

# Advanced Load Flow Analysis and Cost Optimization in Electrical Transmission Networks

Yasser HassanElhaddad<sup>1</sup>, Adel Alsanad<sup>2</sup>

<sup>1</sup>*Department of Electrical Power, The Public Authority for Applied Education and Training, Higher Institute of Energy, Kuwait*

<sup>2</sup>*Department of Electrical Network, The Public Authority for Applied Education and Training, Higher Institute of Energy, Kuwait*

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## I. Introduction

Transmission network expansion is critical to fulfilling the rising energy demands of urban areas. As cities grow, so does the need for a vital, dependable, long-term electricity source. However, the problem is optimizing the current infrastructure and strategically expanding it to ensure uninterrupted energy distribution. This study dives into the intricacies of transmission network expansion planning, addressing the critical need for cost-effective measures in the face of increased energy consumption. This project intends to create solutions that balance the demand-supply relationship by investigating novel approaches and incorporating cutting-edge technologies. The primary goal is to develop informed growth plans that encourage network capacity, environmental sustainability, economic feasibility, and long-term energy resilience.

## II. Literature Review

The discipline of wind turbine monitoring for conditions has made great progress because to the groundbreaking work of Pandit and Infield (2018) on Gaussian process operating curves. Through the application of advanced methodologies, they yielded significant insights into the health and operating efficiency of wind turbines, ultimately augmenting the dependability and efficiency of wind power systems. The seminal work on electric power systems by Weedy et al. (2012) is still essential since it offers a thorough grasp of system integration and operation. Their research has acted as a foundation for other studies, directing the creation and upkeep of resilient electrical grids across the globe.

A major paradigm shift is represented by Barzkar and Ghassemi's (2020) research on electric power systems in airplanes, which reflects the aviation industry's continual transition toward electrification. Their investigation of the difficulties and possibilities in incorporating electric power systems into airplane frames highlights how electrical technologies have the ability to revolutionize the aviation industry. Furthermore, in the context of sustainability, Talbi et al. (2020) study on resolving environmental and economic power dispatch concerns is essential. They offered practical ways for maximizing power system operations by utilizing the BFGS-AL algorithm, highlighting the significance of striking a balance between environmental awareness and economic feasibility.

A promising future for the field is outlined in Cheng and Yu's (2019) prospective viewpoint on machine learning technology applied to smart energy systems. The overview and insights they provided on the use of AI in energy research illuminated how machine learning algorithms have the ability to completely transform the way power systems are managed, controlled, and monitored. Collectively, these research projects not only advance our knowledge of electrical power systems but additionally open new avenues for creative thinking that will influence the development of sustainable and effective energy networks in the future.

## Load Flow Solutions

Power-flow studies are of great importance in planning and designing the future expansion of power systems as well as in determining the best operation of existing systems. The load flow problem consists of a given transmission network where all lines are represented by a  $\pi$  equivalent circuit and transformers by an ideal voltage transformer. Generators and loads represent the boundary conditions of the problem solution. Mathematically, the load flow requires the solution of a system of simultaneous nonlinear algebraic equation. The function of load flow study can be summarized in the shown figure:

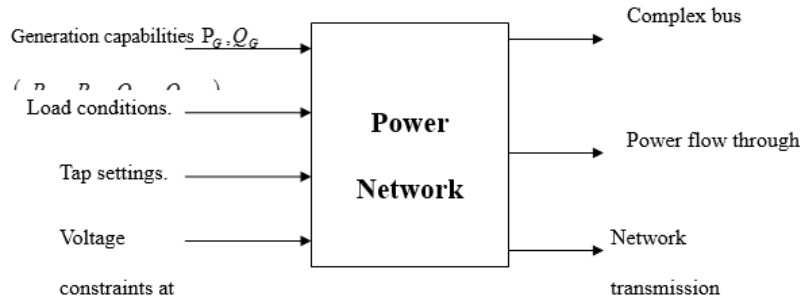


Figure 1: The function of load flow study

Every bus in a power network is associated with four quantities; the real and reactive injected power, the bus voltage magnitude and the phase angle. The general practice in power-flow studies is to identify three types of buses in the network:

**1-slack (reference) or swing bus**

One bus has to be specified as a slack bus. The active and reactive power of the slack bus serve the load and supply the network transmission losses. At this bus, the voltage magnitude and phase angle are specified.

**2-load buses**

In this designated, buses at which both the active and reactive power are specified.

**3-voltage control buses**

Buses at both the voltage magnitude and active power are specified which are termed as P-V buses. Therefore, for a power system comprising a total number of nodes equal (n), and (a) set of [ 2×(n-1)] independent equations may be evaluated. The solution of this set of equations will provide the required load flow results.

**3.1 Power System Equation:**

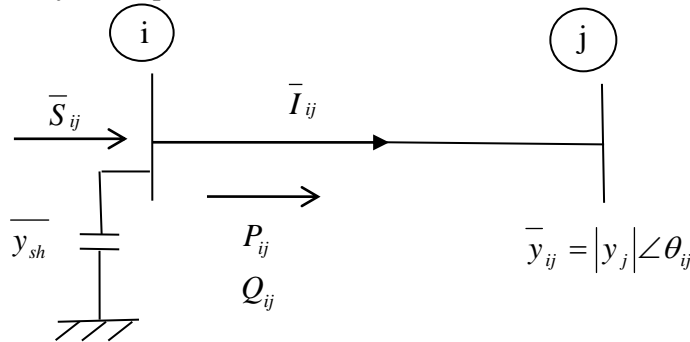


Figure 2: Representation of an Element of Power System

**➤ Polarform:**

$$\begin{aligned} \bar{S}_{ij}^* &= \bar{V}_i^* \bar{I}_{ij} \\ &= \bar{V}_i^* [\bar{V}_i - \bar{V}_j] \bar{Y}_{ij} + V_i^2 \bar{Y}_{sh} \\ &= [\bar{V}_i^* \bar{V}_i - \bar{V}_i^* \bar{V}_j] \bar{Y}_{ij} + V_i^2 \bar{Y}_{sh} \\ &= |V_i|^2 \bar{Y}_{ij} - \bar{V}_i^* \bar{V}_j \bar{Y}_{ij} + V_i^2 \bar{Y}_{sh} \\ \therefore P &= V_i^* Y_{ij} \cos Q_{ij} - V_i V_j Y_{ij} \cos(S_j - S_i + Q_{ij}) \\ Q_{ij} &= -V_i^2 Y_{ij} \sin Q_{ij} + V_i V_j Y_{ij} \sin(S_j - S_i + Q_{ij}) + V_i^2 Y_{sh} \\ \therefore P_{ij} &= V_i V_j Y_{ij} \cos(S_i - S_j - Q_{ij}) - V_i^2 Y_{ij} \cos Q_{ij} + V_i^2 Y_{sh} \\ Q_{ij} &= V_i V_j Y_{ij} \sin(S_i - S_j - Q_{ij}) + V_i^2 Y_{ij} \sin Q_{ij} \end{aligned} \tag{2.1}, (2.2)$$

$Q_{ij}$  is the angle of  $\overline{Y_{ij}}$ .

► **Cartesian form:**

Using the equation:

$$\overline{S_{ij}}^* = V_i^2 \overline{Y_{ij}} - \overline{V_i}^* \overline{V_j} \overline{Y_{ij}} \quad (2.3)$$

substitute  $\overline{Y_{ij}} = -\overline{Y_{ij}}$

$$\therefore \overline{S_{ij}}^* = -V_i^2 \overline{Y_{ij}} + \overline{V_i}^* \overline{V_j} \overline{Y_{ij}}$$

substitute

$$\overline{Y_{ij}} = G_{ij} + jB_{ij}$$

$$\overline{V_i} = e_i + jf_i \quad (2.4)$$

$$\overline{V_j} = e_j + jf_j$$

Then,

$$\begin{aligned} \overline{S_{ij}}^* &= -(e_i^2 + f_i^2)(G_{ij} + jB_{ij}) - (e_i - jf_i)(e_j + jf_j)(G_{ij} + jB_{ij}) \\ &= -(e_i^2 G_{ij} + f_i^2 G_{ij}) - j(e_i^2 B_{ij} + f_i^2 B_{ij}) + (e_i - jf_i)[(e_j G_{ij} - f_j B_{ij}) + j(f_j G_{ij} + e_j B_{ij})] \end{aligned}$$

$$\therefore P_{ij} = -(e_i^2 G_{ij} + f_i^2 G_{ij}) + e_i(e_j G_{ij} - f_j B_{ij}) + f_i(f_j G_{ij} + e_j B_{ij})$$

$$Q_{ij} = (e_i^2 B_{ij} + f_i^2 B_{ij}) + e_i(f_j G_{ij} + e_j B_{ij}) + f_i(e_j G_{ij} - f_j B_{ij})$$

Also, as for as the injected power is concerned, we have:

$$P_i = \sum_{j=1}^n e_i(e_j G_{ij} - f_j B_{ij}) + f_i(f_j G_{ij} + e_j B_{ij}) \quad (2.5), (2.6)$$

$$Q_i = \sum_{j=1}^n f_i(e_j G_{ij} - f_j B_{ij}) - e_i(f_i G_{ij} + e_j B_{ij})$$

### 3.3 Solution Techniques:

#### 1) Gauss Iterative Method using $\overline{Y}$ bus:

Using the network performance equation in bus frame of reference. That is :

$$\overline{I}_{bus} = [Y_{bus}] \overline{E}_{bus}$$

for a particular bus p, we have:

$$\overline{I}_p = \sum_{q=1}^n \overline{Y}_{pq} \times \overline{E}_q \quad (2.7)$$

$$\overline{I}_p = \overline{Y}_{pp} \times \overline{E}_p + \sum_{q=1, q \neq p}^n \overline{Y}_{pq} \times \overline{E}_q \quad (2.8)$$

$$\therefore \overline{E}_p = \frac{1}{\overline{Y}_{pp}} \left[ \overline{I}_p - \sum_{q=1, q \neq p}^n \overline{Y}_{pq} \times \overline{E}_q \right] \quad (2.9)$$

$$p = 1, 2, \dots, n$$

$$p \neq s$$

when,

$$\overline{I}_p = \frac{\overline{S}_p^*}{\overline{E}_p} \quad (2.10)$$

when  $s$  is the slack bus.

The sequence of the calculation steps are as follows:

1. set an initial guess to bus voltages  $|E_i|$ ,  $i=1, 2, \dots, n \neq s$ .
2.  $p = 1$ .
3. if  $p = s$  set  $p = p + 1$ .
4. calculate  $\overline{I}_p$  using (1).
5. calculate  $\overline{E}_p$  using (2).
6. If  $p = n$  go to step 7; otherwise, go to set  $p = p + 1$  and go to step 3.
7. If the difference between the new set and initial guess of bus voltage is less than a prescribed tolerance stops; otherwise, set the new set as the initial guess and go to step 2.

**2) Voltage controlled buses (P-V buses):**

In voltage controlled buses both the bus voltage magnitude and real power are specified. The Gauss Seidel iterative technique, mentioned before, has to be slightly modified to allow dealing with buses. The sequence of the calculation steps are as follows:

1. set  $p = 1$ .
2. if  $p = s$ , set  $p = p + 1$ .
3. if bus  $p$  is not a voltage controlled bus go to step 8; otherwise, move to next step.
4. calculate  $S_p^{(k+1)} = \tan^{-1} \frac{f_p^k}{e_p^k}$ .
5. calculate  $e_p^{(k+1)} = |E_p| \cos S_p^{(k+1)}$ ,  $f_p^{(k+1)} = |E_p| \sin S_p^{(k+1)}$ .
6. calculate  $Q_p^{(k+1)}$  using power flow equations.
7. if  $Q_p^{(k+1)} > Q_p^{(max)}$   $\rightarrow$  set  $Q_p^{(k+1)} = Q_p^{(max)}$ ,  
if  $Q_p^{(k+1)} < Q_p^{(min)}$   $\rightarrow$  set  $Q_p^{(k+1)} = Q_p^{(min)}$ , if  $Q_p^{(k+1)}$  is within limits  
( $Q_p^{(max)} > Q_p^{(k+1)} > Q_p^{(min)}$ ).
8. calculate  $(\overline{I}_p)^{k+1}$  using (1).
9. calculate  $\overline{E}_p^{(k+1)}$  using (2).
10. if  $p = n$  go to step (11); otherwise, set  $p = p + 1$  and go to step (2).
11. check convergence.

Notes: The process starts with an initial guess to  $e_p, f_p$  and is repeated until convergence is achieved. Therefore, at any iteration constant  $k$ , we have;

$$\overline{E}_p^{(k)} = \frac{1}{Y_{pp}} \left[ \overline{I}_p^{(k)} - \sum_{q=1, q \neq p}^n Y_{pq} \overline{E}_q^{(k-1)} \right] \tag{2.11}$$

when  $\overline{E}_p^{(k)} = \frac{S_p^*}{\overline{E}_p^{(k+1)}} \quad p = 1, 2, \dots, n \neq s$ . (2.12)

**I. Gauss Seidel Iterative Method using  $\overline{Y}$  bus:**

In this method, the new calculated bus voltage  $\overline{E}_p^{(k)}$  immediately replaces  $\overline{E}_p^{(k-1)}$  and is used right away in the solution of the subsequent calculation. Doing this, convergence will be obtained in less number of iterations than that of the Gauss's method. Therefore, at any iteration constant  $k$ , the voltage at bus  $p$  is given by:

$$\overline{E}_p^{(k+1)} = \frac{1}{\overline{Y}_{pp}} \left[ \overline{I}_p^{(k+1)} - \sum_{q=1}^{p-1} \overline{Y}_{pq} \overline{E}_p^{(k+1)} - \sum_{q=p+1}^n \overline{Y}_{pq} \overline{E}_q^{(k+1)} \right] \quad (2.13)$$

when  $\overline{I}_p^{(k)} = \frac{\overline{S}_p^*}{\overline{E}_p^{(k+1)}}$  . (2.14)

II. **Newton Raphson Method using  $\overline{Y}$  bus:**

a) **Mathematical Background:**

The Newton Raphson method is concerning with finding the roots of a nonlinear function  $f(x)$  . Therefore, the objection is to find solve to the following equation:

$$f(\overline{x}) = 0 \quad \text{when } \overline{x} = (x_1, x_2, \dots, x_n)$$

For simplicity, we will assume that  $f(x)$  is a scale function, whose independent variable  $x$  is also a scale.

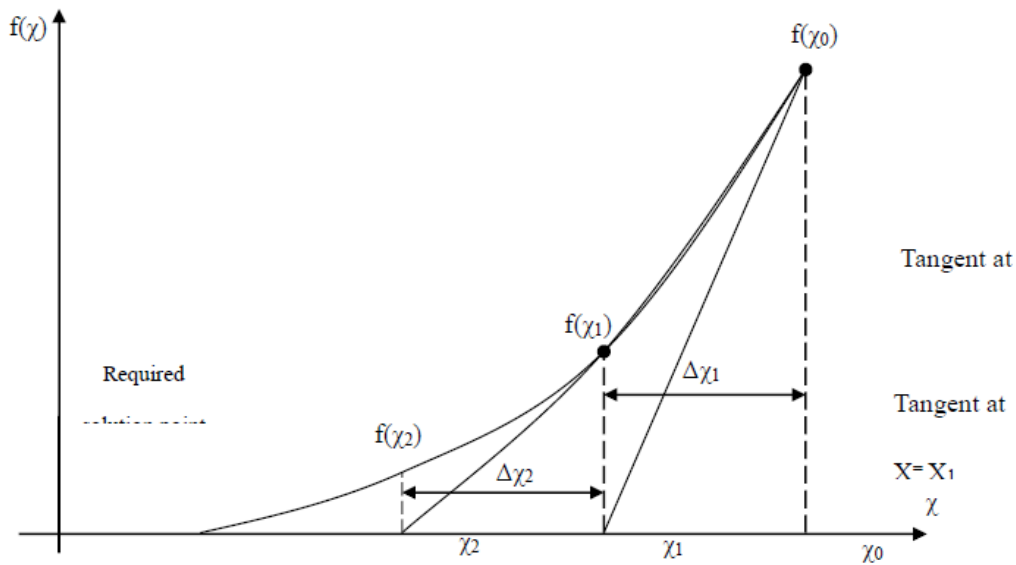


Figure 3: 3 Newton-Raphson Method

From the shown curve, we have:

$$f(x_1) \cong f(x_0) + \left( \frac{df}{dx} \right)_{x=x_0} \Delta x_1 \quad (2.15)$$

$$\Delta x_1 = x_1 - x_0$$

$$f(x_2) \cong f(x_1) + \left( \frac{df}{dx} \right)_{x=x_1} \Delta x_2 \quad (2.16)$$

$$f(x^k) \cong f(x^{k-1}) + \left( \frac{df}{dx} \right)_{x=x^{k-1}} \Delta x^k \quad (2.17)$$

Therefore,

$$\Delta x^k = + \frac{[f(x^k) - f(x^{k-1})]}{\left( \frac{df}{dx} \right)_{x=x^{k-1}}} = \frac{-f(x^{k-1})}{\left( \frac{df}{dx} \right)_{x=x^{k-1}}} \quad (2.18)$$

and  $x^k = x^{k-1} + \Delta x^k$  (2.19)

This calculation procedure is repeated iteratively until  $|\Delta x^k| \leq \epsilon$ , when  $\epsilon$  is a prescribed tolerance. Therefore, Newton Raphson method involves the idea of an ever in a function  $f(x)$  being driven to zero by making adjustments  $\Delta x$  to the independent variable associated with the function.

**b) Generalization for a vectorial function:**

Consider the vectorial function  $f(x)$ . It is required to solve the following equation:

$$f(x) = 0.0 \quad \text{when,}$$

$$f(x) = [f_1(x) f_2(x) \dots f_n(x)]^T \tag{2.20}$$

and  $\bar{x} = [x_1, x_2, \dots, x_n]^T$ .

Using Taylor's expansion, one can get the following set of equations:

$$f(\bar{x}_1) = f(\bar{x}_0) + [H]_{x_0} \Delta \bar{x}_1$$

$$f(\bar{x}_2) = f(\bar{x}_0) + [H]_{x_1} \Delta \bar{x}_2 \tag{2.21}$$

$$f(\bar{x}_k) = f(\bar{x}_{k-1}) + [H]_{x_{k-1}} \Delta \bar{x}_k$$

Ultimately, when convergence towards the required solution is achieved,  $f(\bar{x}_k) \rightarrow 0.0$  Therefore, at the  $k^{th}$  iteration:

$$\Delta \bar{x}_k = -[H]_{x_{k-1}}^{-1} f(\bar{x}_{k-1}) \tag{2.22}$$

when  $[H] =$  Jacobian matrix and defined as follows:

$$[H] = \begin{bmatrix} \frac{df_1}{dx_1} & \dots & \frac{df_1}{dx_n} \\ \dots & \dots & \dots \\ \frac{df_n}{dx_1} & \dots & \frac{df_n}{dx_n} \end{bmatrix} \tag{2.23}$$

$$\bar{x}_{k-1} = \bar{x}_k + \Delta \bar{x}_k \tag{2.24}$$

**2.3 Application of N-R method to load flow analysis:**

**1) Cartesian form:**

Let  $\bar{E}_p = e_p + jf_p$  and  $\bar{Y}_{pq} = G_{pq} - jB_{pq}$ . If we assume that all system buses and P-Q buses; therefore, at every bus (except slack), the following two quantities are defined:

$$P_p - \sum_{q=1}^n e_p (e_p G_{pq} + f_p B_{pq}) + f_p (f_q G_{pq} - V_q B_{pq}) = 0 \tag{2.25}$$

$$Q_p - \sum_{q=1}^n f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) = 0 \tag{2.26}$$

when  $P_p$  and  $Q_p$  represent the specified (scheduled) active and reactive power at bus p. Note that:

$$P_{p(sch.)} = P_{p(Generation)} - P_{p(load)} \tag{2.27}$$

and  $Q_{p(sch.)} = Q_{p(Generation)} - Q_{p(load)} \tag{2.28}$

when  $Q_{p(load)}$ : + for inductive reactive load  
 : \_ for capacitive reactive load.

Apply the N-R method to solve the above set of  $[2 \times (n - 1)]$  equations, one can get the following:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_n \\ \dots \\ \Delta Q_2 \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{dP_2}{de_2} & \frac{dP_2}{de_n} & \frac{dP_2}{df_2} & \frac{dP_2}{df_n} \\ \dots & \dots & \dots & \dots \\ \frac{dQ_n}{de_2} & \frac{dQ_n}{de_n} & \frac{dQ_n}{df_2} & \frac{dQ_n}{df_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_n \\ \dots \\ \Delta f_2 \\ \Delta f_n \end{bmatrix} \quad (2.29)$$

$$\begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \end{bmatrix} = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{e} \\ \Delta \bar{f} \end{bmatrix}$$

$\Delta \bar{P}$  or  $\Delta \bar{Q}$  represent the difference between scheduled and calculated active and reactive power respectively. The sequence of calculation steps is as follows:

1. Initialize  $\bar{e}^{(0)}, \bar{f}^{(0)}$ .
2. Calculate  $\Delta \bar{P}, \Delta \bar{Q}$ .
3. Evaluate the Jacobian matrix at the current values of  $\bar{e}, \bar{f}$ .
4. Solve the matrix equation  $[A] \Delta \bar{e}, \Delta \bar{f}$ .
5. Modify  $\bar{e}, \bar{f}$  when:

$$\begin{aligned} \bar{e}^{(1)} &= \bar{e}^{(0)} + \Delta \bar{e}^{(0)} \\ \bar{f}^{(1)} &= \bar{f}^{(0)} + \Delta \bar{f}^{(0)} \end{aligned} \quad (2.30)$$

6. Check convergence: if  $\Delta \bar{e}, \Delta \bar{f}$  are all less than a prescribed tolerance step. Otherwise, go to (2.) and repeat calculations.

Notes:

- At any iteration constant K, we have: (2.3.7)
- The convergence given in step (6.) may be applied to  $\bar{e}$  and  $\bar{f}$  rather than for  $e, f$ .

**2.4 Elements of the Jacobian matrix:**

Since:

$$\begin{aligned} P_p &= (e_p G_{pp} + f_p B_{pp}) + f_p (f_p G_{pp} - e_p B_{pp}) \\ &+ \sum_{q=1 \neq p}^n e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \quad p = 1, 2, \dots, n-1 \end{aligned} \quad (2.31)$$

Therefore:

$$\frac{dP_p}{de_q} = e_p G_{pq} - f_p B_{pq} \quad q \neq p \quad (2.32)$$

$$\frac{dP_p}{de_p} = 2e_p G_{pp} + \sum_{q=1 \neq p}^n (e_p G_{pq} + f_q B_{pq}) \quad J_1$$

$$\frac{dP_p}{df_q} = e_p B_{pq} + f_p G_{pq} \quad q \neq p$$

$$\frac{dP_p}{df_p} = 2f_p G_{pp} + \sum_{q=1 \neq p}^n (f_q G_{pq} - e_q B_{pq}) \quad J_2$$

since,

$$Q_p = f_p (e_p G_{pp} + f_p B_{pp}) - e_p (f_p G_{pp} - e_p B_{pp}) + \sum_{q=1 \neq p}^n \{f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})\} \quad (2.33)$$

Therefore,  $\frac{dQ_p}{de_q} = e_p B_{pq} + f_p G_{pq} \quad (2.34)$

$$\frac{dQ_p}{de_p} = 2e_p B_{pp} - \sum_{q=1 \neq p}^n (f_q G_{pq} - e_q B_{pq}) \quad J_3$$

$$\frac{dQ_p}{df_q} = -e_p G_{pq} + f_p B_{pq} \quad (2.35)$$

$$\frac{dQ_p}{df_p} = 2f_p B_{pp} + \sum_{q=1 \neq p}^n (e_p G_{pq} + f_q B_{pq}) \quad J_4$$

2) **Polar form:**

Following the same concepts of previous section, the load flow solution in polar form may be represented as follows:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{dP_2}{dV_2} & \dots & \frac{dP_2}{dV_n} & \frac{dP_2}{d\delta_2} & \dots & \frac{dP_2}{d\delta_n} \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots \\ \frac{dP_n}{dV_2} & \dots & \frac{dP_n}{dV_n} & \frac{dP_n}{d\delta_2} & \dots & \frac{dP_n}{d\delta_n} \\ \frac{dQ_2}{dV_2} & \dots & \frac{dQ_2}{dV_n} & \frac{dQ_2}{d\delta_2} & \dots & \frac{dQ_2}{d\delta_n} \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots \\ \frac{dQ_n}{dV_2} & \dots & \frac{dQ_n}{dV_n} & \frac{dQ_n}{d\delta_2} & \dots & \frac{dQ_n}{d\delta_n} \end{bmatrix} \begin{bmatrix} \Delta V_2 \\ \vdots \\ \Delta V_n \\ \Delta \delta_2 \\ \vdots \\ \Delta \delta_n \end{bmatrix} \quad (2.36)$$

In a compact form:

$$\begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{V} \\ \Delta \bar{\delta} \end{bmatrix} \quad (2.37)$$

and the solution is given by:

$$\begin{bmatrix} \Delta \bar{V} \\ \Delta \bar{\delta} \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \end{bmatrix} \quad (2.38)$$

The sequence of the calculation steps is same as that presented in previous section. Elements of the Jacobian matrix are as follows:

1.  $J_1 : \frac{dP_i}{dV_i} \quad (2.39)$

$$\text{if } i = j : \sum_{j=1 \neq i}^n V_i Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij}) + 2V_i Y_{ij} \cos \alpha_{ij}$$

$$\text{if } i \neq j : V_j Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij})$$

2.  $J_2 : \frac{dP_i}{d\delta_j} \quad (2.40)$

$$\text{if } i = j : \sum_{j=1 \neq i}^n -V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij})$$



$$\text{if } i \neq j : +V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij})$$

$$3. \quad J_3 : \frac{dQ_i}{dV_j} \quad (2.41)$$

$$\text{if } i = j : \sum_{j=1 \neq i}^n V_j Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij}) - 2V_i Y_{ij} \sin \alpha_{ij}$$

$$\text{if } i = j : V_i Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij})$$

$$4. \quad J_4 : \frac{d\alpha_i}{d\delta_j} \quad (2.42)$$

$$\text{if } i = j : \sum_{j=1 \neq i}^n V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij})$$

$$\text{if } i \neq j : -V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij})$$

#### IV. Load Flow Solution

##### 4.1 Iterative Process

The Newton-Raphson method is used for load flow solution.

Known values:

$$P_2, P_3, P_4, P_5, P_6, P_7 \quad \text{and} \quad Q_2, Q_3, Q_4, Q_5, Q_6, Q_7$$

$$V_1, \delta_1 (\bar{V}_1 = V_1 \angle \delta_1)$$

values to be found:

$$V_2, \delta_2, V_3, \delta_3, V_4, \delta_4, V_5, \delta_5, V_6, \delta_6, V_7, \delta_7$$

Iterative process:

I.

- Values  $V_2^{(0)}, \delta_2^{(0)}, V_3^{(0)}, \delta_3^{(0)}, V_4^{(0)}, \delta_4^{(0)}, V_5^{(0)}, \delta_5^{(0)}, V_6^{(0)}, \delta_6^{(0)}, V_7^{(0)}, \delta_7^{(0)}$  are estimated

- Using known values and estimated values active and reactive powers in (1),(2),(3),(4),(5),(6),and (7) are calculated:

$$P_i = \sum_{j=1}^7 V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij}) \quad (5.1)$$

$$Q_i = \sum_{j=1}^7 V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij}) \quad (5.2)$$

$$P_2^{(0)} = V_2^{(0)2} \times Y_{22} \times \cos(-\alpha_{22}) - V_2^{(0)} \times V_1 \times Y_{12} \times \cos(\delta_2^{(0)} - \delta_1 - \alpha_{12})$$

$$- V_2^{(0)} \times V_3 \times Y_{23} \times \cos(\delta_2^{(0)} - \delta_3 - \alpha_{23}) - V_2^{(0)} \times V_4 \times Y_{24} \times \cos(\delta_2^{(0)} - \delta_4 - \alpha_{24})$$

$$- V_2^{(0)} \times V_5 \times Y_{25} \times \cos(\delta_2^{(0)} - \delta_5 - \alpha_{25}) - V_2^{(0)} \times V_6 \times Y_{26} \times \cos(\delta_2^{(0)} - \delta_6 - \alpha_{26})$$

$$- V_2^{(0)} \times V_7 \times Y_{27} \times \cos(\delta_2^{(0)} - \delta_7 - \alpha_{27})$$

$$Q_2^{(0)} = V_2^{(0)2} \times Y_{22} \times \sin(-\alpha_{22}) - V_2^{(0)} \times V_1 \times Y_{12} \times \sin(\delta_2^{(0)} - \delta_1 - \alpha_{12})$$

$$- V_2^{(0)} \times V_3 \times Y_{23} \times \sin(\delta_2^{(0)} - \delta_3 - \alpha_{23}) - V_2^{(0)} \times V_4 \times Y_{24} \times \sin(\delta_2^{(0)} - \delta_4 - \alpha_{24})$$

$$- V_2^{(0)} \times V_5 \times Y_{25} \times \sin(\delta_2^{(0)} - \delta_5 - \alpha_{25}) - V_2^{(0)} \times V_6 \times Y_{26} \times \sin(\delta_2^{(0)} - \delta_6 - \alpha_{26})$$

$$- V_2^{(0)} \times V_7 \times Y_{27} \times \sin(\delta_2^{(0)} - \delta_7 - \alpha_{27})$$

- Power differences are calculated:

$$\Delta P_2^{(0)} = P_2 - P_2^{(0)}$$

$$\Delta P_3^{(0)} = P_3 - P_3^{(0)}$$

$$\Delta P_4^{(0)} = P_4 - P_4^{(0)}$$

$$\Delta P_5^{(0)} = P_5 - P_5^{(0)}$$

$$\Delta P_6^{(0)} = P_6 - P_6^{(0)}$$

$$\Delta P_7^{(0)} = P_7 - P_7^{(0)}$$

$$\Delta Q_2^{(0)} = Q_2 - Q_2^{(0)}$$

$$\Delta Q_3^{(0)} = Q_3 - Q_3^{(0)}$$

$$\Delta Q_4^{(0)} = Q_4 - Q_4^{(0)}$$

$$\Delta Q_5^{(0)} = Q_5 - Q_5^{(0)}$$

$$\Delta Q_6^{(0)} = Q_6 - Q_6^{(0)}$$

$$\Delta Q_7^{(0)} = Q_7 - Q_7^{(0)}$$

where:  $i = 2, 3, 4, 5, 6, 7$

$V_i, V_j$ : voltage magnitudes.

$\delta_i, \delta_j$ : voltage angles.

$Y_{ij}$ : admittance magnitude.

$\alpha_{ij}$ : admittance angle.

$$V_i = V_i \angle \alpha_i, \quad \text{and} \quad Y_{ij} = Y_{ij} \angle \alpha_{ij}$$

$$\begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_7 \\ \Delta Q_2 \\ \vdots \\ \Delta Q_7 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_2}{\partial V_7} & \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_7} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial P_7}{\partial V_2} & \cdots & \frac{\partial P_7}{\partial V_7} & \frac{\partial P_7}{\partial \delta_2} & \cdots & \frac{\partial P_7}{\partial \delta_7} \\ \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_2}{\partial V_7} & \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_7} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial Q_7}{\partial V_2} & \cdots & \frac{\partial Q_7}{\partial V_7} & \frac{\partial Q_7}{\partial \delta_2} & \cdots & \frac{\partial Q_7}{\partial \delta_7} \end{bmatrix} \times \begin{bmatrix} \Delta V_2 \\ \vdots \\ \Delta V_7 \\ \Delta \delta_2 \\ \vdots \\ \Delta \delta_7 \end{bmatrix} \quad (5.11)$$

**4.2 Elements of the Jacobian:**

- Elements of Jacobian are calculated (using known and estimated values).

$$\left\{ \begin{aligned} \frac{\partial P_i}{\partial V_i} &= 2V_i Y_{ii} \cos \alpha_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^7 V_i Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij}) \\ \frac{\partial P_i}{\partial V_j} &= V_i Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij}) \end{aligned} \right. \quad (5.12), (5.13)$$

$$\left\{ \begin{aligned} \frac{\partial P_i}{\partial \delta_i} &= -\sum_{\substack{j=1 \\ j \neq i}}^7 V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij}) \\ \frac{\partial P_i}{\partial \delta_j} &= V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij}) \end{aligned} \right. \quad (5.14), (5.15)$$

$$\left\{ \begin{aligned} \frac{\partial Q_i}{\partial V_i} &= -2V_i Y_{ii} \sin \alpha_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^7 V_j Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij}) \\ \frac{\partial Q_i}{\partial V_j} &= V_i Y_{ij} \sin(\delta_i - \delta_j - \alpha_{ij}) \end{aligned} \right. \quad (5.16), (5.17)$$

$$\left\{ \begin{aligned} \frac{\partial Q_i}{\partial \delta_i} &= \sum_{\substack{j=1 \\ j \neq i}}^7 V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij}) \\ \frac{\partial Q_i}{\partial \delta_j} &= -V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \alpha_{ij}) \end{aligned} \right. \quad (5.18), (5.19)$$

- Voltage magnitude and angle differences are calculated:

$$\begin{bmatrix} \Delta V_2^{(0)} \\ \vdots \\ \Delta V_7^{(0)} \\ \Delta \delta_2^{(0)} \\ \vdots \\ \Delta \delta_7^{(0)} \end{bmatrix} = [J]^{-1} \times \begin{bmatrix} \Delta P_2^{(0)} \\ \vdots \\ \Delta P_7^{(0)} \\ \Delta Q_2^{(0)} \\ \vdots \\ \Delta Q_7^{(0)} \end{bmatrix} \quad (5.20)$$

- Voltage magnitudes and angles after the 1<sup>st</sup> iteration are calculated (more precise values than the estimated ones ).

$$V_2^{(1)} = V_2^{(0)} + \Delta V_2^{(0)}$$

$$V_3^{(1)} = V_3^{(0)} + \Delta V_3^{(0)}$$

$$V_4^{(1)} = V_4^{(0)} + \Delta V_4^{(0)}$$

$$V_5^{(1)} = V_5^{(0)} + \Delta V_5^{(0)}$$

$$V_6^{(1)} = V_6^{(0)} + \Delta V_6^{(0)}$$

$$V_7^{(1)} = V_7^{(0)} + \Delta V_7^{(0)}$$

$$\delta_2^{(1)} = \delta_2^{(0)} + \Delta \delta_2^{(0)}$$

$$\delta_3^{(1)} = \delta_3^{(0)} + \Delta \delta_3^{(0)}$$

$$\delta_4^{(1)} = \delta_4^{(0)} + \Delta \delta_4^{(0)}$$

$$\delta_5^{(1)} = \delta_5^{(0)} + \Delta\delta_5^{(0)}$$

$$\delta_6^{(1)} = \delta_6^{(0)} + \Delta\delta_6^{(0)}$$

$$\delta_7^{(1)} = \delta_7^{(0)} + \Delta\delta_7^{(0)}$$

**II.** Values  $V_2^{(0)}, V_3^{(0)}, V_4^{(0)}, V_5^{(0)}, V_6^{(0)}, V_7^{(0)}$  and  $\delta_2^{(0)}, \delta_3^{(0)}, \delta_4^{(0)}, \delta_5^{(0)}, \delta_6^{(0)}, \delta_7^{(0)}$  are replaced by:

$$V_2^{(1)}, V_3^{(1)}, V_4^{(1)}, V_5^{(1)}, V_6^{(1)}, V_7^{(1)} \text{ and } \delta_2^{(1)}, \delta_3^{(1)}, \delta_4^{(1)}, \delta_5^{(1)}, \delta_6^{(1)}, \delta_7^{(1)}$$

$$\Delta V_2^{(1)}, \Delta V_3^{(1)}, \Delta V_4^{(1)}, \Delta V_5^{(1)}, \Delta V_6^{(1)}, \Delta V_7^{(1)} \text{ and } \Delta\delta_2^{(1)}, \Delta\delta_3^{(1)}, \Delta\delta_4^{(1)}, \Delta\delta_5^{(1)}, \Delta\delta_6^{(1)}, \Delta\delta_7^{(1)}$$

then we go to calculate:

$$V_2^{(2)}, V_3^{(2)}, V_4^{(2)}, V_5^{(2)}, V_6^{(2)}, V_7^{(2)} \text{ and } \delta_2^{(2)}, \delta_3^{(2)}, \delta_4^{(2)}, \delta_5^{(2)}, \delta_6^{(2)}, \delta_7^{(2)}$$

**III.** The process is repeated until the difference between the values in two following iterations is less than a given value (which means that their difference can be neglected):

$$\left| V_i^{(k+1)} - V_i^{(k)} \right| < \varepsilon \tag{5.21}$$

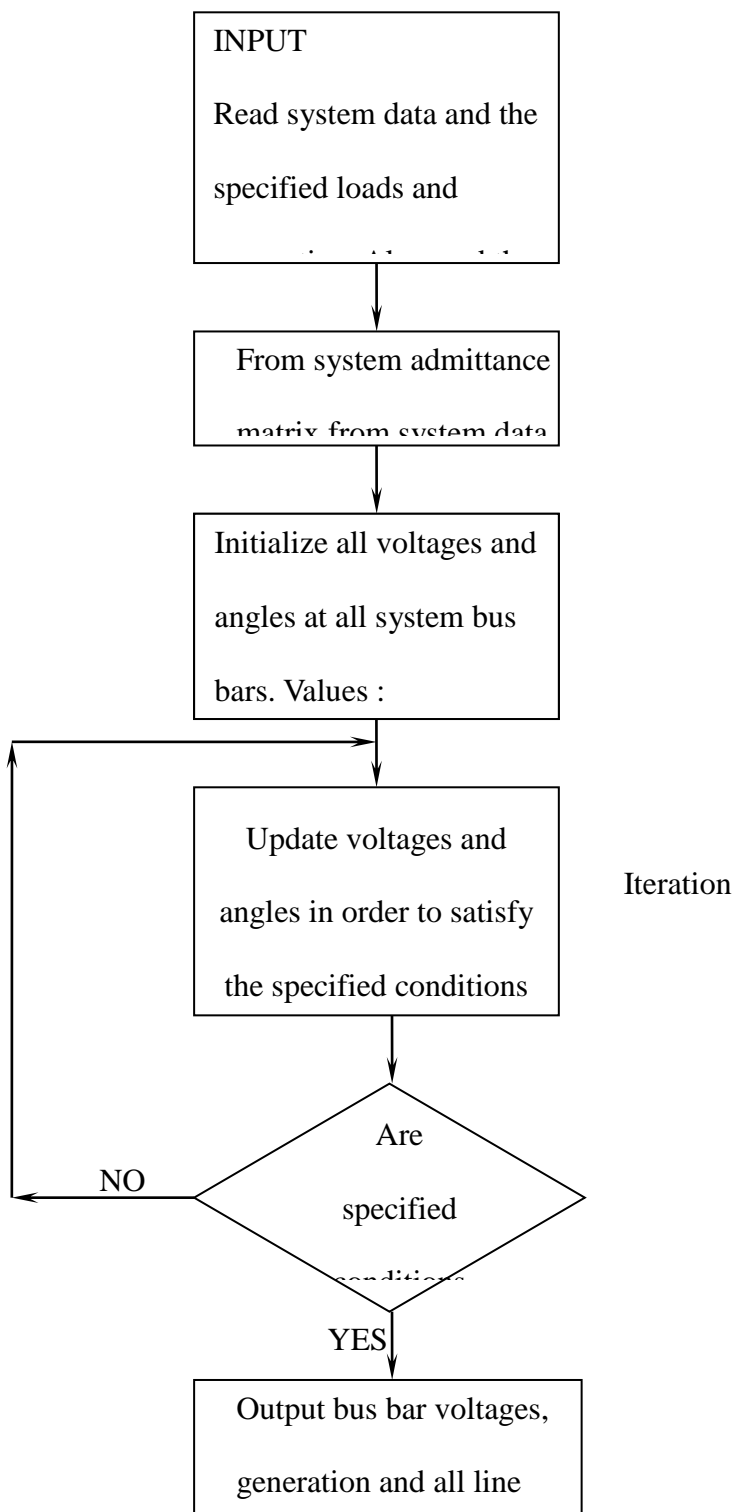


Figure 3: Iterative Process

**5.3.2 Nodal Admittance Matrix**

$$[\bar{Y}] = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \dots & \bar{Y}_{1i} & \dots & \bar{Y}_{1j} \\ \bar{Y}_{21} & \bar{Y}_{22} & \dots & \bar{Y}_{2i} & \dots & \bar{Y}_{2j} \\ \vdots & \vdots & & \vdots & & \vdots \\ \bar{Y}_{i1} & \dots & \dots & \bar{Y}_{ii} & \dots & \bar{Y}_{ij} \\ \vdots & & & \vdots & & \vdots \\ \bar{Y}_{j1} & \dots & \dots & \bar{Y}_{ji} & \dots & \bar{Y}_{jj} \end{bmatrix} \quad (5.22)$$

$\bar{Y}_{ii}$  - sum of admittances of all branches connected to the i-th node

$\bar{Y}_{ij}$  - sum admittances of the branches between i-th and j-th node with (-) sign

➤ The matrix is symmetrical:

$$\bar{Y}_{ij} = \bar{Y}_{ji} \quad (5.23)$$

➤ Respecting shunt admittance:

Equivalent circuit,  $\Pi$ -equivalent circuit

$\bar{y}_{12}$  - series admittance of the line between nodes 1 and 2 ( $\bar{y}_{12} = \frac{1}{\bar{Z}_{12}}$ ,  $\bar{Z}_{12}$ -series impedance).

$\bar{y}_{120}$  - shunt admittance of the line between nodes 1 and 2.

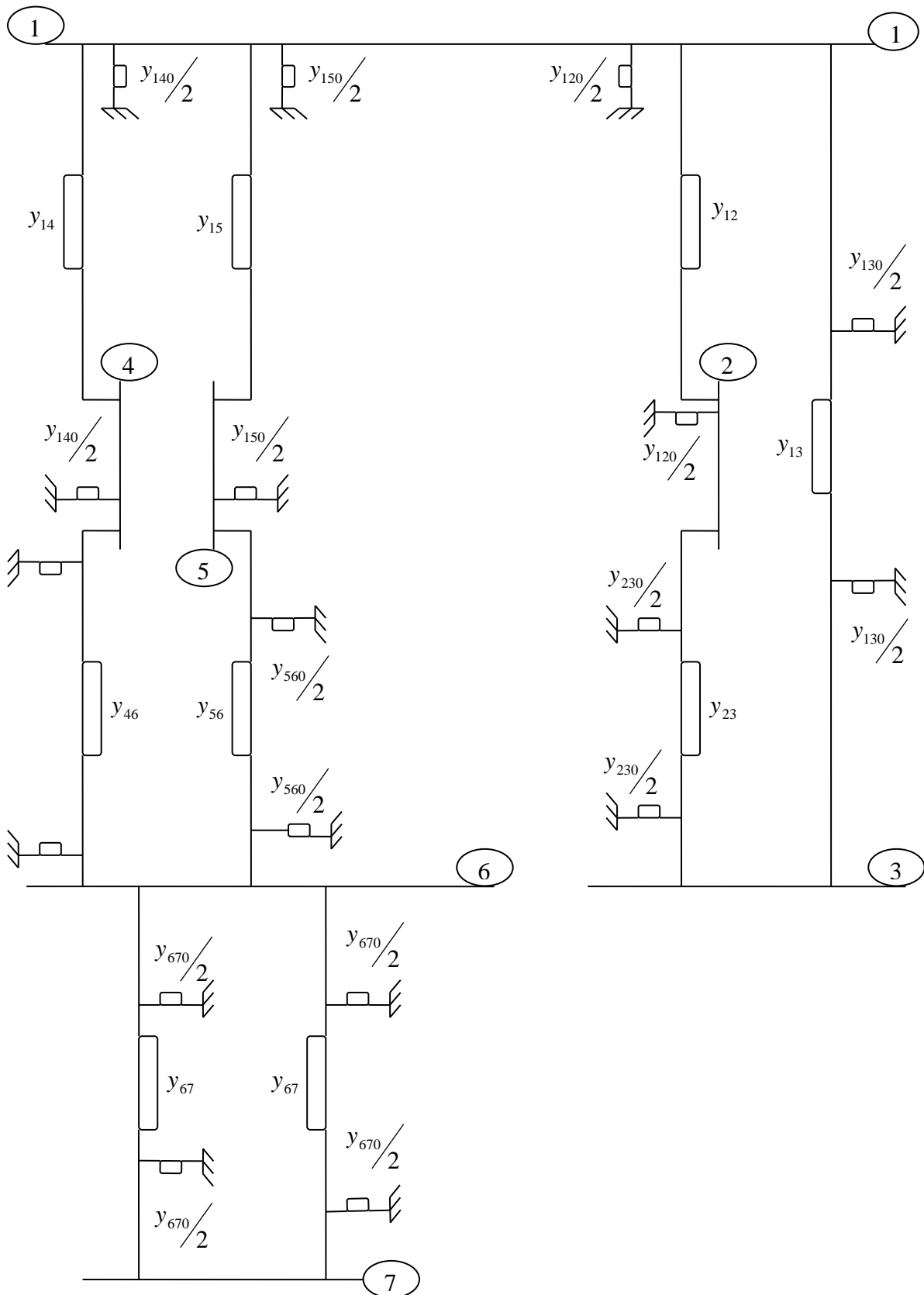


Figure 4: Scheme of the Existing Network with Lines Replaced by  $\Pi$ -Equivalent Circuits

The Series Admittance of lines:

$$\bar{Z}_{12} = \bar{Z}_k \times l_{12} = (R_k + X_{ki}) \times l_{12}$$

$$\text{then, } \bar{y}_{12} = \frac{1}{\bar{Z}_{12}}$$

$$\bar{Z}_{13} = \bar{Z}_k \times l_{13} = (R_k + X_{ki}) \times l_{13}$$

$$\text{then, } \bar{y}_{13} = \frac{1}{\bar{Z}_{13}}$$

$$\bar{Z}_{14} = (R_k + X_{ki}) \times l_{14}$$

$$\text{then, } \bar{y}_{14} = \frac{1}{\bar{Z}_{14}}$$

$$\bar{Z}_{15} = (R_k + X_{ki}) \times l_{15}$$

$$\text{then, } \bar{y}_{15} = \frac{1}{\bar{Z}_{15}}$$

$$\bar{Z}_{16} = (R_k + X_{ki}) \times l_{16}$$

$$\text{then, } \bar{y}_{16} = \frac{1}{\bar{Z}_{16}}$$

$$\bar{Z}_{23} = (R_k + X_{ki}) \times l_{23}$$

$$\text{then, } \bar{y}_{23} = \frac{1}{\bar{Z}_{23}}$$

$$\bar{Z}_{46} = (R_k + X_{ki}) \times l_{46}$$

$$\text{then, } \bar{y}_{46} = \frac{1}{\bar{Z}_{46}}$$

$$\bar{Z}_{56} = (R_k + X_{ki}) \times l_{56}$$

$$\text{then, } \bar{y}_{56} = \frac{1}{\bar{Z}_{56}}$$

$$\bar{Z}_{67} = (R_k + X_{ki}) \times l_{67}$$

$$\text{then, } \bar{y}_{67} = \frac{1}{\bar{Z}_{67}}$$

others:

$$\bar{y}_{17} = 0$$

$$\bar{y}_{24} = 0$$

$$\bar{y}_{25} = 0$$

$$\bar{y}_{26} = 0$$

$$\bar{y}_{27} = 0$$

$$\bar{y}_{34} = 0$$

$$\bar{y}_{35} = 0$$

$$\bar{y}_{36} = 0$$

$$\bar{y}_{37} = 0$$



$$\begin{aligned} \bar{y}_{45} &= 0 \\ \bar{y}_{47} &= 0 \\ \bar{y}_{57} &= 0 \end{aligned}$$

all these admittances equal zero because there is no connection between these nodes.  
The shunt admittances of the lines:

$$\begin{aligned} \bar{y}_{120} &= Y_k \times l_{12} = (G_k + B_{ki}) \times l_{12} \\ \bar{y}_{130} &= (G_k + B_{ki}) \times l_{13} \\ \bar{y}_{140} &= (G_k + B_{ki}) \times l_{14} \\ \bar{y}_{150} &= (G_k + B_{ki}) \times l_{15} \\ \bar{y}_{160} &= (G_k + B_{ki}) \times l_{16} \\ \bar{y}_{230} &= (G_k + B_{ki}) \times l_{23} \\ \bar{y}_{460} &= (G_k + B_{ki}) \times l_{46} \\ \bar{y}_{560} &= (G_k + B_{ki}) \times l_{56} \\ \bar{y}_{670} &= (G_k + B_{ki}) \times l_{67} \end{aligned}$$

Table 1: Lines Series and Shunt Admittances

i-j	l <sub>i-j</sub> [km]	z <sub>i-j</sub> [Ω]	y <sub>i-j</sub> [S]	y <sub>i-j-0</sub> [S]	y <sub>i-j-0/2</sub> [S]
1-2	26	4.056+10.4i	0.03255-0.08346i	7.28×10 <sup>-5</sup> i	3.64×10 <sup>-5</sup> i
1-3	50	7.8+20i	0.01692-0.04340i	1.4×10 <sup>-4</sup> i	7×10 <sup>-5</sup> i
1-4	26	4.056+10.4i	0.03255-0.08346i	7.28×10 <sup>-5</sup> i	3.64×10 <sup>-5</sup> i
1-5	6	0.936+2.4i	0.14105-0.36166i	1.68×10 <sup>-5</sup> i	8.4×10 <sup>-6</sup> i
2-3	20	3.12+8i	0.04231-0.10850i	5.6×10 <sup>-5</sup> i	2.8×10 <sup>-5</sup> i
4-6	2	0.312+0.8i	0.42314-1.08498i	5.6×10 <sup>-6</sup> i	2.8×10 <sup>-6</sup> i
5-6	27	4.212+10.8i	0.03134-0.08037i	7.56×10 <sup>-5</sup> i	3.78×10 <sup>-5</sup> i
6-7	7	1.092+2.8i	0.12090-0.30999i	1.96×10 <sup>-5</sup> i	9.8×10 <sup>-6</sup> i

$$\begin{aligned} Y_{11} &= y_{12} + \frac{y_{120}}{2} + y_{13} + \frac{y_{130}}{2} + y_{14} + \frac{y_{140}}{2} + y_{15} + \frac{y_{150}}{2} \\ Y_{12} &= Y_{21} = -\bar{y}_{12} \\ Y_{13} &= Y_{31} = -\bar{y}_{13} \\ Y_{14} &= Y_{41} = -\bar{y}_{14} \\ Y_{15} &= Y_{51} = -\bar{y}_{15} \\ Y_{16} &= Y_{61} = 0 \\ Y_{17} &= Y_{71} = 0 \\ Y_{22} &= y_{12} + \frac{y_{120}}{2} + y_{23} + \frac{y_{230}}{2} \\ Y_{23} &= Y_{32} = -\bar{y}_{23} \\ Y_{24} &= Y_{42} = 0 \\ Y_{25} &= Y_{52} = 0 \\ Y_{26} &= Y_{62} = 0 \end{aligned}$$

$$Y_{27} = Y_{72} = 0$$

$$Y_{33} = y_{13} + \frac{y_{130}}{2} + y_{23} + \frac{y_{230}}{2}$$

$$Y_{34} = Y_{43} = 0$$

$$Y_{35} = Y_{53} = 0$$

$$Y_{36} = Y_{63} = 0$$

$$Y_{37} = Y_{73} = 0$$

$$Y_{44} = y_{14} + \frac{y_{140}}{2} + y_{46} + \frac{y_{460}}{2}$$

$$Y_{45} = Y_{54} = 0$$

$$Y_{46} = Y_{64} = -\bar{y}_{46}$$

$$Y_{47} = Y_{74} = 0$$

$$Y_{55} = y_{15} + \frac{y_{150}}{2} + y_{56} + \frac{y_{560}}{2}$$

$$Y_{56} = Y_{65} = -\bar{y}_{56}$$

$$Y_{57} = Y_{75} = 0$$

$$Y_{66} = y_{46} + \frac{y_{460}}{2} + y_{56} + \frac{y_{560}}{2} + y_{67} + \frac{y_{670}}{2}$$

$$Y_{67} = Y_{76} = -\bar{y}_{67}$$

$$Y_{77} = y_{67} + \frac{y_{670}}{2} + y_{67} + \frac{y_{670}}{2}$$

Table 2: Elements of the Nodal Admittance Matrix for 9 lines

	Admittance	Magnitude	Angle
	Y [S]	Y [S]	$\alpha$ [rad]
Y <sub>11</sub>	0.2231-0.5718i	0.6138	-1.1988
Y <sub>12</sub>	-0.0325+0.0835i	0.0896	1.9427
Y <sub>13</sub>	-0.0169+0.0434i	0.0466	1.9427
Y <sub>14</sub>	-0.0325+0.0835i	0.0896	1.9427
Y <sub>15</sub>	-0.1410+0.3617i	0.3882	1.9427
Y <sub>16</sub>	0	0	0
Y <sub>17</sub>	0	0	0
Y <sub>22</sub>	0.0749-0.1919i	0.2060	-1.1988
Y <sub>23</sub>	-4.2314+0.1085i	0.1165	1.9427
Y <sub>24</sub>	0	0	0
Y <sub>25</sub>	0	0	0
Y <sub>26</sub>	0	0	0
Y <sub>27</sub>	0	0	0
Y <sub>33</sub>	0.0592-0.1518i	0.1629	-1.1987
Y <sub>34</sub>	0	0	0
Y <sub>35</sub>	0	0	0
Y <sub>36</sub>	0	0	0
Y <sub>37</sub>	0	0	0
Y <sub>44</sub>	0.4557-1.1684i	1.2541	-1.1989
Y <sub>45</sub>	0	0	0
Y <sub>46</sub>	-0.4231+1.0850i	1.1646	1.9427
Y <sub>47</sub>	0	0	0

Y <sub>55</sub>	0.1724-0.4419i	0.4744	-1.1989
Y <sub>56</sub>	-0.0313+0.080i	0.0863	1.9427
Y <sub>57</sub>	0	0	0
Y <sub>66</sub>	0.6963-1.7853i	1.9162	-1.1989
Y <sub>67</sub>	-0.2418+0.6200i	0.6655	1.9427
Y <sub>77</sub>	0.2418-0.6200i	0.6654	-1.1989

$$[Y] = \begin{bmatrix} 0.6138 & 0.0896 & 0.0466 & 0.0896 & 0.3882 & 0.0728 & 0 \\ 0.0896 & 0.2060 & 0.1165 & 0 & 0 & 0 & 0 \\ 0.0466 & 0.1165 & 0.1629 & 0 & 0 & 0 & 0 \\ 0.0896 & 0 & 0 & 1.2541 & 0 & 1.1646 & 0 \\ 0.3882 & 0 & 0 & 0 & 0.4744 & 0.0863 & 0 \\ 0.0728 & 0 & 0 & 1.1646 & 0.0863 & 1.9162 & 0.6655 \\ 0 & 0 & 0 & 0 & 0 & 0.6655 & 0.6654 \end{bmatrix}$$

$$[\alpha] = \begin{bmatrix} -1.1988 & 1.9427 & 1.9427 & 1.9427 & 1.9427 & 0 & 0 \\ 1.9427 & -1.1988 & 1.9427 & 0 & 0 & 0 & 0 \\ 1.9427 & 1.9427 & -1.1987 & 0 & 0 & 0 & 0 \\ 1.9427 & 0 & 0 & -1.1989 & 0 & 1.9427 & 0 \\ 1.9427 & 0 & 0 & 0 & -1.1989 & 1.9427 & 0 \\ 0 & 0 & 0 & 1.9427 & 1.9427 & -1.1989 & 1.9427 \\ 0 & 0 & 0 & 0 & 0 & 1.9427 & -1.989 \end{bmatrix}$$

**V. Results from MATLAB Model of Load flow solution**

```
n=7
V=[63509; 63509; 63509; 63509; 63509; 63509; 63509];
d=[0; 0; 0; 0; 0; 0; 0];
Y= [0.6138 0.0896 0.0466 0.0896 0.03882 0.0728 0; ...
0.0896 0.2060 0.1165 0 0 0 0;...
0.0466 0.1165 0.1629 0 0 0 0;...
0.0896 0 1.2541 0 1.1646 0;...
0.3882 0 0 0.4744 0.0863 0;...
0.0728 0 1.1646 0.0863 1.9162 0.6655;...
0 0 0 0.6655 0.6654];
a= [-1.1988 1.9427 1.9427 1.9427 1.9427 0 0 ;...
1.9427 -1.1988 1.9427 0 0 0 0;...
1.9427 1.9427 -1.1987 0 0 0 0 ;...
1.9427 0 0 -1.1989 0 1.9427 0;...
1.9427 0 0 -1.1989 1.9427 0;...
0 0 1.9427 1.9427 -1.1989 1.9427;...
0 0 0 0 1.9427 -1.989];
Pgiv=[0; -40e6; -30.3e6; -20.6e6; -25.5e6; -38.7e6; -45.3e6];
Qgiv=[0; -3e6; -5.2e6; -5.1e6; -6e6; -8e6; -10.1e6];
Pgiv_ph=Pgiv/3;
Qgiv_ph=Qgiv/3;
difPQ=max(max(abs(Pgiv_ph)),max(abs(Qgiv_ph)));
dif=100;
while difPQ>=dif
for i=2:n
prevP(i)=0;
```

```

    prevQ(i)=0;
    for j=1:n
        P(i)=prevP(i)+V(i)*V(j)*Y(i,j)*cos(d(i)-d(j)-a(i,j));
        prevP(i)=P(i);
        Q(i)=prevQ(i)+V(i)*V(j)*Y(i,j)*sin(d(i)-d(j)-a(i,j));
        prevQ(i)=Q(i);
    end
    P(i);
    Q(i);
end
P;
Q;
dPQ=zeros(2*(n-1),1);
for i=2:n
    dPQ(i-1,1)=Pgiv(i)-P(i);
    dPQ(i+n-2,1)=Qgiv(i)-Q(i);
end
dPQ;

difPQ=max(max(abs(dPQ(1:n-1,1))),max(abs(dPQ(n:2*(n-1),1))));
if difPQ<dif
    break
else
%Partial Derivatives dP/dV
for i=2:n
    prevPVsum(i)=0;
    for j=[1:i-1,i+1:n]
        PVsum(i)=prevPVsum(i)+V(j)*Y(i,j)*cos(d(i)-d(j)-a(i,j));
        prevPVsum(i)=PVsum(i);
    end
    PVdiag(i)=2*V(i)*Y(i,i)*cos(a(i,i))+PVsum(i);
end
PV=zeros(2*(n-1));
for i=2:n
    for j=i+1:n
        PV(i,j)=V(i)*Y(i,j)*cos(d(i)-d(j)-a(i,j));
    end
end
%Partial Derivatives dP/dd
for i=2:n
    prevPddiag(i)=0;
    for j=[1:i-1,i+1:n]
        Pddiag(i)=prevPddiag(i)-V(i)*V(j)*Y(i,j)*sin(d(i)-d(j)-a(i,j));
        prevPddiag(i)=Pddiag(i);
    end
end
for i=2:n
    for j=i+1:n
        Pd(i,j)=V(i)*V(j)*Y(i,j)*sin(d(i)-d(j)-a(i,j));
    end
end
%Partial Derivatives dQ/dV
for i=2:n
    prevQVsum(i)=0;
    for j=[1:i-1,i+1:n]
        QVsum(i)=prevQVsum(i)+V(j)*Y(i,j)*sin(d(i)-d(j)-a(i,j));
        prevQVsum(i)=QVsum(i);
    end
    QVdiag(i)=-2*V(i)*Y(i,i)*sin(a(i,i))+QVsum(i);
end

```

```

end
QV=zeros(2*(n-1));
for i=2:n
    for j=i+1:n
        QV(i,j)=V(i)*Y(i,j)*sin(d(i)-d(j)-a(i,j));
    end
end
%Partial Derivatives dQ/dd
for i=2:n
    prevQddiag(i)=0;
    for j=[1:i-1,i+1:n]
        Qddiag(i)=prevQddiag(i)+V(i)*V(j)*Y(i,j)*cos(d(i)-d(j)-a(i,j));
        prevQddiag(i)=Qddiag(i);
    end
end
for i=2:n
    for j=i+1:n
        Qd(i,j)=-V(i)*V(j)*Y(i,j)*cos(d(i)-d(j)-a(i,j));
    end
end
J=zeros(2*(n-1));
for i=2:n
    J(i-1,i-1)=PVdiag(i);
    J(i-1,i+n-2)=Pddiag(i);
    J(i+n-2,i-1)=QVdiag(i);
    J(i+n-2,i+n-2)=Qddiag(i);
    for j=i+1:n
        J(i-1,j-1)=PV(i,j);
        J(j-1,i-1)=J(i-1,j-1);
        J(i-1,j+n-2)=Pd(i,j);
        J(j-1,i+n-2)=J(i-1,j+n-2);
        J(i+n-2,j-1)=QV(i,j);
        J(j+n-2,i-1)=J(i+n-2,j-1);
        J(i+n-2,j+n-2)=Qd(i,j);
        J(j+n-2,i+n-2)=J(i+n-2,j+n-2);
    end
end
J;
detJ=det(J);
Jinv=inv(J);
dVd=Jinv*dPQ;
for i=2:n
    Vnew(i)=V(i)+dVd(i-1);
    V(i)=Vnew(i);
    dnew(i)=d(i)+dVd(i+n-2);
d(i)=dnew(i);
end
V;
d;
dPQ;
end
pause
end
V
d
difPQ
I=zeros(n);
for i=1:n
    for j=i+1:n

```

```

I(i,j)=sqrt((((V(i)*sin(d(i))-V(j)*sin(d(j))))^2)+(((V(i)*cos(d(i))-V(j)*cos(d(j))))^2))*abs(Y(i,j));
end
end
I
dltPij=zeros(n);
dltQij=zeros(n);
for i=1:n
    for j=i+1:n
        if Y(i,j)==0
            dltPij(i,j)=0;
            dltQij(i,j)=0;
        else
            dltPij(i,j)=3*abs(1/Y(i,j)*cos(-a(i,j)))*I(i,j)^2;
            dltQij(i,j)=3*abs(1/Y(i,j)*sin(-a(i,j)))*I(i,j)^2;
        end
    end
end
dltPij
dltQij
dltPtotal=sum(sum(dltPij))
dltQtotal=sum(sum(dltQij))
P1=-sum(Pgiv)+dltPtotal
Q1=-sum(Qgiv)+dltQtotal

```

## VI. Conclusion

In summary, this study met its objectives by carefully exploring transmission network growth planning procedures. The study realized ideal ways to manage growing energy demands in urban areas by meticulously reviewing present infrastructures and innovative methodologies. Significant contributions have been made to electrical transmission network planning, comprising data-driven insights, better modeling tools, and protocols to sustainable development. By incorporating these insights, this research paper not only enhances existing procedures but also lays the framework for future breakthroughs in the industry. The proposed recommendations should be implemented in real time, using future technologies such as IoT and AI for dynamic network management. Furthermore, research into integrating renewable energy sources and technology for smart grids would contribute to the progress of developing robust and sustainable transmission networks.

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