The International Journal of Engineering and Science (IJES)
|| Volume || 13 || Issue || 4 || Pages || PP 89-97 || 2024 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

Position Control and Stability Analysis of Ball & Beam Mechanism Using Fractional Order Controller

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ABSTRACT
One of the frequently employed systems for analyzing a wide range of actual systems and control schemes is the ball and beam system (BB). Generally, it is an open-loop, fundamentally unstable nonlinear system. The ball and beam system can be represented linearly, and PI-PID control scheme can be is applied. This work does research on fractional-order PI-D controllers. Developing controllers to control the location of the ball while taking into account the gear angle and beam. The Lagrange technique is used to determine the system's ball position. The system's energy balance is the foundation for the system. The design of the open-loop system and closed-loop system is based on the transfer function and state space model. Fuzzy logic is used to tune control parameters for both fractional order and traditional PID controllers. In terms of a bode plot and step response, a fractional-order PID controller's stability study is conducted. The objective of this project is intended in terms of controlling and balancing a ball at a specific location on the beam and stability analysis of the ball's position. This Ball and Beam Model is implemented in MATLAB Simulink for PID and Fuzzy. This work focuses on stability analysis of BB system using PI-PID controllers, fuzzy controller and also using FOTF viewer.

Keywords
Ball and Beam, PID, Fuzzy, FOTF Viewer, Fractional Order Controller, Position Control, Stability

Date of Submission: 05-04-2024 Date of acceptance: 17-04-2024

I. Introduction

The Ball and beam systems are the most basic types of conventional physical systems. Ball and beam systems are frequently used in control system laboratories at academic institutions and commercial settings[1]. This mechanical tool for investigating control systems is simple and portable. In general, it has a physical relation to stabilising the automatic system, such as horizontal stabilising an airliner during take-off and landing in turbulent air flow, in the aerospace, power generation, and chemical process industries, etc. Numerous control strategies can be employed to study the system's instability and govern it by providing it with the proper input. The ball & beam technique consists of Ball, beam, gear, make up a ball and beam system. An open-loop unstable system is the ball and beam system. The design of a ball and beam system is to adjust the ball location by adjusting the beam angle. A controller is always needed to stabilize this system because its open loop response is unstable. In order to stabilize the ball and beam system, numerous controllers, including the fuzzy controller have been utilized. These controllers have been tuned using a variety of conventional tuning methods.

As seen in the illustration below, a ball is placed on a beam and given 1 degree of freedom to roll along its length[2]. The beam is coupled to a lever arm at one end and a servo gear at the other. The servo gear rotates at an angle theta while the lever changes the beam's angle by an alpha value. If the angle is changed from horizontal, gravity compels the ball to roll along the beam. A controller will be developed for this system so that the ball's position can be altered.This technology has real-world applications such aircraft roll control, chemical facilities, and lifting luggage.

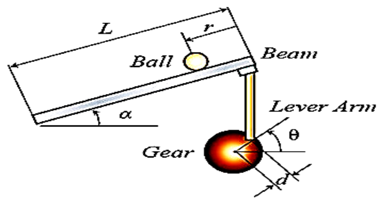


Fig.1. Diagram of the Ball and Beam System

1. System Description

One of the examples and systems frequently used to explain various engineering concepts is the ball and beam system (control, mechanics, sensors etc.). To understand how the ball and beam system works, one first needs to have a working knowledge of system modelling. The ball will be stabilised at the desired location, or the set point, by applying a control system to the gear with the aid of this modelling. The positioning can be changed. The beam has one degree of rolling freedom along its length. A servo motor and a lever arm are fixed to the ends of the beam, respectively. In this section, the arm rotates at a particular angle that matches a particular change in the motor's rotational angle. When the angle is changed from horizontal to any other angle, the ball rolls along the beam. Using a controller, the ball's location can be adjusted. The ball rolls on the beam in the ball and beam system because of the gravitational pull of the slope of the beam[8]. By adjusting the beam's angle up and down, the ball's precise location can be altered.

Mathematical Model:

This section consists of mathematical modeling of the transfer function of the ball and beam system between the gear angle ((s)) and ball position (P(s)). Figure 1 contains the system's schematic diagram. Table 1. lists the values of the ball and beam system's parameters.

The Lagrangian equation of motion is used to calculate the ball's linear acceleration along the mounted beam.

$$(\frac{J}{R^2}+m)\ddot{p}\text{+mgsin}\alpha\text{-mp}\dot{\alpha}^2 \quad ....(1)$$

where the ball's mass, **M**, is represented by the radius, **R,** of the ball. The gravitational acceleration is denoted by **g. J** stands for the ball's moment of inertia.The position of the ball is represented by **p**.The definition of is the beam angle **α**

Equation (1)'s linearization around = 0 can be expressed as follows:

$$(\frac{J}{R^2}+m)\ddot{p}=-mg\alpha \quad ....(2)$$

Also,

$$\alpha=(\frac{d}{L}) \quad ....(3)$$

where,

The length of the beam is indicated by **L**.

The joint of the lever arm's distance from the gear's center is measured in terms of **d**.

Substituting equation (3) into equation (2),

$$(\frac{J}{R^2}+m)\ddot{p}=-mg(\frac{d}{L})\theta \quad ....(4)$$

If we apply the Laplace transform to equation (4), we get,

$$G_{BB}(s)=\frac{P(S)}{\theta(S)}=-\frac{mgd}{L(\frac{J}{R^2}+m)}\frac{1}{s^2} \quad ....(5)$$

$G_{BB}(s)$is the ball and beam system's transfer function

Table-1

Parameters used for Ball-Beam System

Sl.	*Parameter*	*Units*	*Size / Capacity*
1.	Mass of the ball (m)	Kg	0.12
2.	Ball radius (R)	m	0.015
3.	Gravitational Acceleration (g)	m/s^2	9.8
4.	Beam Length (L)	m	0.5
5.	Gear Radius (d)	m	0.05
6.	Ball Inertia (J)	$kg.m^2$	$150*10^{-3}$

Table 1. Parameters used for Ball and Beam System[5]

G_{BB} (s) can be expressed as after substituting parameter values from Table 1 in equation 5 as follows:

$$G_{BB}(s)=\frac{P(S)}{\theta(S)}=\frac{0.7}{s^2} \quad ....(6)$$

2. Simulink Models

The Simulink model for the ball and beam system was developed in MATLAB software 2016b version

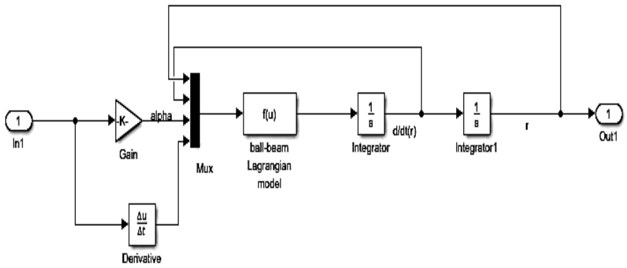


Fig 2. Ball and beam system open loop simulation

The developed mathematical model of the system being implemented in MATLAB-simulink as indicated in fig 2.

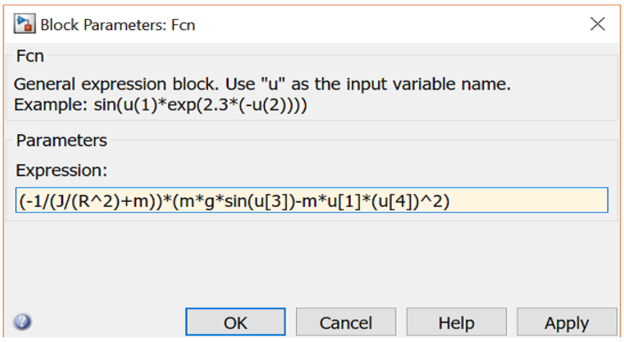


Fig 3. function block of open-loop model

- Open-loop response of BB Model

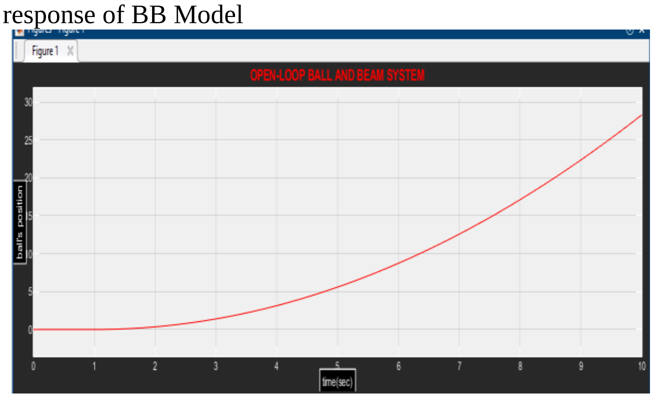


Fig 4. open-loop response of ball and beam system

In this above graph shown, the x-axis indicatethe time taken by the ball to settle in seconds, and the y-axis indicate the ball's position. Because the ball fell off at the end of the beam, it is clear from this plot that thesystem is unstable in an open loop mode[6]. Consequently, a method of regulating the ball's position is required in this system[7].

<u>Simulation Diagram With Different Controllers:</u>

The Simulink model for the ball and beam system with different control modes was developed using MATLAB software

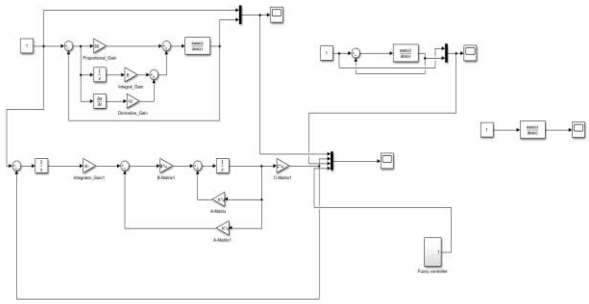


Fig 5. simulation of ball and beam system without controller,with PID controller,with Fuzzy controller

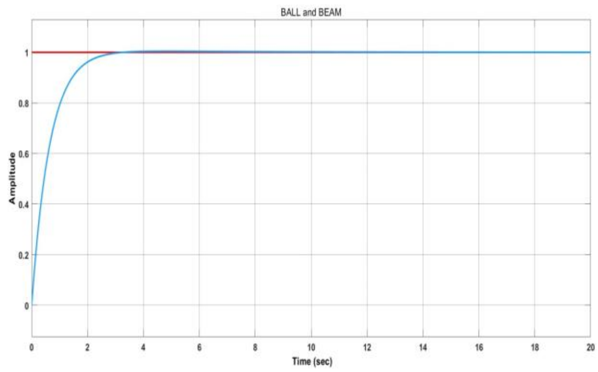


Fig 6. open- loop response of the ball and beam system

In this plot, x-axis represents time in seconds that ball taken to settle and Y-axis represents position of the ball.This plot makes it obvious that the system is in an open-loop,The ball takes 0.28sec to settle on the beam.

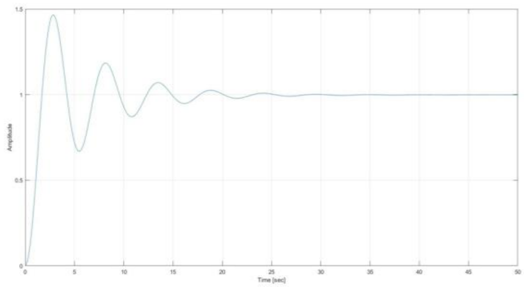


Fig 7. System response withk_p=15,k_d=1.3,$k_{i=}$ 5

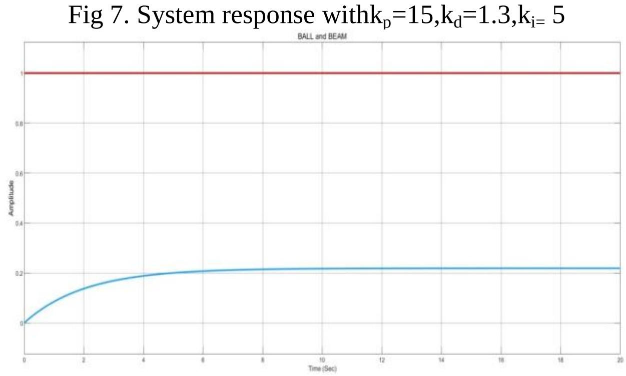


Fig 8. Closed-loop response of the ball and beam system

In this plot, x-axis represents time in seconds that ball taken to settle and Y-axis represents position of the ball.This plot makes it obvious that the system is closed-loop, The ball takes 0.22sec to settle on the beam.

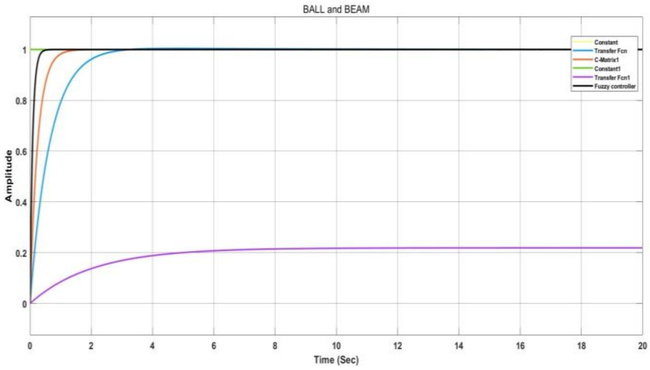


Fig 9.Comparison of step Responses of ball and beam without controller,with PID controller,with Fuzzy controller

In this plot, the x-axis indicate the time taken by the ball to settle in seconds, and the y-axis measures its current location on the beam. This plot clearly compares the step responses of the ball and beam without controller and fuzzy controllers for position control and ball settling time. Therefore, a method of regulating the position of the ball is used in this system.

3. Result Analysis

Sl.	Control Mode	Final value indicating position control	Settling time indicating equilibrium in seconds
1	Open-loop	0.28	12 seconds for a constant magnitude of 0.28
2	Closed-loop	0.22	10 seconds for a constant magnitude of 0.22
3	PID Controller	1	3 seconds
4	State space approach	1	1.7 seconds
5	Fuzzy controller	1	0.45 seconds

Table2. Comparative result analysis of ball and beam system from simulink model

The table 2 represent the result analysis of ball and beam system using different control modes such as open-loop,closed-loop,PID controller, state space approach, fuzzy controller for position contol and settling time of ball on the beam. It can be verified that using controllers, the position control of the BB system is much accurate over open-loop mode. Also, the fuzzy control topology is much efficient method that can be employed for accurate position control applications like BB system.

FOMCON
(Fractional Order Modelling and Control) :

- The Fomcon toolbox for MATLAB is fractional order calculus toolbox for system modelling and control design
- Which is advance way or method for illustrate fractional calculus easier
- Due to the availability of graphical user interfaces and extensive capabilities, it is a product ideal for both novices and more demanding users.
- It focuses on extending traditional control techniques with fractional calculus notions.

Steps for FOTF viewer Use :

i. Download and Install the "FOTF viewer" toolbox from any searching tool.
ii. After successful installation process go to matlab Command prompt
iii. Type "Loadset" then enter in the command prompt
iv. It will indicate "Load test set successfully"
v. Now Type FOTF viewer in the command prompt, it will redirect into the file given below as shown in fig. 4.
vi. There we can check sability of any order transfer function with out applying any calculations.

The transfer function of ball and beam is

$$G(s) = \frac{7.1631s^{1.8036}+143151\,s^{0.9191}+1.001}{s^{2.9191}+7.1631\,s^{1.8036}+14.3150\,s^{0.9191}+1.001} \ldots(7)$$

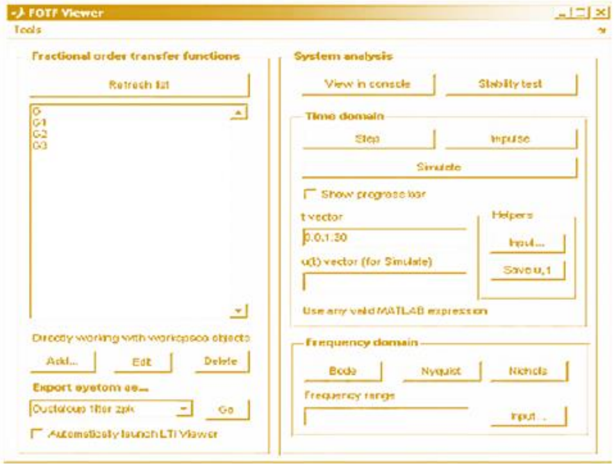


Fig 10. First view of FOTF viewer

After processing in FOTF viewer, the stability test results in declaring the system to be stable or unstable. For the considered system, the BB system is stable for q=0.1 (0<q<1) as shown in Fig 10. indicating the system as STABLE.

Fig 11. Stability test through FOTF Viewer

4.3 Stability Analysis

Stability analysis of non linear ball and beam system is also done by using Root locus,Nyquist plot, Bode plot.These three plots signifies that the non-linear ball and beam system is stable.

i. ***Root Locus Plot***

Based on the fractional order transfer function of the BB system,it can be observed that the root locus has two branches that depend on the difference between the orders of the numerator and denominator. The two root locus branches will orgibate from the respective open-loop poles and converge at the available open-loop zeroes.

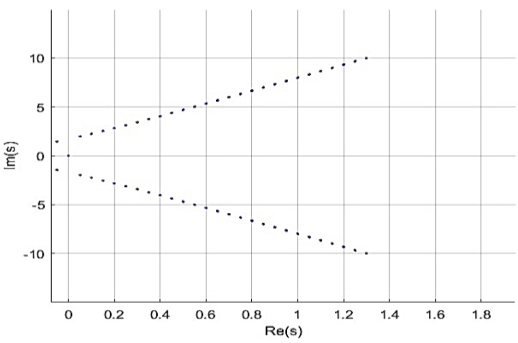


Fig 12. Root locus plot

ii. ***Nyquist Plot***

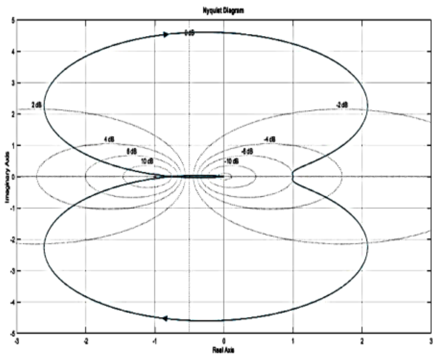


Fig 13. Nyquist plot

For the approximated second order transfer function of the considered system, there are no closed loop poles pocated in the right half region. Therefore, there must not be any encirclement around the point (-1+j0), which can be observed in the plot shown in Fig. 13.

Nyquist stability criterion states the number of encirclements about the critical point (1+j0) must be equal to the poles of characteristic equation, which is nothing but the poles of the open loop transfer function in the right half of the ‘s’ plane. The shift in origin to (1+j0) gives the characteristic equation plane.

Based on the stability conditions through Nyquist plot, we can conclude that the considered BB system is stable.

iii. ***Bode Plot:***

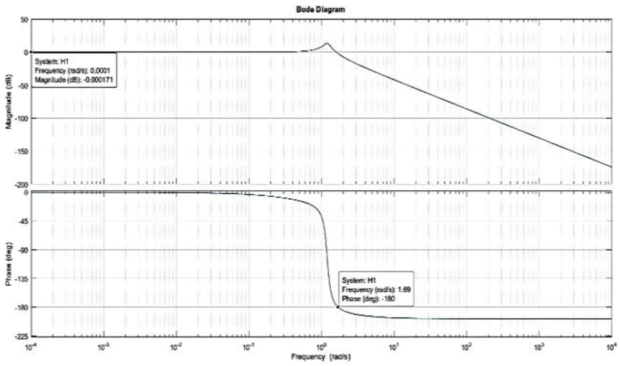


Fig 14. Bode plot

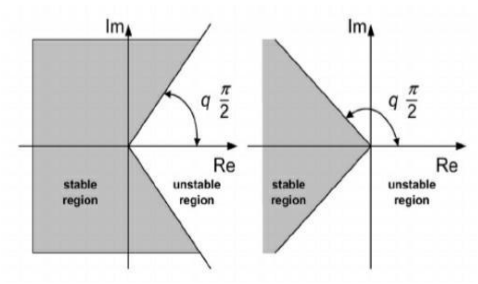
Referring to Fig. 14, it can be observed that the phase crossover frequency (ω_{gc}=1.8 rad/sec) is greater than the gain crossover frequency (ω_{pc}=0.0001 rad/sec), which is the primary condition to judge on stability through bode plot. Therefore, it can concluded that, the considered BB system is stable with reference to gain and phase crossover frequencies. Also, since both phase and gain margins are positive, the system with the approximated transfer function is stable in the sense of Bode plot.

<u>Stability of Fractional Ordered System:</u>

When dealing with dynamical systems and their behaviours, one of the most frequently used concepts in literature is "stability." Stability theory, in mathematical terms, deals with the converging solutions of differential or difference equations. A system (LTI) is considered to be stable if the roots of the polynomial features have a negative real part. The stability of a fractional order system (LTI) is not the same as that of an integer one. The roots of a fractional order system may lie on the right half of the complex plane[3].

a) $0<q<1$ b) $1<q<2$

In order to maintain a system's stability, we know that only the poles are significant. As a result, just the denominator is used to measure stability, and the stability of a FOTF is not affected by the numerator.



According to this x-axis is real and y-axis is imaginary of pole zero map, hence it shown that the fractional order system G(s) is stable. The function is checked is stable the denominator of G(s) with q = 0.1 . Here we can see that G(s) has more stability region. Now it easy to summarize here that the system is stable even if its poles lines on the right side of the imaginary axis.

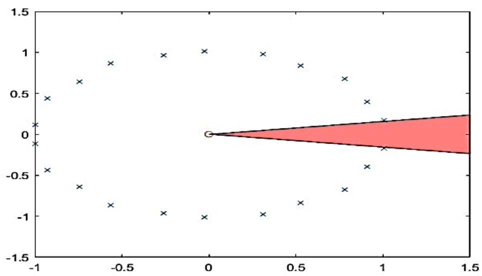


Fig 15. Poles position in complex plane for G (s)

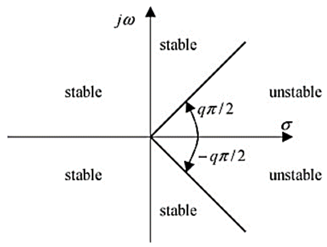


Fig 16. Stability region of FO-LTI system with order $0 < q \leq 1$[4].

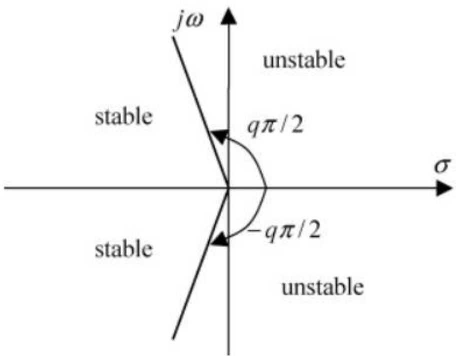


Fig 17. Stability region of FO-LTI system with order $1 \leq q < 2$[4].

The fractional ordered transfer function of the consideredball and beam system [3] is given in eq. no. (7). With reference to the obtained non-linear equality, the stability analysis has been studeied using FOTF Viewer in MATLAB, version 2023b.

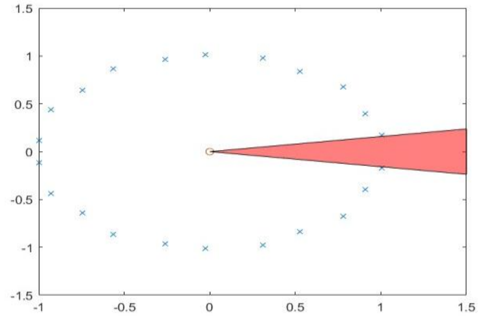


Fig 18. Stability test through FOTF Viewer for 0<q<1

II. Conclusions

A ball and beam system's physical dynamics and operation have been investigated. The system's mathematical model was constructed by taking into account the parameters of a real-time ball-beam system. The simulation was run in open-loop mode, and closed-loop mode was implemented in MATLAB Simulink using a PID controller and a fuzzy logic controller for the ball and beam system. Then, a comparison of closed loop activities using conventional and fuzzy controllers was performed in order to validate fractional controller effectiveness and deliver superior performance than other controllers employed. To avoid difficulties in mathematical fractional calculus, the system's stability is examined using the FOTF viewer advance method hence verified system is stable. The simulation findings for the BB system's fuzzy controller validate its improved performance over other controllers.

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