A new method for the construction of Partially balanced n-ary block design

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Abstract

The concept of partially balanced n-ary block (PBNB) designs was first introduced by Mehata, Agarwal and Nigam (1975) as a generalization of balanced n-ary block (BIB) designs. In this paper an attempt is made to propose a new method for the construction of partially balanced n-ary block designs using balanced n-ary block designs. The method is also illustrated with a suitable example.

Key words: Balanced n-ary Block Design; Partially Balanced n-ary block design.

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I. INTRODUCTION

Incomplete block designs were introduced to eliminate heterogeneity to a greater extent than is possible with randomized blocks and Latin squares when the number of treatments is large. The arrangement of ‘v’ treatments in ‘b’ blocks, each of sizes $k_1$, $k_2$, …, $k_b$, each of the treatment appears $r_1$, $r_2$, …, $r_v$ blocks such that some pairs of treatments occur in $\lambda_1$ blocks, some pairs of treatments occur in $\lambda_2$ blocks, soon some rest of pairs of treatments occur in $\lambda_m$ blocks the design is said to be a “General Incomplete Block Design”.

The total number of treatments are $\Sigma r_i = \Sigma k_j$, where $i=1, 2, \ldots, v$; and $j=1, 2, \ldots, b$. If each treatment occurs at most once in blocks then the design is binary and if it occurs at most (n-1) times the design is said to be n-ary design. Balanced n-ary block designs were introduced by Tocher (1952) as a generalization of balanced incomplete block binary designs by allowing a treatment to occur more than once in a block.

DEFINITION 1.1: A balanced n-ary block design (BnBD) is one whose incidence matrix $N_{BV}$ has $n_{ij}$ ($j = 1, 2, \ldots, B$, $i = 1, 2, \ldots, V$), as elements where $n_{ij}$ takes any one of the n-distinct values $0, 1, \ldots, n-1$ and the variance of the comparison between any two treatment is the same.

For such a design, V treatments are arranged in B blocks each of size K such that every treatment is replicated R times and the sum of products $n_{ij}n_{ij}$ is constant ($\Sigma n_{ij}n_{ij} = \pi$ say). The quantities V, B, R, K, and $\pi$ are called the parameters of the balanced n-ary block design.

DEFINITION 1.2: A block design with V treatments, B blocks is said to be partially balanced n-ary block design with $p$- associate classes if

(i) The incidence matrix $N_{BV}$ has $n$ entries 0,1,2,..,n-1
(ii) The row sum $N_{B}$ is $K$
(iii) The column sum of $N_{B}$ is $R$ and the column sum of squares is $\delta$
(iv) The inner product of any two columns of $N_{BV}$ is $\pi_{\alpha}$, if $\theta$ and $\phi$ are mutually $\alpha$th associates $\alpha=1, 2, \ldots, p$
(v) There exists a relationship between the treatments defined as
(a) Any two treatments are either 1st, 2nd, or $p$th associate being symmetrical,
(b) Each treatment $\theta$ has $n_\alpha \cdot \alpha$ associates. If $\theta$ and $\phi$ are $\alpha^{th}$ associates the number of treatments that are $j^{th}$ associates of $\theta$ and $k^{th}$ associates of $\phi$ is $p_{ijk}$

In particular, in the incidence matrix $N_{BxV'}$, elements $n_{ij}$ takes three values 0,1,2 the design corresponding to the incidence matrix is called ‘partially balanced ternary design (PBTD)’ and. $n_{ij}$ takes four values, 0,1,2,3 the corresponding design is called ‘partially balanced quarternary design (PBQD)’. In this paper an attempt is made propose a new method of constructions of partially balanced n-ary block design.

II. METHOD OF CONSTRUCTION OF PARTIALLY BALANCED N-ARY BLOCK DESIGN

Theorem 2.1 : If $N_{V\times B}$ is the incidence matrix of Balanced Ternary Design with parameters $V$, $B$, $R$, $K$ and $\pi$ AND where $J$ is matrix of unities, then $N^* = \begin{bmatrix} N & J \\ J & N \end{bmatrix}$ is the incidence matrix of Partially Balanced Ternary Design with parameters $V'=2V$, $B'=2B$, $R'=R+B$, $K'=V+K$ and $\pi_1=2R$.

The method is illustrated in the example 2.1

EXAMPLE 2.1: Consider a BTD with $V = 4$, $B = 12$, $K = 4$, $R = 12$, $\pi=10$ with incidence matrix $N$.

where $N_{V\times B}^* = \begin{bmatrix} 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$

The resulting incidence matrix of PBTD with $V'=8$, $B'=24$, $R'=24$, $K'=8$, $\pi_1=22$ and $\pi_2=24$ is

$N_{B\times V'}^* = \begin{bmatrix} 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

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REFERENCES:


