The International Journal Of Engineering And Science (IJES)
|| Volume || 4 || Issue || 7 || Pages || PP.001-05|| July - 2015 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

STRONGLY b-δ-CONTINUOUS FUNCTIONS

Dr. S.Anuradha, S.Padmanaban

Prof & Head, P G & Research Dept. of Mathematics, Hindusthan College of Arts & Science
Assistant Professor, Department of Science & Humanities, Karpagam Institute of Technology

ABSTRACT

Generalized open sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general topology and real analysis concerns the various modified forms of continuity, separation axioms etc., by utilizing generalized open sets. In this paper, we introduce and study a new class of function called strongly b-δ- continuous function by using the notions of b-δ-open sets and b-δ-closed sets. We investigate some of the fundamental and basic properties of this strongly b-δ- continuous function.

KEYWORDS - *δ - open sets, b-open set, b-δ-open sets, strongly b-δ-open function.*

Date of Submission: 20-June-2015 Date of Accepted: 20-July-2015

I. INTRODUCTION

In 1961, Levine [9] introduced the concept of weak continuity as a generalization of continuity. Latter in 1963, Levine [10] also introduced the concept of semi open sets in topological space. Since then numerous applications have been found in studying different types of continuous like maps and separation of axioms.. In 1966, Hussain [7] introduced almost continuity as another generalization of continuity and Andrew and Whittlesy [1] introduced the concept of closure continuity which is stronger than weak continuity. The concept of δ -interior, δ -closure, θ-interior and θ-closure operators were first introduced by Velico [22] in 1968, for the purpose of studying the important class of H-closed spaces. These operators have since been studied intensively by many authors. The collection of all δ -open sets in a topological space (X, τ) forms a topology.

In 1970, Levine [11] initiated the study of generalized closed sets, i.e., the sets whose closure belongs to every open superset and defined the notation of $T_{1/2}$ space to be one in which the closed sets and generalized closed sets coincide. In 1980, the notion of δ -continuous function was introduced and studied by Noiri [16]. Latter in 1982, Mashhour.et.al., [12] introduced the concept of pre open sets. In 1986, the notion of semi-pre open set was introduced by Andrijevic [2]. Latter in 1996, Andrijevic [3] introduced a class of generalized open sets in a topological space, so called b-open sets. The class of b-open sets is contained in the class of semi preopen sets and contains all semi-open sets and pre-open sets. In 2003, Ganguly.et al.,[6] introduced the notion of strongly δ -continuous function in topological spaces. Latter, El.Atik [5] introduced and studied the notion of b-continuous function. He also introduced and studied a new class of functions called b-irresolute function.
This notion has been studied extensively in recent years by many topologists. The notions of b-δ -closed Sets was introduced and studied by Padmanaban [19]. The purpose of this paper is to introduce and investigate the function of strongly b-δ- continuous function. We investigate some of the fundamental properties of this class of functions. We recall some basic definitions and known results.

II. PRELIMINARY

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space (X, τ). We denote closure and interior of A by cl(A) and int(A), respectively. The set A is said to be regular open (resp. regular closed) [21] if A = int(cl(A))(resp. A = cl(int(A))).The family of all regular open(resp. Regular closed) sets of (X, τ) is written by RO(X, τ) (resp. RC(X, τ)).This family is closed under the finite intersections (resp. finite unions). The δ –closure of A [22] is the set of all x in x such that the interior of every closed neighbourhood of x intersects A non trivally. The δ –closure of A is denoted by cl_δ (A) or δ-cl(A) . The δ –interior of a subset A of X is the union of all regular open sets of X contained in A and is denoted by δ-int(A).The subset A is called δ-open if A= δ-int(A). i.e, a set is δ-open if it is the union of regular open sets.The complement of δ-open set is δ-closed. Alternatively, a set $A \subset (X, \tau)$ is called δ-closed if A= δ-cl(A), where $\delta\text{-cl}(A) = \{x \in X : \text{int}(\text{cl}(U)) \cap A \neq \varphi, U \in \tau \text{ and } x \in X\}$.

A subset A is said to be b-open [3] if $A \subset \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$.The complement of b-open is said to be b-closed. The intersection of b-closed sets of X containing is called b-closure of A and denoted by b-cl(A).The union of all b-open sets of X contained in A is called b-interior and is denoted by b-int(A).The subset A is b-regular if it is b-open and b-closed. The family of b-open (b-closed, b-regular) sets of X is denoted by BO(X) (resp. BC(X), BR(X)) and family of all b-open (b-regular) sets of X containing a point $x \in X$ is denoted by BO(X, x) (resp.BR(X, x)).

Let A be a subset of a topological space (X,τ) . A point x of X is called a b-δ-cluster point [19] of A if int (b-cl(U)) $\cap$ A $\neq$ φ for every b-open set U of X containing x. The set of all b-δ-cluster point of A is called b-δ-closure of A and is denoted by b-δ-cl(A). A subset A of a topological space (X, τ) is said to be b-δ-closed, if A = b-δ-cl(A). The complement of a b-δ-closed set is said to be b-δ-open set.The b-δ-interior of a subset A of X is defined as the union of all b-δ-open sets contained in A and is denoted by b-δ-int(A). Alternatively, a point x in X is called b-δ-interior point of A, if there exists a b-open sets containing x such that int(b-cl(U)) ⊆ A. The set of all b-δ-interior points of A is called b-δ-interior of A. The family of all b-δ-open sets of the space (X,τ) is denoted by BδO(X) and the family of all b-δ-closed sets of the space (X, τ) is denoted by BδC(X). A function f: (X, τ) $\rightarrow$ (Y, σ) is said to be b-δ -continuous (briefly b- δ -c) if for each x$\in$ X and each open set V of (Y, σ) containing f(x), there exists a b-open set U in (X, τ) containing x such that f(int (b-cl(U))) $\subseteq$ cl(V). A subset A of a space X is said to be α-open [15] (resp. semi-open [10], preopen [12], β-open or semi-preopen [2]) if A ⊊ int(cl(int(A))) (resp. A ⊊ d(int(A)), A ⊊ int(cl(A)),A ⊊ cl (int (cl(A))).

A function f: (X, τ) $\rightarrow$ (Y, σ) is said to be

1. α – continuous [13] , if f^{-1}(V) is α - open in (X, τ) for every open set V of (Y, σ),
2. b-continuous [5] , if for each x $\in$ X and each open set V of Y containing f(x), there exists U $\in$ BO(X,x) such that f(U) ⊆ V,
3. δ -continuous function [16], if for each x $\in$ X and each open set V containing f(x), there is an open set U containing (X, τ) such that f(int(cl(U))) ⊆ int(cl(U)),
4. Strongly continuous [8], if for every subset A of topological space (X, τ), f(cl(A)) ⊆ f(A),
5. Strongly δ –continuous [6], at a point x $\in$ X if and only if for any open neighbourhood V of f(x) in (Y, σ), there exists a δ -open neighbourhood U of x in (X, τ) such that f(U) ⊆ V,
6. Open [8], if f (U) is open in (Y, σ) for every open set U of (X, τ),
7. α –open [13], if f(U) is α - open in (Y, σ) for every open set U of (X, τ),
8. b- open [5], if f(U) is b-open in (Y, σ) for every open set U of (X, τ).

A topological space (X, τ) is said to be

1. b-T_2 [18], if for each pair of distinct points x and y in (X, τ),there exists U $\in$ BO(X,x) and V $\in$ BO(X,y) such that U ∩ V = ϕ . i.e., If every two distinct points of (X, τ) can be separated by disjoint b-open sets.
2. almost regular [20], if for any regular open set U ⊆ X and each point x $\in$ U, there is a regular open set V of X suchthat x $\in$ V ⊆ cl(V) ⊆U,
3. almost b-regular [17], if for any regular closed set F ⊆ X and any point x $\in$ X-F, there exist disjoint b-open sets U and V such that with x $\in$ U and F ⊆ V,
4. b-compact [5], or γ -compact if every cover of (X, τ) by b-open sets has a finite subcover.

Let A be subset of a topological space (X, τ)). The collection of subsets of X in (X, τ) is said to cover [4], of A if and only if every point of A belongs to atleast one of these subsets.

Lemma 2.1. [14] In a topological space (X, τ),

1. The intersection of an open set and a b-open set is b-open set,

2. The intersection of an α-open set and a b-open set is b-open set.

Lemma 2.2. [14] If X_0 is α-open in topological space (X, τ), then, BO(X_0) = BO(X) ∩ X_0 .

Lemma 2.3. [18] If A ⊆ X_0 ⊆ X and X_0 is an α-open in topological space (X, τ), then b-cl(A) ∩ X_0 = b-cl $_{X0}$ (A) ,where b-cl $_{X0}$ (A) denotes the b-closure of A in the subspace X_0 .

Lemma 2.4. [14] Let A be subset of topological space (X, τ) and B be subset of (Y,σ). If A ∈ BO(X) and B ∈ BO(Y), then A x B ∈ BO(X x Y).

Lemma 2.5. [5] If f: (X, τ) → (Y, σ) is an α-continuous, α-open function and V is b-open(resp. b-closed) set of (Y, σ), then f^{-1} (V) is b-open (resp. b- closed) in (X, τ).

Lemma 2.6. [18] For a topological space (X, τ), then the following are equivalent:

1. X is b-regular,
2. For each point x ∈ X and for each open set U of (X, τ) containing x, there exists V ∈ BO(X) such that x ∈ V ⊆ b-cl (V) ⊆ U ,
3. For each subset A of X and each closed set F such that A ∩ F = φ , there exist disjoint U, V ∈ BO (X) such that A ∩ U ≠ φ and F ⊆ V ,
4. For each closed set F of X, F = ⋂ { b-cl(V): F ⊆ V and V ∈ BO (X)}.

III. STRONGLY b - δ-CONTINUOUS FUNCTIONS.

Definition 3.1. A function f: (X, τ) → (Y, σ) is said to be strongly b-δ -continuous (briefly st. b- δ -c) if for each x∈ X and each open set V of (Y, σ) containing f(x), there exists a b-open set U in (X, τ) containing x such that f(int (b-cl(U))) ⊆ V.

Theorem 3.2. Every strongly b-δ -continuous function is b-δ -continuous function.
Proof. Obvious.
The Converse of the above theorem need not be true as shown in the following example.

Example 3.3. Let X = {a, b, c,} and τ= {φ, {a}, {b}, {a, b}, X}. Define a function f: (X, τ) → (X, σ) by f(a) = b, f(b) = c and f(c) = c. Then we have BO(X) = {φ, {a}, {b}, {a, b},{a, c}, {b, c}, X}. Then f is b- δ-continuous function but not strongly b- δ -continuous function, since for V = {b} and V ={a, b} there exists no U ∈ BO(X, x) for x=a such that f(int(b-cl(U))) ⊆ V.

Theorem 3.4. Let (Y, σ) be a regular space. Then f: (X, τ) → (Y, σ) is strongly b- δ -continuous if and only if f is b-continuous.
Proof. Let x ∈ X and V an open set of (Y, σ) containing f (x). Since (Y, σ) is regular, there exists an open set W such that f (x) ∈ W ⊆ cl (W) ⊆ V, by Lemma 2.6. . If f is b-continuous, there exists U ∈ BO (X, x) such that f (U) ⊆ W. Suppose that y ∉ cl (W). Then there exists an open set G containing y such that G ∩ W = φ. Since f is b-continuous, f^{-1} (G) ∈ BO(X) and f^{-1}(G) ∩ U = φ. Hence f^{-1} (G) ∩ b-cl (U) = φ . Therefore, we obtain G ∩ f (int(b-cl (U))) = φ and y ∉ f (int(b-cl (U))). Consequently, we have f (int(b-cl (U))) ⊆ cl (W) ⊆ V . The converse is obvious.

Theorem 3.5. If f: (X, τ) → (Y, σ) is strongly b- δ -continuous and X_0 is an α-open subset of X , then the restriction f | $_{X0}$: X_0 → Y is strongly b- δ -continuous.
Proof. For any x∈ X_0 and any open set V of (Y, σ) containing f (x), there exists U ∈ BO(X, x) such that f (int(b-cl (U))) ⊆ V since f is strongly b- δ -continuous. Put U_0 = U∩X_0 then by Lemma 2.2. and Lemma2.3., U_0 ∈ BO (X_0 ,x) and b-cl$_{X0}$ (U_0) ⊆ b-cl (U_0). Therefore, we obtain (f| $_{X0}$)(int(b-cl$_{X0}$(U_0)))=f(int(b-cl$_{X0}$(U_0)) ⊆ f(int(b-cl(U_0))) ⊆ f(int(b-cl(U))) ⊆ V. This shows that f| $_{X0}$ is strongly b- δ -continuous.

Lemma 3.7. If f : (X, τ) → (Y, σ) is an α-continuous α-open function and V is a b- δ -open set of (Y, σ) , then f^{-1} (V) is b- δ -open in (X, τ).

Proof. Let V be a b- δ -open set of (Y, σ), and $x \in f^{-1}(V)$. There exists W ∈ BO (Y) such that f (x) ∈ W ⊆ b-cl (W) ⊆ V. By Lemma2.5., $f^{-1}(W) \in BO(X)$ and b-cl $(f^{-}(W)) \subseteq f^{-1}$(b-cl (W)). Therefore, we have $x \in f^{-1}(W) \subseteq$ b-cl $(f^{-1}(W)) \subseteq f^{-1}(V)$ and $f^{-1}(V)$ is b- δ -open in (X, τ).

Theorem 3.8. If f: (X, τ) → (Y, σ) is b-continuous and (Y, σ) is almost regular, then f is strongly b- δ -continuous.

Proof. Let x∈ X and V be any open set in (Y, σ) containing f(x). Since (Y, σ) is almost regular, there exists a regular open set W such that f(x) ∈ W⊆ cl(W) ⊆ V. Since f is b-continuous, there exists U ∈ BO(X) containing X such that f(U) ⊆ W. Suppose y ∉ cl(W). Then there exists an open neighborhood G of Y such that G ∩ W =φ. Since f is b-continuous, $f^{-1}(G) \in BO(X)$ and $f^{-1}(G) \cap U = \varphi$. Hence $f^{-1}(G) \cap$ int(b-cl(U)) = φ. Therefore, we obtain G ∩ f(int(b-cl(U))) = φ and y ∉ f(int(b-cl(U))). This shows that f(int(b-cl(U))) ⊆ cl(W) ⊆ V. Hence f is strongly b-δ-continuous function.

Theorem 3.9. If f : (X,τ) →(Y,σ) is a strongly b-δ-continuous injection and (Y, σ) is T_0, then (X,τ) is b -T_2 .

Proof. Let x and y be any distinct points of (X,τ) . Since f is injective, f (x) ≠ f (y) and since Y is T_0, there exists an open set V containing f (x) but not containing f (y) or an open set W containing f (y) but not containing f (x). If the first case holds, then since f is strongly b-δ-continuous, there exists U ∈BO(X, x) such that f (int(b-cl (U))) ⊆ V .Therefore, we obtain f (y) ∉f (int(b-cl (U))). If the second case holds, then we obtain a similar result. Therefore, (X, τ) is b -T_2.

Theorem 3.10. If f : (X,τ) →(Y,σ) is a strongly b-δ-continuous function and (Y, σ) is Hausdorff, then the subset E = { (x, y) : f (x) = f (y)} is b-δ-closed in X x X .

Proof. Suppose that (x, y) ∉ E. Then f(x) ≠ f (y). Since (Y, σ) is Hausdorff , there exist open sets V and W of (Y, σ) containing f (x) and f (y) respectively, such that V ∩ W = φ. Since f is strongly b-δ-continuous, there exist U ∈ BO(X, x) and G ∈ BO(X, y) such that f (int(b-cl (U))) ⊆V and f (int(b-cl (G))) ⊆ W . Set D=b-cl (U) x b-cl (G). It follows that (x, y) ∈D ∈BR(X x X) and D ∩ E = φ. This means that b-δ-cl(E) ⊆ E and therefore, E is b-δ-closed in X x X .

Theorem 3.11. If f : (X,τ) →(Y,σ) is a strongly b-δ-continuous function and A is b-closed relative to X , then f (A) is a compact set of (Y, σ) .

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of f (A) by open sets of (Y, σ) . For each point x ∈ A, there exists α(x) ∈ Λ such that $f(x) \in V_{\alpha(x)}$. Since f is strongly b-δ-continuous, there exists $U_x \in BO(X, x)$ such that f (int(b-cl(U_x))) ⊆ $V_{\alpha(x)}$. The family $\{U_x : x \in A\}$ is a cover of A by b-open sets of X and hence there exists a finite subset A_0 of A such that $A \subseteq \bigcup_{x\in A0}$ b-cl (U_x). Now since U_x ∈BR(X,x), U_x is b-closed. Then U_x = b-cl(U_x). Therefore we have $f(A) \subseteq \bigcup_{x\in A0} V_{\alpha(x)}$. This shows that f (A) is compact.

Corollary 3.12. Let f: (X,τ) →(Y,σ) be a strongly b-δ-continuous surjection. Then the following hold:
1. If X is b-closed, then (Y, σ) is compact,
2. If X is countably b-closed, then (Y, σ) is countable compact.

IV. CONCLUSION

An attempt has been made in this paper, to introduce a new class of function called strongly b-δ-continuous function. Some fundamental and basic properties of this strongly b-δ- continuous function is investigated. We also derived the relationship between strongly b-δ- continuous function and b-δ-continuous and also with b-continuous function. Further we can extend this paper by finding relationship of strongly b-δ- continuous function with δ- continuous function , strongly δ- continuous function and etc..

REFERENCES

[1] D. R. Andrew and E. K. Whittlesy, Closure continuity, *Amer. Math. Month., (73)*,1966, 758 - 759.
[2] D. Andrijevic, Semi-preopen sets, *Mat. Vesnik, (18),* 1986, 24 - 32.
[3] D. Andrijevic, On b-open sets, *Mat. Vesnik, (48),* 1996, 59 - 64.
[4] R. David Wilkins, *Algebraic topology,* (2008).

[5] A.A.El.Atik, A study of some types of mappings on topological spaces, M.Sc. Thesis,Faculty of science,Tanta University,Tanta, Egypt, (1997).
[6] S.Ganguly and K.Dutta, Strongly δ -continuous function and topology on function space, *Bull.Stiint. Univ. Buaia Mare, Romania.,(1),* 2002, 39 - 52.
[7] T.Husain, Almost continuous functions, *Prace Mat., (10),* 1996, 01 - 07.
[8] N. Levine, Strong continuity in topological spaces, *Amer. Math. Month., (67).* 1960, 266 - 269.
[9] N. Levine, A decomposition of continuity in topological spaces, *Amer. Math. Month., (68),*1961, 44 - 46.
[10] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly, (70),* 1963, 36 - 41.
[11] N. Levine, Generalized closed sets in topology, *Rend Circ Mat Palermo, (19),* 1970, 89 - 96.

[12] A. S. Mashhour, M. E. Abd El-Monsef, and S.N. El-Deeb, On precontinuous and weak precontinuous functions, *Proc. Math. Phys. Soc. Egypt,(53),* 1982, 47 - 53.
[13] A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, α -continuous and α -open mappings, *Acta Math. Hungar., (41),* 1982, 213 - 218.
[14] A. A. Nasef, On b-locally closed sets and related topic, Chaos, *Solitons and Fractals, (12),* 2001, 1909 - 1915.
[15] O. Njastad, On some classes of nearly open sets, *Pacific J. Math., (15),* 1965, 961 - 970.
[16] T. Noiri, On δ -Continuous functions, *J.Korean Math Soc., (16),* 1980, 161 - 166.
[17] T. Noiri and S. M. Kang, On almost strongly θ -continuous functions, *Indian J. Pure. Appl. Math., 15*(1), 1984, 1 – 8
[18] J.H.Park, Strongly θ-b-continuous, *Acta Math. Hunger.,110*(4), 2006, 347-359.
[19] S.Padmanaban, On b- δ-open sets in topological spaces, *Int Jr. of Mathematical Sciences & Applications,(3),* 2013, 365-371.
[20] M. K. Singal and S. P. Arya, On almost regular spaces, *Glas.Mat.Ser.III4*(24), 1969, 89 - 99.
[21] M. Stone,Applications of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc., (41),* 1937, 375 - 381.
[22] N. V. Velicko, H-closed topological spaces, *Amer. Math. Soc. Transl., (78),* 1968, 103 - 118.

BIOGRAPHIES AND PHOTOGRAPHS

Dr.S.Anuradha *is working as Prof & Head, PG & Research Dept of Mathematics, Hindusthan Arts College. She has more than 20 years teaching experience and 15 years of research experience. She has published 25 research articles in reputed international journals and attended 62 Seminars / Workshops / Conference and presented the papers in the same. She guided 13 M.Phil students and guiding 6 Ph.D students.*

S.Padmanaban *is working as Assistant Professor in Department of Science & Humanities, Karpagam Institute of Technology. He is having more than 19 years of teaching experience in engineering colleges and published 6 papers in reputed international journals.*