

Computer Based Free Vibration Analysis of Isotropic Thin Rectangular Flat CCCC Plate

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ABSTRACT

The paper is aim at the development of a computer program for free vibration analysis of an all-round clamped (i.eCCCC) rectangular thin isotropic plate. Polynomial-based shape function for a CCCC plate, was used in Ritz energy equation to formulate an equation in term of a non-dimensional parameter 'k' for calculating the fundamental natural frequency of the plate. Computer program based on the formulated equation was develop in Matlab language for calculating fundamental natural frequency of a CCCC rectangular thin isotropic plate subjected to free vibration. To validate the values of the fundamental natural frequency obtained from the program, comparison was made between the values obtained and those available in relevant existing literatures. And the average percentage difference was 0.079% which is acceptable. Hence, it can be deduce that the computer program can be adopted as a better and faster alternative to the former rigorous approaches for obtaining the fundamental natural frequency of CCCC plate.

Keywords: Computer Program, Free Vibration, Fundamental Frequency, MATLAB, Thin Rectangular Isotropic Plate.

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I. INTRODUCTION

What is important in the analysis of free vibration is the fundamental natural frequency which is the value of an externally induced frequency of a vibrating plate that causes it to resonate. Structural elements, such as plates frequently undergo vibration which is usually undesirable. Several scholars have carried out free vibration analysis of CCCC rectangular plates using classical and approximate methods based on Fourier or trigonometric series. Among them are Chakraverty (2009); Lee (2004); Misra(2012); Ventsel & Krauthammer (2001); Werfalli & Karaid(2005); Sakata et al (1996).

Due to the rigorousness of the classical and approximate methods using Fourier series, recent researchers such as Ibearugbulem et al.(2013); Njoku et al.(2013); Ebirim et al.(2014); Ezeh et al.(2014); Ibearugbulem et al. (2014), have gone ahead to use Taylor Maclaurin's series for the analysis of vibration of rectangular plates. The approach they adopted is less rigorous but still full of lengthy and time consuming computations.

None of the scholars have to the knowledge of the writers; develop computer program(s) based on this new approach that uses polynomial series. Thus, this work concerned with designing a computer program for analyzing CCCC thin rectangular isotropic flat plate that uses polynomial series.

II. THEORETICAL BACKGROUND

Consider a plate clamped all round (i.e CCCC) shown in Fig. 1. The total potential energy functional for free vibration of rectangular isotropic plate as developed by Ibearugbulem(2013) using Ritz method, is given in (1) as

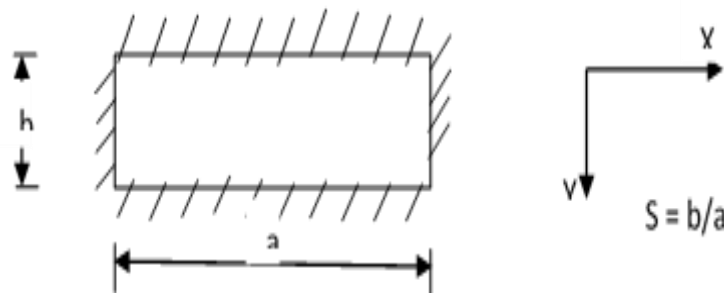


Figure 1: Schematic Representation of CCCC Rectangular Plate.

where a and b are plate dimensions, s is aspect ratio, while X and Y are axis

$$\Pi = \frac{D}{2b^2} \iint \left[\frac{1}{p^3} (w''^R)^2 + \frac{2}{p} (w''^{RQ})^2 + p (w''^Q)^2 \right] \partial R \partial Q - p \rho h \omega^2 b^2 \iint w^2 \partial R \partial Q \quad (1)$$

where $p = a/b$ is the aspect ratio, D is flexural rigidity, w is plate deflection, $R = X/a$ and $Q = Y/b$ are non-dimensional parameters, and $w''^R = \frac{\partial^2 w}{\partial R^2}$; $w''^Q = \frac{\partial^2 w}{\partial Q^2}$; $w''^{RQ} = \frac{\partial^2 w}{\partial R \partial Q}$, ρ is the specific gravity of the plate material use (steel), h is plate thickness and ω is the fundamental natural frequency. The double integrals are within $0 \leq R \leq 1$ and $0 \leq Q \leq 1$

Assuming $w = Ak$, then total potential energy functional, Π , becomes

$$\Pi = \frac{DA^2}{2b^2} \iint \left[\frac{1}{p^3} (k''^R)^2 + \frac{2}{p} (k''^{RQ})^2 + p (k''^Q)^2 \right] \partial R \partial Q - p \rho h \omega^2 b^2 A^2 \iint k^2 \partial R \partial Q \quad (2)$$

where A is amplitude of deflection, and k is the R and Q components of the shape function.

Minimizing (2) and making ω^2 the subject of the formula, gives (3)

$$\omega^2 = \frac{\iint \left[\frac{1}{p^3} (k''^R)^2 + \frac{2}{p^2} (k''^{RQ})^2 + (k''^Q)^2 \right] \partial R \partial Q}{\iint k^2 \partial R \partial Q} \cdot \frac{D}{\rho h b^4} \quad (3)$$

For aspect ratio $s = b/a = 1/p$, the (3) becomes (4)

$$\omega^2 = \frac{\iint \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^R)^2 \right] \partial R \partial Q}{\iint k^2 \partial R \partial Q} \cdot \frac{D}{\rho h a^4} \quad (4)$$

$$\text{Let } f_s^2 = \frac{\iint \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^R)^2 \right] \partial R \partial Q}{\iint k^2 \partial R \partial Q} \quad (5)$$

Therefore, f_s is given by (6)

$$f_s = \left[\frac{\iint \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^R)^2 \right] \partial R \partial Q}{\iint k^2 \partial R \partial Q} \right]^{1/2} \quad (6)$$

From (4), the square of the fundamental natural frequency can be obtained as:

$$\omega^2 = f_s^2 \cdot \frac{D}{\rho h a^4} \quad (7)$$

$$\text{Hence, } \omega = \frac{f_s}{a^2} \sqrt{\frac{D}{\rho h}} \quad (8)$$

where f_s (or f_s) is the coefficient of fundamental natural frequency.

But for CCCC plate, Adah(2016) gave k value asin (9)

$$k = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) = UV \quad (9)$$

(9) can be substituted into (7), and the resulting equation is used to develop a Matlab program for easy and quick analysis.

III. DEVELOPMENT OF COMPUTER PROGRAM

To ease and quicken the analysis, a program was developed to aid free vibration analysis of CCCC plate using Matlab. According to Chapra (2012), Matlab offers better advantage as a result of its ability to handle complex mathematical manipulations and have a user friendly environment. Matlab codes were applied to the derived expression systematically to develop the program for the analysis of free vibration of CCCC rectangular isotropic plate. Fig. 2 shows the flow chart of the develop program.

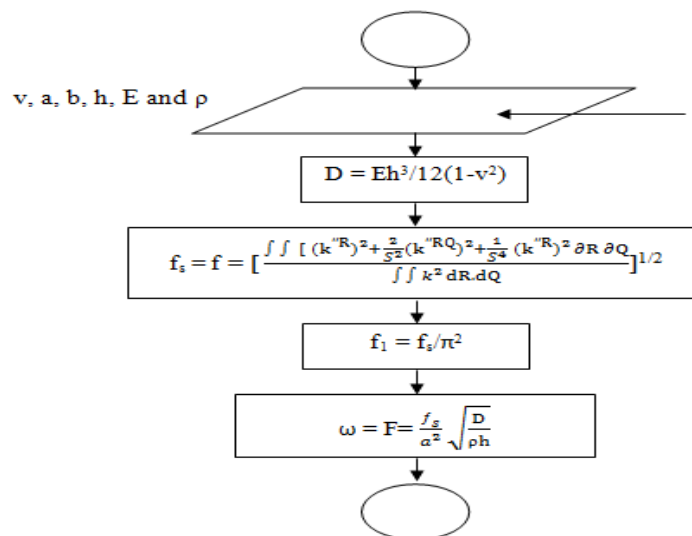


Figure 2: Flow Chart

The program is presented in Appendix and the values of the coefficients of the fundamental natural frequency 'f_s' obtained from the program are presented on Table 1.

IV. RESULTS AND DISCUSSIONS

When the developed program was executed by aspect ratio s = b/a, the values of the coefficient of the fundamental natural frequency, f_s, given in Table 1 were obtained.

Table 1: Coefficients 'f_s' of Fundamental Natural frequency

Aspect Ratio s= b/a	CCCC	
	$\omega = \frac{f_s}{a^2} \sqrt{\frac{D}{\rho h}}$ f_s	$\omega = \frac{f_{1s}}{a^2} \sqrt{\frac{D}{\rho h}}$ $f_{1s} = f_s \pi^2$
1.0	36.000	3.648
1.2	30.774	3.118
1.5	27.047	2.740
1.6	26.333	2.668
2.0	24.648	2.497

4.1 Comparison of Coefficients of Fundamental Natural Frequency, F.

The values of the fundamental natural frequency 'f' obtained from the executed program were validated by carrying out a comparative analysis in Table 2, with those of Njoku et al.(2013) and Ibearugbulem et al.(2014). A close look at Table 2, shows that the percentage differences are all less than 0.1% when compared with values obtained by Njoku et al. (2013). The implication of this is that the values of, 'f', obtained from this program, are very close to those values obtained by Njoku et al. (2013). Also, comparing with Ibearugbulem et al.(2014), the percentage differences ranges from the lowest value of 0.040% to the highest value of 0.110%, which are all negligible. And so, the computer results closely agree with the results obtained by Ibearugbulem et al. (2014).

V. CONCLUSION

Since, the values of the results from the computer program, are very close, the computer program written in MATLAB, can predict accurately fundamental natural frequency, ω. Thus, the conclusion is that, the use of the develop computer program, is an easier and quicker approach for accurate analysis of free vibration of a thin isotropic rectangular plate clamped all round.

Table 2: Comparison of computer values of coefficients of fundamental natural frequency 'f_s' with those of Njoku et al. (2013) and Ibearugbulem et al. (2014)

Aspect Ratio S = b/a S	Computer Values of Coeff.	Njoku et al (2013)	% difference 100(f ₁ ,f ₂)/f ₂	Computer Values of Coeff.	Ibearugbulem et al (2014)	% difference 100(f ₃ ,f ₄)/f ₄
	f ₁	f ₂		f ₃	f ₄	
1.0	36.000	35.9709	0.081	3.648	3.644	0.110
1.2	30.774	30.7508	0.075	3.118	3.115	0.096
1.5	27.047	27.0305	0.061	2.740	2.738	0.073
1.6	26.333	26.3175	0.059	2.668	2.666	0.075
2.0	24.648	24.6377	0.041	2.497	2.496	0.040
Aver. %diff.			0.063			0.079

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APPENDIX

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clc
%PROGRAM FOR ANALYZING FUNDAMENTAL NATURAL FREQUENCY OF CCCC PLATE
v = input('Enter value of poisson ratio v:');
a = input('Enter plate dimension along x-axis -length- a(m):');
b = input('Enter plate dimension along y-axis -width- b(m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
syms r q
U = r^2-2*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
Y6 = y2*z1;
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)

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