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A new method for the construction of Partially balanced n-ary block design

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ABSTRACT
The concept of partially balanced n-ary block (PBnB) designs was first introduced by Mehata, Agarwal and Nigam (1975) as generalization of balanced n-ary block (BIB) designs. In this paper an attempt is made to propose a new method for the construction of partially balanced n-ary block designs using balanced n-ary block designs. The method is also illustrated with a suitable example.
Key words: *Balanced n-ary Block Design; Partially Balanced n-ary block design.*

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I. INTRODUCTION

Incomplete block designs were introduced to eliminate heterogeneity to a greater extent than is possible with randomized blocks and Latin squares when the number of treatments is large. The arrangement of 'v' treatments in 'b' blocks, each of sizes k_1, k_2, … , k_b, each of the treatment appears r_1, r_2, … , r_v blocks such that some pairs of treatments occur in λ_1 blocks, some pairs of treatments occur in λ_2 blocks , soon some rest of pairs of treatments occur in λ_m blocks the design is said to be a "General Incomplete Block Design". The total number of treatments are $\Sigma r_i = \Sigma k_j$, where i=1, 2, … , v; and j= 1, 2, … , b. If each treatment occurs at most once in blocks then the design is binary and if it occurs at most (n-1) times the design is said to be n-ary design. Balanced n-ary block designs were introduced by Tocher (1952) as generalization of balanced incomplete block binary designs by allowing a treatment to occur more than once in a block.

DEFINITION 1.1: A balanced n-ary block design (BnBD) is one whose incidence matrix N_{BxV} has n_{ij} (j = 1,2, … , B, i= 1,2,…,V), as elements where n_{ij} takes any one of the n-distinct values 0, 1, … , n-1 and the variance of the comparison between any two treatment is the same.

For such a design, V treatments are arranged in B blocks each of size K such that every treatment is replicated R times and the sum of products $n_{ij}n_{ij'}$ is constant ($\Sigma n_{ij}n_{ij'} = \pi$ say). The quantities V,B, R, K, and π are called the parameters of the balanced n-ary block 'design.

DEFINITION 1.2: A block design with V treatments, B blocks is said to be partially balanced n-ary block design with p- associate classes if

(i) The incidence matrix N_{BxV} has n entries 0,1,2, ..n-1
(ii) The row sum N_{BxV} is K
(iii) The column sum of N_{BxV} is R and the column sum of squares is δ
(iv) The inner product of any two columns of N_{BxV} is π_α, if θ and ϕ are mutually α^{th} associates α=1, 2, …, p
(v) There exists a relationship between the treatments defined as
  (a) Any two treatments are either 1^{st} , 2^{nd}, or p^{th} associate being symmetrical,

(b) Each treatment θ has n_α- α associates. If θ and ϕ are α^{th} associates the number of treatments that are j^{th} associates of θ and k^{th} associates of ϕ is p_{jk}

In particular, in the incidence matrix N_{BxV}, elements n_{ij} takes three values 0,1,2 the design corresponding to the incidence matrix is called 'partially balanced ternary design (PBTD)' and. n_{ij} takes four values 0,1,2,3 the corresponding design is called 'partially balaced quarternary design (PBQD)'. In this paper an attempt is made propose a new method of constructions of partially balanced n-ary block design.

II. METHOD OF CONSTRUCTION OF PARTIALLY BALANCED N-ARY BLOCK DESIGN

Theorem 2.1 : If N_{VxB} is the incidence matrix of Balanced Ternary Design with parameters V, B, R, K and π AND where J is matrix of unities, then $N^* = \begin{bmatrix} N & J \\ J & N \end{bmatrix}$ is the incidence matrix of Partially Balanced Ternary Design with parameters V'=2V, B'=2B, R'=R+B, K'=V+K and π_1=2R.

The method is illustrated in the example 2.1

EXAMPLE 2.1: Consider a BTD with V = 4, B = 12, K = 4, R = 12, π=10 with incidence matrix N.

$$\text{where} \qquad N'_{VXB} = \begin{bmatrix} 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

The resulting incidence matrix of PBTD with V'=8, B'=24, R'=24, K'=8, π_1=22and π_2=24 is

$$N^*_{B'xV'} =$$

$$\begin{bmatrix} 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

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