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Studying the presence of the Fibonacci sequence and the Golden Ratio in nature along with the general public's perception of the concept.

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ABSTRACT

This paper studies the concept of Fibonacci's sequence and the Golden Ratio alongside its presence in nature and the public's perception of this mathematical concept. The latter part is done by examining a collection of data from an online survey consisting of 10 questions, 5 being multiple choice style questions and the other 5 being short answer type questions. This research shows that although approximate instances of the Fibonacci Sequence and the Golden Ratio are present in nature, the majority of the public have limited knowledge of the concept. It is also observed that those involved in more technical fields involving some form of mathematics know more about the concept than those who are involved in fields with less of a need for mathematics.

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I. Introduction

The Fibonacci Sequence. Everybody has heard of it, everybody has seen it but do they really know what it is? This sequence, although not widely known about in detail, can be found everywhere around us in nature, manmade art and even in living creatures. Fibonacci's sequence is a series of numbers in which a given number is the sum of the preceding two in the series and the Golden Ratio, subsequently, is a relationship between two successive numbers in the Fibonacci series wherein the ratio of two values is roughly equal to the ratio of the sum of the two to the larger number out of them. These concepts are explored further in the contents of this paper alongside the extent of public knowledge and the public's perspective on said concepts.

II. Review of Literature

Fibonacci's sequence

The Fibonacci sequence is a series of numbers where each number is the sum of its two preceding numbers. The numbers part of this series are known as Fibonacci Numbers.
The most common/simplest Fibonacci series consists of the numbers:
1,1,2,3,5,8,13,21,34,55,89,144,etc

The Fibonacci sequence's formula through which you can determine the terms of such a sequence is given by:
$\mathbf{F_n = F_{n-1} + F_{n-2}}$**, where n > 1**

Fibonacci's rabbits

Fibonacci's rabbit problem is a method to represent the Fibonacci sequence and an application of it as well. The question posed by the problem is how many total pairs of rabbits will you have after 1 year if you start with 1 pair and they each take 1 month to mature and produce 1 other pair each month afterwards. The table that gives the solution to said problem is given below:

month	J	F	M	A	M	J	J	A	S	O	N	D	J
juvenile	1	0	1	1	2	3	5	8	13	21	34	55	89
adult	0	1	1	2	3	5	8	13	21	34	55	89	144
total	1	1	2	3	5	8	13	21	34	55	89	144	233

Thus as can be seen, the number of total rabbit pairs given by the table is 1,1,2,3,5,8,etc or otherwise, the Fibonacci sequence. Thus the solution to said problem is given by the relation:
$\mathbf{F_n = F_{n-1} + F_{n-2}}$**, where n > 1**

The Golden Ratio

Two quantities are said to be in golden ratio when their ratio is equivalent to the ratio of the sum of the two quantities to the larger of the two. ie:
a/b = (a+b)/a when a>b

The Golden Ratio is denoted by the greek letter phi (***Φ***). ***Φ*** is an irrational number with the value

$\Phi = (1 + \sqrt{5})/2 = 1.61803398….$

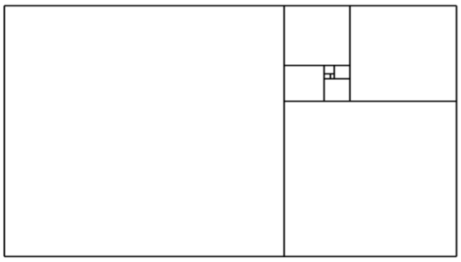
Another thing to be noted is that the irrational constant ***Φ*** satisfies the quadratic equation:
$\Phi^2 = \Phi + 1$

Binet's Formula
Binet's Formula was derived by the mathematician Jacques Philippe Marie Binet. It is used to find the nth term of the Fibonacci sequence. Binet's Formula is given by:

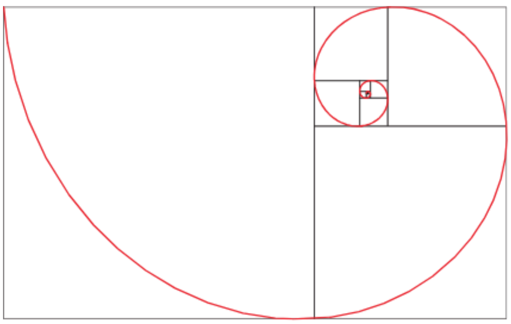
$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

The Golden Rectangle
The Golden Rectangle, known to some as the perfect rectangle, is a rectangle in which the ratio of its length to its width is the Golden Ratio. It appears in many works of art and architecture due to being known as the most visually pleasing rectangle.The Parthenon of ancient Greece is the most famous example of the use of the golden rectangle.

You can divide a golden rectangle into another golden rectangle and a square. You can repeat this pattern with each smaller rectangle



If arcs are drawn along the squares as shown below, it forms a spiral of sorts.



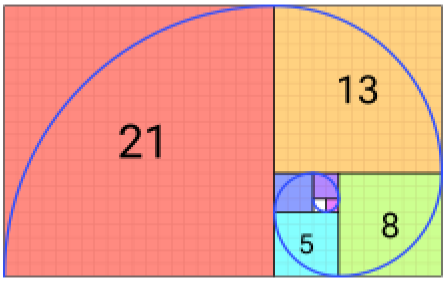
A golden rectangle can be constructed with only a straightedge and a compass in four steps:

1) Drawing a square
2) Drawing a line from the midpoint of one side of the square to an opposite corner
3) Using that line as the radius to draw an arc that defines the height of the rectangle

4) Completing the golden rectangle

Golden Spirals

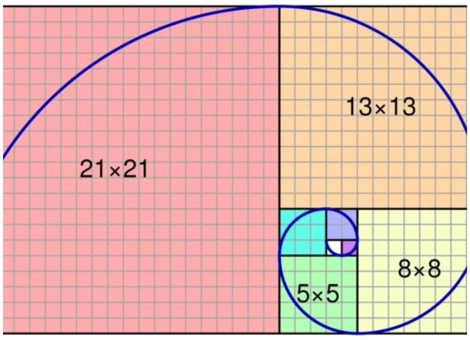
In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio. That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes.



The Golden spiral is defined by creating a spiral of golden rectangles, each with a length to width ratio of the golden ratio (1.618…), and each expanding in size by a golden ratio of the one before it.

Fibonacci Spirals

The Fibonacci spiral is defined by creating a spiral of squares that increase in size by the numbers of the Fibonacci sequence, so 1, 1, 2, 3, 5, 8, 13, 21,.....



Due to the close relationship between the Fibonacci series and the Golden Ratio, these spirals are almost identical as you progress in the Fibonacci sequence. Though, It is convenient to note that while the Golden Spiral diverges from the Golden Ratio, the Fibonacci Spiral converges on it.

Fibonacci Numbers and the Golden Ratio in nature

Human bodies have the golden ratio, from the navel to the floor and the top of the head to the navel. You'll also find it in the shape of hurricanes, elephant tusks, star fish, sea urchins, ants and honeybees. While not in every structure or pattern, it is a significant discovery by Leonardo Fibonacci. The Golden Ratio can be seen in drawings and scriptures from over 4000 years ago, proving its use has been active long before we think.

More examples of the Golden Ratio in nature and the universe are given by:

1) Seed heads. The head of a flower produces seeds at the center which migrate towards the outside in a spiral pattern to fill all the space.
2) Pineapples, Romanesco broccoli and cauliflowers. These also follow the Fibonacci process.
3) Pine cones. The seed pods on a pinecone are arranged in a spiral pattern, each cone has a pair of spirals which spiral up in opposite directions.
4) Tree branches. The sequence is seen in the way tree branches form or split: the trunk grows until it produces a branch, which creates two growth points. Then one of the new stems branches into two, leaving the other dormant.
5) Chameleon Tails and Snail Shells
6) The eye, fins and tail fall at golden ratio sections.
7) Animal flight patterns. When a hawk approaches its prey, its sharpest view is at an angle to their direction of flight – an angle that's the same as the spiral's pitch.
8) Spiral galaxies. The Milky Way has several spiral arms, each a logarithmic spiral of about 12 degrees.

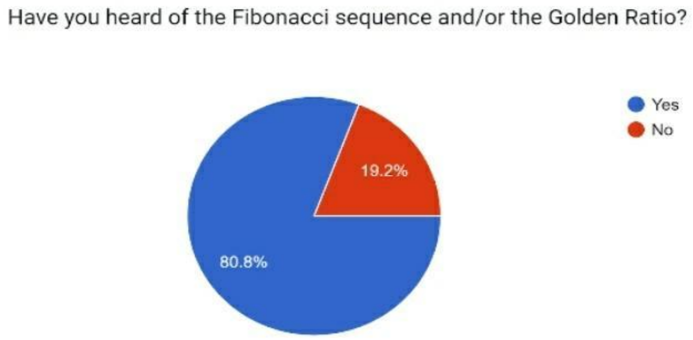
III. Methodology

The Methodology employed to gather data was in the form of a survey sent out online. The survey consisted of 10 questions(consisting of 5 Multiple Choice Questions and 5 Short Answer Questions) asking the participant about the extent of their knowledge on the topic of Fibonacci Numbers and the Golden Ratio as well as how useful said topic would be in their chosen field of study or career. The number of participants recorded was 50. The ages of the participants ranged from 15 to 50 years of age.

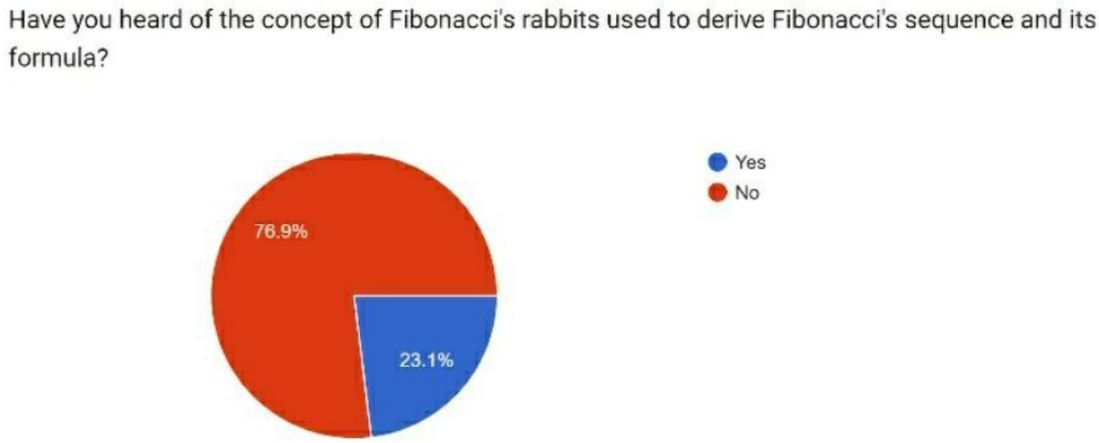
IV. Results and Conclusion

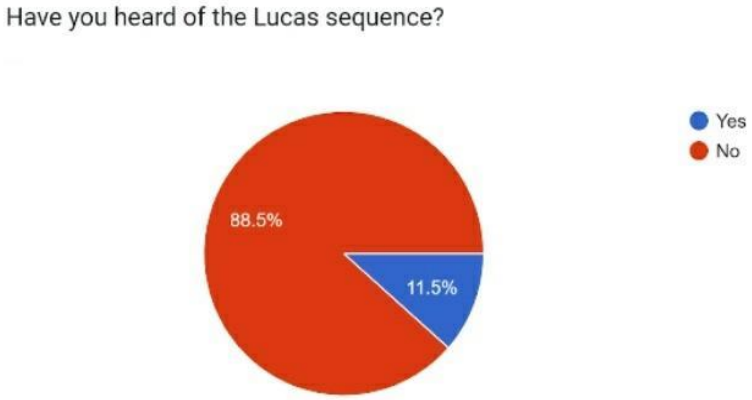
In this section, all the data collected from the survey will be analyzed.

The first question was about whether the participant had ever heard of the Fibonacci Sequence and/or the Golden Ratio. Out of the 50 participants, 80.8% had heard of the concept.



In contrast to the majority of the participants having heard of the Fibonacci Sequence, only 23.1% knew of the concept of Fibonacci's rabbits used to derive the sequence and even fewer, 11.5%, knew of the concept of Lucas Numbers which is a number sequence closely related to the Fibonacci Sequence.





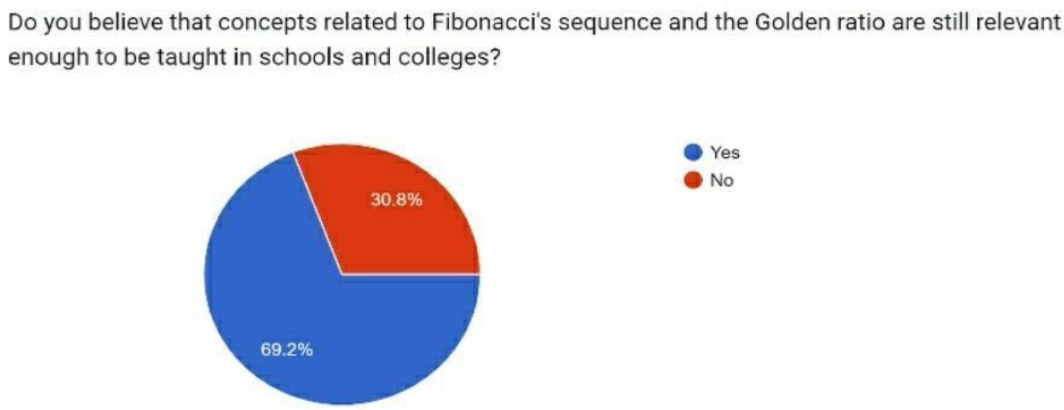
From the series of Short Answer Questions used to assess the participant's extent of knowledge on the Fibonacci Series and its presence in the natural world, it was found that most of the participants possessed a vague knowledge of its presence around us, with only 27 participants being able to name specific applications of the concept. The detailed answers of the participants in said questions will be analyzed in the following paragraphs.

When the participants were asked what career they are pursuing or are planning to pursue and if the concept of the Fibonacci Sequence might help them one way or another in said career, it was found that most of the participants who chose a career heavily or loosely related to Mathematics (such as Investment Banking, Computer Science, Mechanical Engineering, Physics, etc) answered that yes, knowing the concept would help them in their field while most of the participants who chose a field mostly unrelated to Mathematics (such as Law, Political Science, Real Estate, etc) believed that knowing the concept would serve no use to them professionally. There were two outliers in this Question, with one participant answering that no, it wouldn't be useful to them although they were planning to pursue Electrical Engineering, a field in which Math is heavily involved and the other being a participant pursuing Law claiming that the concept of Fibonacci Numbers would be useful to them.

On being asked to name specific applications of the Fibonacci Sequence, it was found that most participants, regardless of the field of study they were pursuing, were unable to name too many. Out of the 20 that were able to answer this question, 90%(18 out of the 20) were either Engineering or Computer Science aspirants, naming one common application which was that it could be used to help create various computer algorithms and with pattern tracing and prediction.

The overall consensus was that the Fibonacci Sequence and the Golden Ratio is found and used in nature and other professional fields involving mathematics. This lines up with the article from TreeHugger that states that approximate, but not exact (since it is an irrational number), instances of the Golden Ratio can be seen in nature such as in Snail Shells and Ocean Waves as mentioned above. Similarly, a 2020 article from the University of Edinburgh Science Media also states that the Ratio cannot be applied precisely in nature, with it being an irrational number, but rather as an approximation using the example of The Great Pyramid being only an approximate golden pyramid, not a precise one.

Interestingly, 79.2% of the participants believed the concepts of Fibonacci Sequence and the Golden ratio are still relevant enough to be taught in schools and other educational institutes such as colleges.



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