7 showed the Portmanteau test (Ljung-Box Q(18)) for 18 years (Jan.1990 - Dec.2007).

According to Ljung and Box(1978), the Ljung-Box Test for lack of fit is a diagnostic tool used to test the lack of fit of a time series model. The test is applied to the residuals of a time series after fitting an ARIMA(p,q) model or ARIMA(p,d,q) to the data. The test examines auto correlations of the residuals. If the auto correlations are very small, we conclude that the model does not exhibit significant lack of fit. The Ljung-Box test is implemented using a residual time series, see Ljung-Box test website for more details.

In general, the Ljung-Box test is defined as:

H_0:The model does not exhibit lack of fit.

H_a:The model exhibits lack of fit.

Test Statistic: Given a time series Y of length n, the test statistic is defined as:

$$Q = n(n+2)\sum_{k=1}^{m} \frac{\hat{r}_k^2}{n-k}$$

where $\hat{r}_k$ is the estimated autocorrelation of the series at lag k, and m is the number of lags being tested.

For significance level 0, and the critical region, the Ljung-Box test rejects the null hypothesis (indicating that the model has significant lack of fit) if $Q > X^2_{1-\alpha,h}$ which gives the chi-square distribution table value with h degrees of freedom and significance level α.

Because the test is applied to residuals, the degrees of freedom must account for the estimated model parameters so that $h = m - p - q$, where p and q indicate the number of parameters from the ARIMA (p,d,q) model fit to the data.

IV. RESULTS

Figure 1 is the empirical plot of the NASI MULT showing number (amount in millions) versus date (year). The plot of the residual of ACF and PACF for the multiplicative trend of NASI MULT is shown in Figure 2 while Figure 3 showed its corresponding case for simple seasonal with 24 lags. Figures 2 and 3 appeared to be similar yet not exactly the same. Each of the residual ACF and PACF had mean zero and unit variance for 24 lags of NASI MULT. Also, Figure 4 showed a typical case for the residual ACF and PACF plots for ARIMA(0,1,0). The trend type of NASI MULT is simple seasonal. Tables 1 and 2 respectively showed ACF and PACF of NASI MULT while Table 3 showed the model statistic for ARIMA(1,1,1) of NASI MULT. Considering the trend summary of model fit statistic for simple seasonal of NASI MULT, the descriptive chat is shown in Table 4 with zero predictors and outliers. The model fit statistic had stationary R-squared value as 0.069, and R-squared value as 0.938, while Ljung-Box *Q(18)* statistics, degree of freedom and significance level are 10.620, 16 and 0.832 respectively. The NASI MULT model fit statistic summary chat of ARIMA (0,1,0) of Table 5 showed various fit statistic with respective mean, minimum, maximum, with no standard error. In Table 6, each fit statistic showed a constant value for the respective percentile range from 5 to 95 for the model fit statistic for ARIMA (0,1,0) of NASI MULT. The model fit statistic and Ljung-Box (18) for ARIMA (0,1,0) of NASI MULT in Table 7, showed the Ljung-Box Q(18) estimated statistics, degree of freedom and significance level as 15.728, 18 and 0.612 respectively. The model fit statistic has R-squared value as 0.944 and zero values for both predictors and outliers.

V. CONCLUSION

The residual ACF and PACF for autoregressive fractional integrated moving average (ARFIMA) models may give rise to persistent and anti-persistent behaviour in financial markets similar to fractional noise. This is possible if the time series is broken up into blocks of size m so that the partial sum are regressed on an arbitrary line within each block. The residual of this regression produces an equation. Then for each block the sample variance of the residual is computed where the average of this sample variance over all the blocks becomes proportional to m^{2H}. The value of the Hurst parameter, *H*, obtained in a long-range dependence where *H* >0.5 could be useful in characterising the fractional noise process. Figure 4 is an example of an ARFIMA (0,1,0) (or ARIMA (0,1,0)) process, confirming the idea of the fBm of [17],[18],[19] and [20]. Since the more general ARFIMA (p,d,q) process may include short memory autoregressive (AR) or moving average (MA) processes over a long memory process, it has potential in describing markets. Hence, the result of the analysis in a long-range dependence phenomenon showed that finding the residual functions in financial data could help in further understanding of the financial market trends in a given economy.

Table 1: The Autocorrelations of the NASI MULT

Lag	Autocorrelation	Standard *Error*[a]	Box-Ljung Statistic		
			Value	df	*Sig*[b].
1	.879	.115	57.981	1	.000
2	.764	.115	102.374	2	.000
3	.659	.114	135.952	3	.000
4	.628	.113	166.843	4	.000
5	.569	.112	192.547	5	.000
6	.521	.111	214.455	6	.000
7	.495	.110	234.511	7	.000
8	.467	.110	252.643	8	.000
9	.435	.109	268.682	9	.000
10	.414	.108	283.383	10	.000
11	.393	.107	296.863	11	.000
12	.373	.106	309.194	12	.000
13	.341	.105	319.697	13	.000
14	.297	.104	327.773	14	.000
15	.230	.103	332.720	15	.000
16	.175	.103	335.627	16	.000

a. The underlying process assumed is independence (white noise);
b. Based on the asymptotic Chi-square approximation.

Table 2: PACF of NASI MULT

Lag	**Partial Autocorrelation**	**Standard Error**
1	.879	.118
2	-.039	.118
3	-.016	.118
4	.259	.118
5	-.142	.118
6	.041	.118
7	.157	.118
8	-.104	.118
9	.030	.118
10	.101	.118
11	-.080	.118
12	.035	.118
13	-.006	.118
14	-.138	.118
15	-.097	.118
16	.012	.118

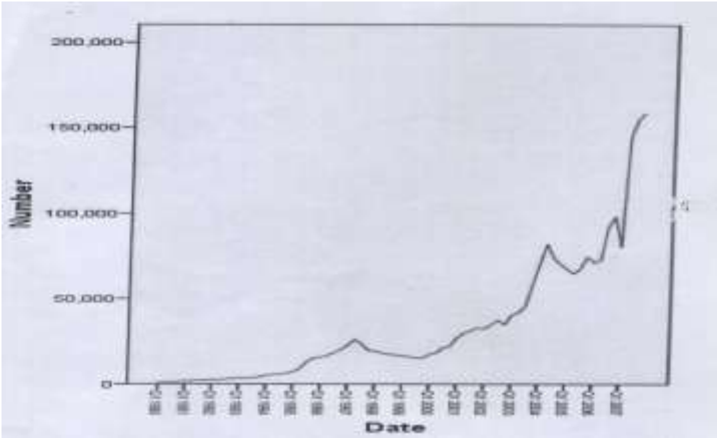


Figure1: NASI MULT showing number (amount) versus Date year)

Table 3: The Model Statistic for ARIMA(1,1,1) of NASI MULT

Model	Number of Predictors	Model Fit Statistic		Ljung-Box *Q(18)*			Number of Outliers
		Stationary R-squared	R-squared	Statistics	DF	Sig.	
NASI MULT	0	.006	.942	10.266	16	.852	0

Table 4: The Model Fit Statistic for Simple Seasonal of NASI MULT

Model	Number of Predictors	Model Fit Statistic		Ljung-Box *Q(18)*			Number of Outliers
		Stationary R-squared	R-squared	Statistics	DF	Sig.	
NASI MULT	0	.069	.938	10.620	16	.832	0

Table 5: The Model Fit Statistic for ARIMA(0,1,0) of NASI
MULT showing mean, SE, minimum and maximum

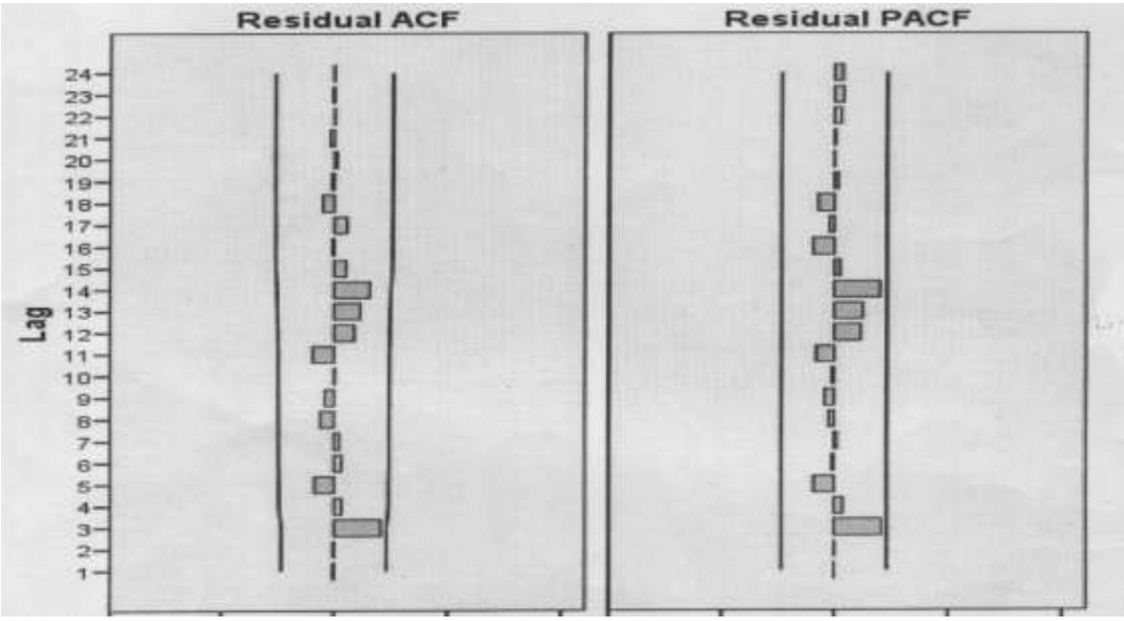
Fit Statistic	Mean	SE	Minimum	Maximum
Stationary R-squared	-2.0DE-015	-	-2.0DE-015	-2.0DE-015
R-squared	.944	-	.944	.944
RMSE	8601.294	-	8601.294	8601.294
MAPE	8.140	-	8.140	8.140
MaxAPE	40.017	-	40.017	40.017
MAE	3539.471	-	3539.471	3539.471
MaxAE	58005.435	-	58005.435	58005.435
Normalized BIC	18.179	-	18.179	18.179

Table 6: The Model Fit Statistic for ARIMA(0,1,0) of NASI MULTshowing percentile range from 5 to 95

Fit Statistic	Percentile						
	5	10	25	50	75	90	95
Stationary R_squared	-2.0DE-015	-2.0DE-015	-2.0DE-015	-2.0DE-015	-2.0DE-015	-2.0DE-015	-2.0DE-015
R_squared	.944	.944	.944	.944	.944	.944	.944
RMSE	8601.294	8601.294	8601.294	8601.294	8601.294	8601.294	8601.294
MAPE	8.140	8.140	8.140	8.140	8.140	8.140	8.140
MaxAPE	40.017	40.017	40.017	40.017	40.017	40.017	40.017
MAE	3539.471	3539.471	3539.471	3539.471	3539.471	3539.471	3539.471
MaxAE	58005.435	58005.435	58005.435	58005.435	58005.435	58005.435	58005.435
Normalized BIC	18.179	18.179	18.179	18.179	18.179	18.179	18.179

Table 7: The Model Fit Statistic and Ljung-Box(18) for ARIMA (0,1,0) of NASI MULT

Model	Number of Predictors	Model Fit Statistics	Ljung-Box *Q(18)*			Number of Outliers
		R-squared	Statistics	DF	Sig.	
NASI MULT	0	.944	15.728	18	.612	0



-1.0 -0.5 0.0 0.5 1.0 -1.0 0.5 0.0 0.5 1.0

Residual

Figure 2: The plot of the residual of ACF and PACF of NASI MULT

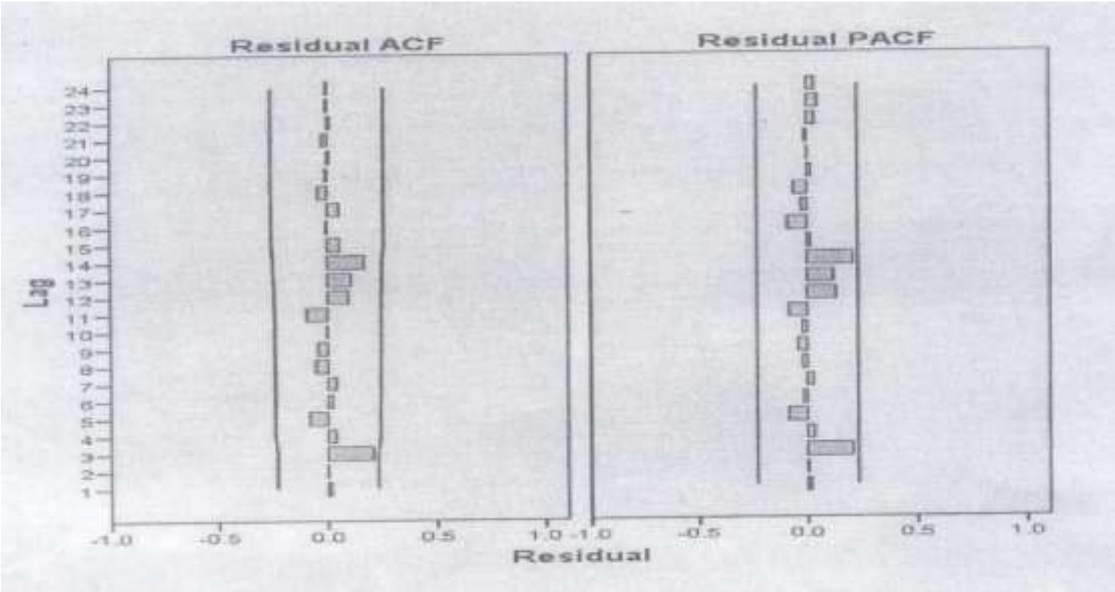


Figure 3: Residual ACF and PACF plots for simple seasonal of NASI MULT

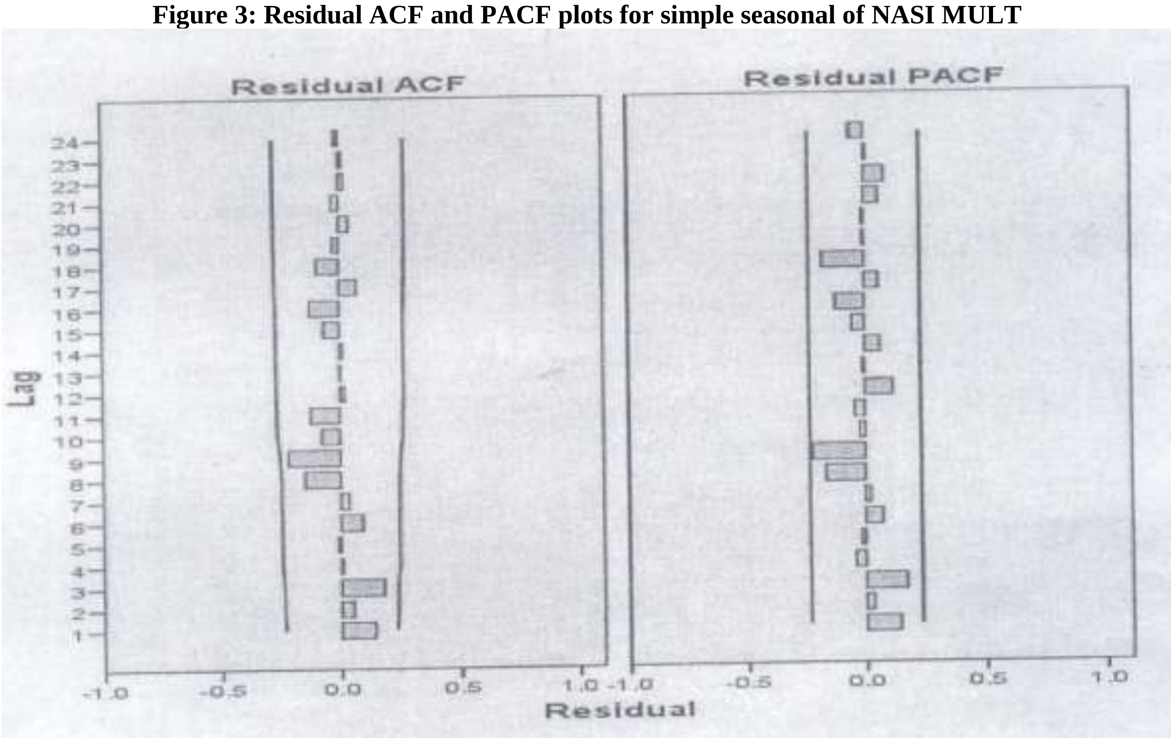


Figure 4: Residual ACF and PACF plots for ARIMA (0,1,0)

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