The International Journal Of Engineering And Science (IJES)
|| Volume || 4 || Issue || 10 || Pages || PP -18-28|| 2015 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

# Heat and Mass Transfer On Unsteady Mhd Free Convection Flow Of Second Grade Fluid Past An Infinite Rotating Vertical Plate

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**ABSTRACT**

*We consider the unsteady MHD free convection flow of an incompressible electrically conducting second grade fluid bounded by an infinite vertical porous surface in the presence of heat source and chemical reaction in a rotating. The flow through porous medium is governed by Brinkman's model for the momentum equation. In the undisturbed state, both the plate and fluid in porous medium are in solid body rotation with the same angular velocity about normal to the infinite vertical plane surface. The vertical surface is subjected to the uniform constant suction perpendicular to it and the temperature on the surface varies with time about a non-zero constant mean while the temperature of free stream is taken to be constant. The exact solutions for the velocity, temperature and concentration are obtained making use of perturbation technique. The velocity expression consists steady state and oscillatory state. It reveals that, the steady part of the velocity field has three layer characters while the oscillatory part of the fluid field exhibits a multi layer character. The influence of various flow parameters on the velocity, temperature and concentration is analysed graphically, and computational results for the skin friction, Nusselt number and Sherwood number are also obtained in the tabular forms.*

**Keywords:** *Heat and mass transfer, MHD flows, infinite vertical plates, porous medium, rotating channels, second grade fluids.*

Date of Submission: 14 October 2015 Date of Accepted: 25 October 2015

## 1. Introduction

In the study of non-Newtonian fluids, it has been mainly motivated to their importance in the problems from applications of engineering and chemical industry. The partial differential equations usually appear in many areas of the natural and physical sciences. They describe different physical systems, ranging from gravitational to fluid dynamics and have been used to solve problems in the chemistry, mathematical biology, solid state physics etc. We mentioned the most interesting studies of second grade fluids [9, 7, 6, , 2, 4]. Veera Krishna.M and S.G. Malashetty [12] discussed unsteady flow of an incompressible electrically conducting second grade fluid through a composite medium in a rotating parallel plate channel and the problem extended for taking the hall currents by Veera Krishna.M and S.G. Malashetty [13]. The heat transfer rates can be controlled by using a magnetic field. The inclusion of magnetic field in the study of second grade fluid flow has many practical applications for example, the cooling of turbine blades. Magneto hydro dynamics provides a mean of cooling the turbine blade and keeping the structural integrity of the nose cone. Anand Rao [1] studied the magneto-convective flow in a Darcian porous medium channel. Ram [10] discussed analytically the transient hydro magnetic natural convection flow with Hall current effects in a Darcian regime and this extended to consider the supplementary effects of mass transfer [11]. Krishna et al. [8] have studied hydro magnetic convection boundary layer heat transfer through porous medium in a rotating parallel plate channel, presenting analytical solutions and discussing the structure of the different boundary layers formed. Zakaria [14] studied on the magneto hydro dynamic transient natural convection flow of a couple stress fluid through porous medium with relaxation effects also using the state space solution approach.
Recently, Bég et al. [3] have studied the oscillatory hydro magnetic convection through porous regime using a perturbation method. El-Kabeir et al. [5] discussed the group transformation method to study transient hydro magnetic convection boundary layer flow through porous medium.
In this paper, the unsteady MHD free convection flow of an incompressible electrically conducting second grade fluid bounded by an infinite vertical porous surface in the presence of heat source and chemical reaction in a rotating system.

### 1.1. Basic Equations:

An incompressible simple fluid is defined as a material whose state of present stress is determined by the history of the deformation gradient without a preferred reference configuration, its constitutive equation can be written in the form of a functional.

$$T(t) = -pI + \sum_{s=0}^{\infty} F_t^{\,t}(S), \tag{1.1.1}$$

Where $pI$ is the undetermined part of the stress tensor and F is the deformation gradient. The constitutive equation for the stress T in an incompressible fluid of second grade is given by

$$T(t) = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1 \tag{1.1.2}$$

Where $\mu$ is the dynamic viscosity $\alpha_1$, $\alpha_2$ are the normal stress moduli and the kinematical tensors $A_1$ and $A_2$ are defined through [Rivlin et.al. [29]].

$$A_1 = (\text{Grad V}) + (\text{gradV})^T,$$

$$A_2 = \frac{dA_1}{dt} + A_1(\text{Grad V}) + (\text{gradV})^T A_1, \tag{1.1.3}$$

Where, V is the velocity, grad the gradient operator and d/dt the material time derivative. Dunn and Fosdick [6] found that if the fluid modelled by (1.1.2) is to be compatible with thermodynamics, in the sense that all motions of the fluid meet the Clausius-Duhem inequality and the assumption that the specific Helmholtz free energy of the fluid takes its minimum value in equilibrium, then the material moduli must satisfy,

$$\mu \geq 0, \ \alpha_1 \geq 0, \ \alpha_1 + \alpha_2 = 0 \tag{1.1.4}$$

This, then, was shown to give to the theory a rather well behaved and pleasant stability and bounded ness structure. It was also shown that if $\alpha_1$ was taken negative, the remainder of (1.1.4) being preserved, then in quite arbitrary flows instability and unbounded ness were unavoidable. However, it is well known that for most non-Newtonian fluids of current rheological interest, conclusions (1.1.4) are contradicted by experiments.

$$\mu \geq 0, \quad \alpha_1 \leq 0, \qquad \alpha_1 + \alpha_2 \neq 0, \tag{1.1.5}$$

Which were supposedly obtained by data reduction from experiments for those fluids which the experimentalists presumed to be constitutively described by (1.1.2) as a second grade fluid, and they showed that such values for the material moduli led to anomalous behavior, thus questioning whether the fluid under consideration in the experiments could be described as a second grade fluid.

The unsteady hydro magnetic flow in a rotating co- ordinate system is governed by the equation of motion, continuity equation and the Maxwell equations in the form.

$$\rho\left(\frac{\partial V}{\partial t} + (V.\nabla)V + 2\Omega \times V + \Omega \times (\Omega \times r)\right) = \nabla .T + J \times B \tag{1.1.6}$$

$$\nabla .V = 0 \tag{1.1.7}$$

$$\nabla .B = 0 \tag{1.1.8}$$

$$\nabla \times B = \mu_m J \tag{1.1.9}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1.1.10}$$

Where, $J$ is the current density, $B$ is the total magnetic field, $E$ is the total electric field, $\mu_m$ is the magnetic permeability and r is radial co-ordinate given by $r^2 = x^2 + y^2$.

## II. Mathematical Formulation and Solution of the Problem:

We consider the unsteady free convection flow of viscous incompressible second grade fluid bounded by a vertical porous surface in a rotating system in the presence of heat source and chemical reaction under the influence of uniform transverse magnetic field of strength $B_0$. The temperature on the surface varies with the time about a non-zero constant mean while the temperature of free stream is taken to be constant. We consider that the vertical infinite porous plate rotates with the constant angular velocity about an axis is perpendicular to the vertical plane surface. The physical configuration of the problem is as shown in Fig. 1.

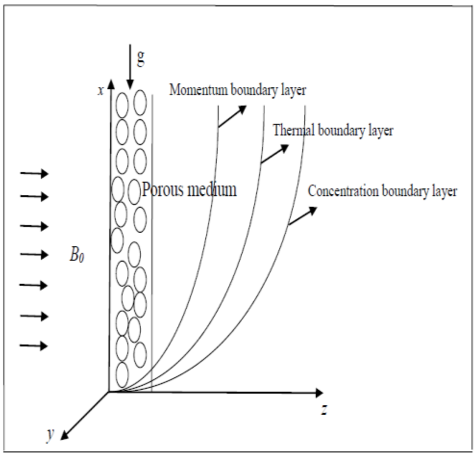


Figure 1: Physical configuration of the problem

We choose a Cartesian co-ordinate system $O(x, y, z)$ such that *x*, *y* axes respectively are in the vertical upward and perpendicular directions on the plane of the vertical porous surface $z = 0$, while z-axis normal to it. The interaction of coriolis force with the free convection sets up a secondary flow in addition to primary flow and hence the flow becomes three dimensional.

With the above frame of reference and assumptions, all the physical variables are functions of *z* and *t* alone. Making use of the governing equation the unsteady hydro magnetic flow in a rotating co- ordinate system is governed by the equation of motion, continuity equation and the Maxwell equation 1.1.6,the constitutive equations and keeping in view of the flow configuration of the problem, the unsteady hydro magnetic flow in a rotating system is governed by the equation of motion for momentum, the conservation of mass, energy and the equation of mass transfer, under usual Boussinesq approximation, are given by

$$\frac{\partial w}{\partial z} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{K_1}u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{2.2}$$

$$\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\sigma B_0^2}{\rho}v - \frac{\nu}{K_1}v \tag{2.3}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\nu}{k}w \tag{2.4}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} + S_1(T - T_\infty) \tag{2.5}$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} - K_C(C - C_\infty) \tag{2.6}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} u &= v = 0, \\ T &= T_w + \varepsilon(T_w - T_\infty)e^{i\omega t} \\ C &= C_w + \varepsilon(C_w - C_\infty)e^{i\omega t} \end{aligned}\right\} \quad \text{at} \quad z = 0 \tag{2.7a}$$

$$\left.\begin{aligned} u &= v = 0. \\ T &= T_\infty \\ C &= C_\infty \end{aligned}\right\} \quad \text{at} \quad z = \infty \tag{2.7b}$$

Where $\varepsilon << 1$ and $\omega$ is the frequency of oscillation. There will be always some fluctuation in the temperature, the plate temperature is assumed to vary harmonically with time. It varies from $T_w \pm \varepsilon(T_w - T_\infty)$ as $t$ varies from $0$ to $\frac{\pi}{2\omega}$. Now there may also occur some variation in suction at the plate due to the variation of the temperature, here we assume that, the frequency of suction and temperature variation are same.

Integrating the equation (2.1), we get

$$w(t) = -w_0(1 + \varepsilon A e^{i\omega t}) \tag{2.8}$$

Where $A$ is the suction parameter, $w_0$ is the constant suction velocity and $\varepsilon$ is the small positive number such that $\varepsilon A \le 1$. The equation (2.4) determines the pressure distribution along the axis of rotation and the absence of $\dfrac{\partial p}{\partial y}$ in the equation (2.3) implies that there is a net cross flow in the $y$ – direction. We choose, $q = u + iv$ and taking into consideration (2.8), the momentum equation (2.2) and (2.3) can be written as

$$\frac{\partial q}{\partial t} - w_0(1+\varepsilon A e^{i\omega t})\frac{\partial q}{\partial z} + 2i\Omega q = \nu\frac{\partial^2 q}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 q}{\partial z^2 \partial t} - \frac{\sigma B_0^2}{\rho} q - \frac{\nu}{K_1} q + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (2.9)$$

Introducing the following non-dimensional quantities:

$$z^* = \frac{w_0 z}{\nu},\ q^* = \frac{q}{w_0},\ T^* = \frac{T - T_\infty}{T_w - T_\infty},\ C^* = \frac{C - C_\infty}{C_w - C_\infty},\ \omega^* = \frac{\nu\omega}{w_0^2},\ t^* = \frac{t w_0^2}{\nu}$$

Making use of non-dimensional quantities (dropping asterisks), the governing equations (2.9), (2.5) and (2.6) can be written as

$$\frac{\partial q}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial q}{\partial z} + 2iRq = \frac{\partial^2 q}{\partial z^2} + \alpha\frac{\partial^3 q}{\partial z^2 \partial t} - \left(M^2 + \frac{1}{K}\right)q + G_r T + G_m C \quad (2.10)$$

$$\frac{\partial T}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial T}{\partial z} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial z^2} + ST \quad (2.11)$$

$$\frac{\partial C}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial C}{\partial z} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} - K_C C \quad (2.12)$$

The corresponding non-dimensional boundary conditions

$$q = 0,\ T = 1 + \varepsilon e^{i\omega t},\ C = 1 + \varepsilon e^{i\omega t} \quad \text{at} \quad z = 0 \quad (2.13)$$

$$q = T = C = 0 \quad \text{at} \quad z = \infty \quad (2.14)$$

In order to reduce the system of partial differential equations (2.10) – (2.12) under their boundary conditions (2.13) and (2.14), to a system of ordinary differential equations in the non-dimensional form, In view of the equation (2.8) and oscillating plate temperature $T$ , The solution form of the equations (2.10), (2.11) and (2.12) are,

$$q(z,t) = q_0(z) + \varepsilon q_1(z) e^{i\omega t} \quad (2.15)$$

$$T(z,t) = T_0(z) + \varepsilon T_1(z) e^{i\omega t} \quad (2.16)$$

$$C(z,t) = C_0(z) + \varepsilon C_1(z) e^{i\omega t} \quad (2.17)$$

These equations (2.15) – (2.17) are valid for small amplitude of oscillation. Substituting from (2.15) to (2.17) into the system of equations (2.10) – (2.12) respectively, and equating the harmonic and non-harmonic terms, we get

$$\frac{d^2 q_0}{dz^2} + \frac{dq_0}{dz} - \left(2iR + M^2 + \frac{1}{K}\right)q_0 = -G_r T_0 - G_m C_0 \quad (2.18)$$

$$(1+\alpha i\omega)\frac{d^2 q_1}{dz^2} + \frac{dq_1}{dz} - \left((2R+\omega)i + M^2 + \frac{1}{K}\right)q_1 = -G_r T_1 - G_m C_1 - A\frac{dq_0}{dz} \quad (2.19)$$

$$\frac{d^2 T_0}{dz^2} + \Pr\frac{dT_0}{dz} + S\Pr T_0 = 0 \quad (2.20)$$

$$\frac{d^2 T_1}{dz^2} + \Pr\frac{dT_1}{dz} - (i\omega - S)\Pr T_1 = -A\Pr\frac{dT_0}{dz} \quad (2.21)$$

$$\frac{d^2 C_0}{dz^2} + Sc\frac{dC_0}{dz} - Sc\, K_C C_0 = 0 \quad (2.22)$$

$$\frac{d^2 C_1}{dz^2} + Sc\frac{dC_1}{dz} - (i\omega + K_C) ScC_1 = -A\, Sc\frac{dC_0}{dz} \quad (2.23)$$

The corresponding boundary conditions

$$\left.\begin{array}{l} q_0 = 0, T_0 = 1, C_0 = 1 \\ q_1 = 0, T_1 = 1, C_1 = 1 \end{array}\right\} \quad \text{at} \quad z = 0 \quad (2.24)$$

$$\left.\begin{array}{l} q_0 = T_0 = C_0 = 0 \\ q_1 = T_1 = C_1 = 0 \end{array}\right\} \quad \text{at} \quad z = \infty \quad (2.25)$$

The solutions of the equations (2.20) and (2.21) using the boundary conditions (2.24) and (2.25), we obtain,

$$T_0(z) = e^{C_5 z},\quad T_1(z) = e^{a_2 z} + \frac{A\Pr C_5}{C_6}(e^{a_2 z} - e^{C_5 z})$$

The equation (2.16) becomes,

$$T(z,t) = e^{C_5 z} + \varepsilon \left( e^{a_2 z} + \frac{A\,\Pr\, C_5}{C_6} (e^{a_2 z} - e^{C_5 z}) \right) e^{i\omega t} \tag{2.26}$$

The solutions of the equations (2.23) and (2.24) using the boundary conditions (2.24) and (2.25), we obtain,

$$C_0(z) = e^{C_2 z}, \quad C_1(z) = e^{a_4 z} + \frac{A\,Sc\,C_2}{C_3}(e^{a_4 z} - e^{C_2 z})$$

The equation (2.16) becomes,

$$C(z,t) = e^{C_2 z} + \varepsilon \left( e^{a_4 z} + \frac{A\,Sc\,C_2}{C_3}(e^{a_4 z} - e^{C_2 z}) \right) e^{i\omega t} \tag{2.27}$$

The solutions of the equations (2.18) and (2.19) using the boundary conditions (2.24) and (2.25), we obtain,

$$q_0(z) = b_1 e^{C_5 z} + b_2 e^{C_2 z} + b_3 e^{a_6 z}$$

$$q_1(z) = C_{17} e^{a_8 z} + C_{12} e^{a_2 z} + C_{13} e^{C_5 z} + C_{14} e^{a_4 z} + C_{15} e^{C_2 z} + C_{16} e^{a_6 z}$$

The equation (2.16) becomes,

$$q(z,t) = b_1 e^{C_5 z} + b_2 e^{C_2 z} + b_3 e^{a_6 z} + \varepsilon \left( C_{17} e^{a_8 z} + C_{12} e^{a_2 z} + C_{13} e^{C_5 z} + C_{14} e^{a_4 z} + C_{15} e^{C_2 z} + C_{16} e^{a_6 z} \right) e^{i\omega t} \tag{2.28}$$

The equation (2.28) reveals that the steady part of the velocity field has three layer character while the oscillatory part of the fluid field exhibits a multilayer character. From equations (2.26) and (2.27), we observe that in case of considerably slow motion of the fluid. i.e., when the viscous dissipation term is neglected, the temperature profiles are mainly affected by Prandtl number (Pr) and Source parameter (*S*): and the concentration profiles are affected by Schimdt number (*Sc*) and chemical reaction parameter ($K_C$) of the fluid respectively. Considering

$q_0 = u_0 + iv_0$ and $q_1 = u_1 + iv_1$

Now it is convenient to write the primary and secondary velocity fields in terms of the fluctuating parts, separating the real and imaginary parts from the equation (2.28) and taking only the real parts as they have physical significance. The velocity distribution of the flow field can be expressed as in fluctuating parts,

$$q(z,t) = q_0(z) + \varepsilon\, q_1(z)\, e^{i\omega t}$$

$$u + iv = (u_0 + iv_0) + \varepsilon\,(u_1 + iv_1)(\cos \omega t + i \sin \omega t)$$

$$= u_0 + iv_0 + \varepsilon\, u_1 \cos \omega t + i\,\varepsilon\, u_1 \sin \omega t + i\,\varepsilon\, v_1 \cos \omega t - \varepsilon\, v_1 \sin \omega t$$

Comparing real and imaginary parts,

$$u(z,t) = w_0 (u_0(z) + \varepsilon\,(u_1 \cos \omega t - v_1 \sin \omega t)) \tag{2.29}$$

$$v(z,t) = w_0 (v_0(z) + \varepsilon\,(u_1 \sin \omega t + v_1 \cos \omega t)) \tag{2.30}$$

Hence the expression for the transient velocity profiles for

$\omega t = \pi / 2$ are given by

$$u\left( z, \frac{\pi}{2\omega} \right) = w_0\,(u_0(z) - \varepsilon\, v_1(z)) \tag{2.31}$$

$$v\left( z, \frac{\pi}{2\omega} \right) = w_0\,(v_0(z) + \varepsilon\, u_1(z)) \tag{2.32}$$

**Skin friction:**

The non-dimensional skin friction at the plate $z = 0$ in term of amplitude and phase angle is given by

$$\tau = \left( \frac{dq}{dz} \right)_{z=0} = \left( \frac{dq_0}{dz} \right)_{z=0} + \varepsilon \left( \frac{dq_1}{dz} \right)_{z=0} e^{i\omega t}$$

$$= C_5 b_1 + C_2 b_2 + C_3 a_6 + \varepsilon\,(a_8 C_{17} + a_2 C_{12} + C_5 C_{13} + a_4 C_{14} + C_2 C_{15} + a_6 C_{16})\, e^{i\omega t} \tag{2.33}$$

The $\tau_{xz}$ and $\tau_{yz}$ components of skin friction at the plate are given by

$$\tau_{xz} = \left( \frac{du_0}{dz} \right)_{z=0} - \varepsilon \left( \frac{dv_1}{dz} \right)_{z=0} \quad \text{and} \quad \tau_{yz} = \left( \frac{dv_0}{dz} \right)_{z=0} + \varepsilon \left( \frac{du_1}{dz} \right)_{z=0}$$

**Rate of heat transfer (Nusselt number):**

The rate of heat transfer co-efficient at the plate $z = 0$ in term of amplitude and phase angle is given by

$$Nu = \left( \frac{dT}{dz} \right)_{z=0} = \left( \frac{dT_0}{dz} \right)_{z=0} + \varepsilon \left( \frac{dT_1}{dz} \right)_{z=0} e^{i\omega t} = C_5 + \varepsilon \left( a_2 + \frac{A\,\Pr\,C_5}{C_6}(a_2 - C_5) \right) e^{i\omega t} \tag{2.34}$$

**Rate of mass transfer (Sherwood number):**

The rate of mass transfer co-efficient at the plate $z = 0$ in term of amplitude and phase angle is given by

$$Sh = \left(\frac{dC}{dz}\right)_{z=0} = \left(\frac{dC_0}{dz}\right)_{z=0} + \varepsilon\left(\frac{dC_1}{dz}\right)_{z=0} e^{i\omega t} = C_2 + \varepsilon\left(a_4 + \frac{A\,ScC_2}{C_3}(a_4 - C_2)\right)e^{i\omega t} \quad (2.35)$$

## III. Results and Discussion:

The unsteady magneto hydro dynamic free convection flow of an incompressible electrically conducting second grade fluid bounded by an infinite vertical porous surface in a rotating system under the of heat source and chemical reaction. The closed form solutions for the velocity $q = u + iv$ , temperature $\theta$ and concentration $C$ are obtained making use of perturbation technique. The velocity expression consists of steady state and oscillatory state. It reveals that, the steady part of the velocity field has three layer characters while the oscillatory part of the fluid field exhibits a multi layer character. The figures (1-3) shows the effects of non-dimensional parameters *M* the Hartmann number, $\alpha$ the second grade fluid parameter, *K* permeability parameter, R rotation parameter, S heat source parameter, Gr Grashof number, Gm mass Grashof number, Kc chemical reaction parameter, Pr the Prandtl number and *t* time; the Figure (4) exhibit the temperature distribution with different variations in the governing parameters S and Pr, and the Figure (5) depicts the concentration profiles with variations in Schmidt number Sc and chemical reaction parameter *Kc*, $\omega$ the frequency of oscillation and time *t*.

It is noticed that, from the Figures 1(a-f) the magnitude of the velocity *u* reduces with increasing the intensity of the magnetic field (Hartmann number M) while it enhances with increasing second grade fluid parameter $(\alpha)$ or permeability of porous medium (*K*) throughout the fluid region. The velocity component *v* enhances with increasing (*M)* or second grade fluid parameter ($\alpha$ ) or permeability of porous medium (*K*). The application of the transverse magnetic field plays the important role of a resistive type force (Lorentz force) similar to drag force (that acts in the opposite direction of the fluid motion) which tends to resist the flow thereby reducing its velocity. The resultant velocity $q$ increasing with second grade fluid parameter $(\alpha)$ , permeability of porous medium (*K*) and reduces with increasing Hartmann number (*M*).

We observe that lower the permeability of porous medium lesser the fluid speed in the entire fluid region. From the Figures 2 (a-h) depicts the velocity component *u* reduces with increasing the rotation parameter (*R*) while it enhances with increasing source parameter (*S*), Grashof number (Gr) and mass Grashof number (Gm). The profiles show the magnitude of the velocity component *v* reverse trend whenever there is increasing rotation parameter (*R*) or source parameter (*S*) or Grashof number (Gr) and mass Grashof number (Gm). The resultant velocity $q$ increases with increasing rotation parameter (*R*) or Grashof number (Gr) and mass Grashof number (Gm) and reduces with increasing source parameter (*S*). Further, it is to observed that from Figures 3 (a-h) the velocity *u* reduces and *v* enhances with increasing Schmidt number (*Sc*), first the velocity *u* increases and then experiences retardation where as *v* reduces in the entire fluid region with increasing chemical reaction parameter (*Kc*). With increasing Prandtl number (Pr) the velocity *u* reduces and *v* enhances in the complete flow field. This implies that an increase in Prandtl number Pr leads to fall the thermal boundary layer flow. This is because fluids with large have low thermal diffusivity which causes low heat penetration resulting in reduced thermal boundary layer. Likewise the velocity *u* enhances and *v* decreases with increasing the frequency of oscillation $(\omega)$ and time (*t*). The resultant velocity reduces with increasing chemical reaction parameter (*Kc*) or Schmidt number (*Sc*) and increases with increasing Prandtl number (Pr) and time (*t*).

The temperature profiles exhibit in the Figures 4(a-d) for different variations in source parameter (*S*), Prandtl number (Pr), the frequency of oscillation $(\omega)$ and time *t*. It is observed that Prandtl number (Pr) leads to decrease the temperature uniformly in all layers being the heat source parameter fixed. It is found that the temperature decreases in all layers with increase in the heat source parameter (*S*). It is concluded that the heat source parameter (*S*) and Prandtl number (Pr) reduces the temperature in all layers. The temperature increases with increasing the frequency of oscillation $(\omega)$ and time *t*.

The concentration profiles are shown in the Figures 5(a-d) for different variations in Schmidt number (*Sc*), the chemical reaction parameter (*Kc*), the frequency of oscillation $(\omega)$ and time (*t*). It is noticed that the concentration decreases at all layers of the flow for heavier species such as $CO_2$, $H_2O$ and $NH_3$ having Schmidt number 0.3, 0.6 and 0.78 respectively. It is observed that for heavier diffusing foreign species, i.e. the velocity reduces with increasing Schmidt number (Sc) in both magnitude and extent and thinning of thermal boundary layer occurs. Likewise, the concentration profiles decrease with increase in chemical reaction parameter (*Kc*). It is concluded that the Schmidt number and the chemical reaction parameter reduces the concentration in all layers. The concentration increases with increasing the frequency of oscillation $(\omega)$ and time *t*.

It is noted from the table 1 that the magnitudes of both the skin friction components $\tau_{xz}$ and $\tau_{yz}$ increase with increase in permeability parameter (*K*), thermal Grashof number (Gr) and mass Grashof number (Gm), and where as it reduces with increase in Hartmann number (*M*), second grade fluid parameter $(\alpha)$, heat source parameter (*S*), Schmidt number (*Sc*), chemical reaction parameter (*Kc*) anf Prandtl number (Pr). Likewise the rotation parameter (*R*) enhances skin friction component $\tau_{xz}$ and reduces skin friction component $\tau_{yz}$. From the table 2 that The magnitude of the Nusselt number (Nu) increases for the parameters heat source parameter (*S*) and Prandtl number (Pr) or time (*t*), and it reduces with the frequency of oscillation $(\omega)$. Also from the table 4, the similar behaviour is observed. The magnitude of the Sherwood number (*Sh*) increases for increasing the parameters Schmidt number (*Sc*) and chemical reaction parameter (*Kc*) or time (*t*) and reduces with increasing the frequency of oscillation $(\omega)$

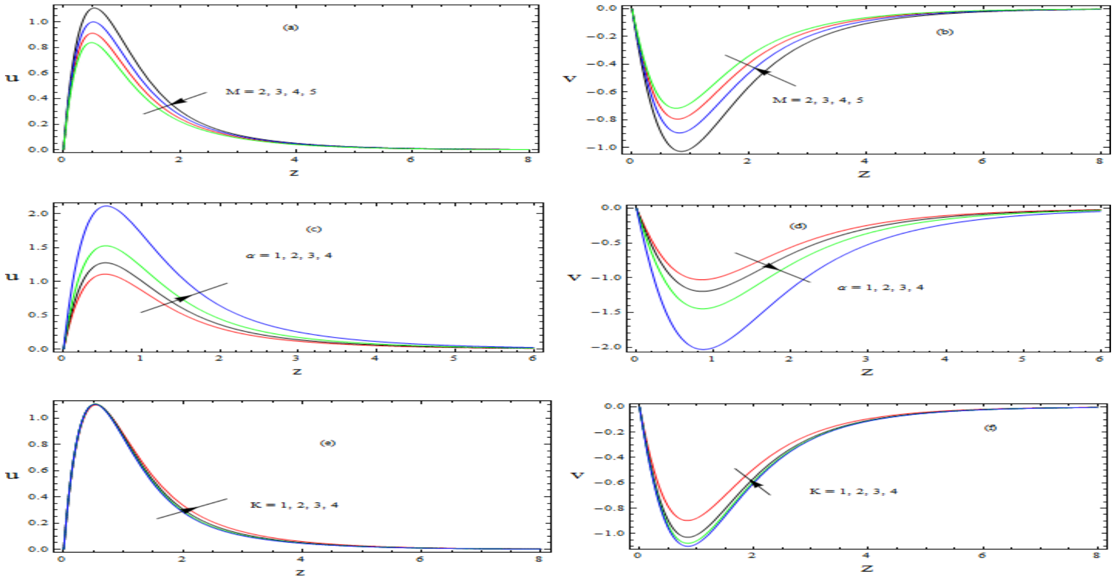


Fig. 1. The velocity profiles for the components *u* and *v* with $A = 0.05$ ; $\omega = 5\pi/2$ ; $\varepsilon = 0.001$ , $t = 0.2$

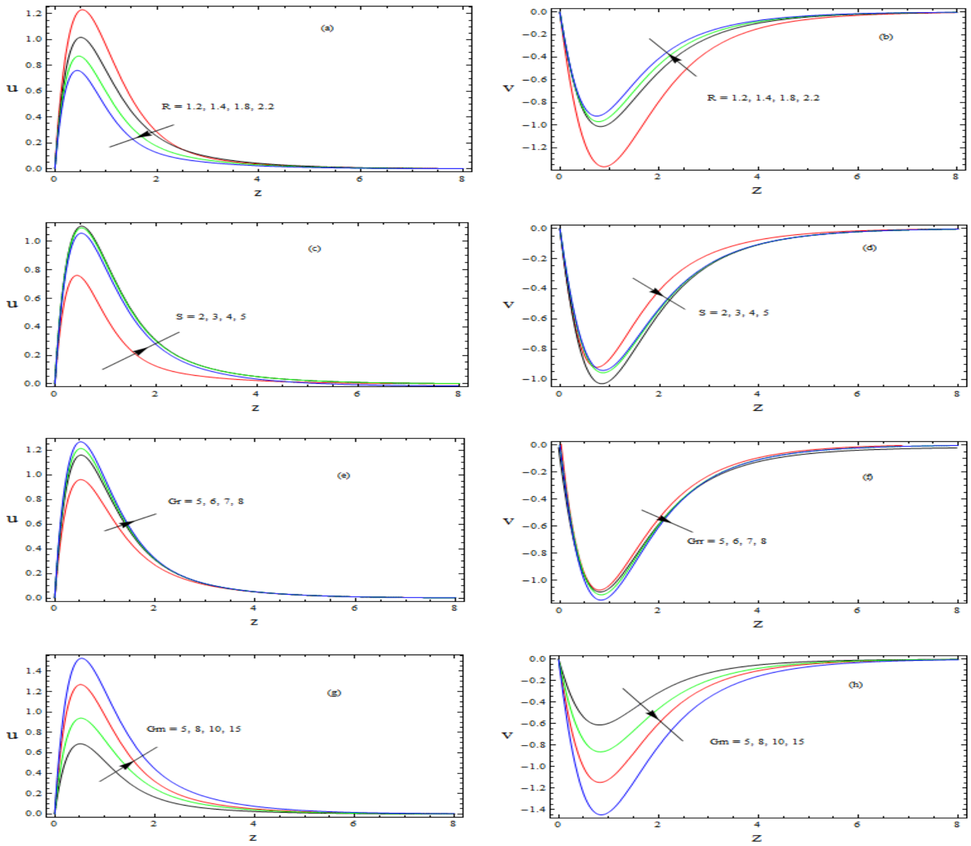


Fig. 2. The velocity profiles for the components $u$ and $v$

with $A = 0.05$ ; $\omega = 5\pi / 2$ ; $\varepsilon = 0.001$ , $t = 0.2$

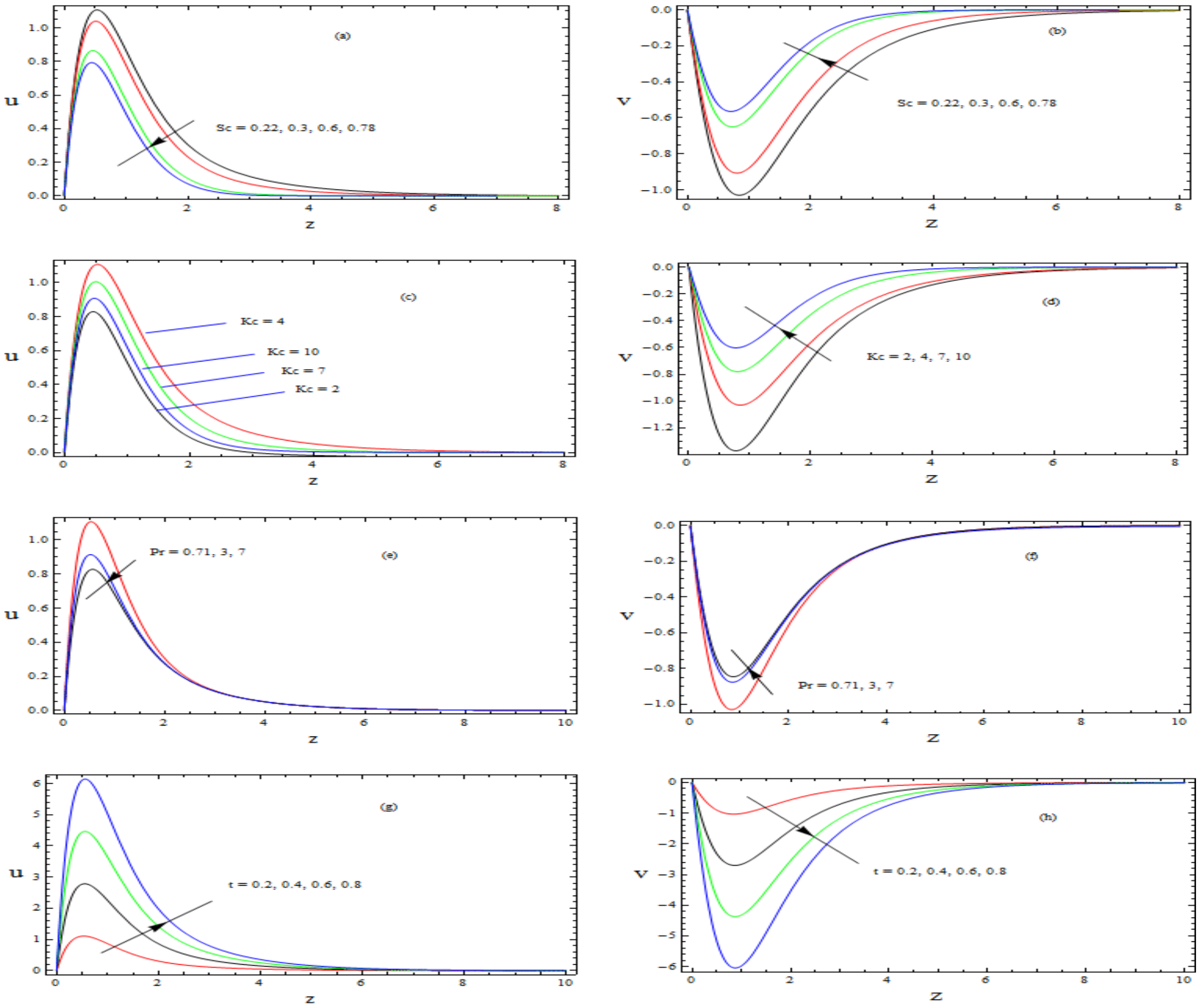


Fig. 3. The velocity profiles for the components $u$ and $v$ with $A = 0.05$ ; $\omega = 5\pi/2$ ; $\varepsilon = 0.001$ , $t = 0.2$

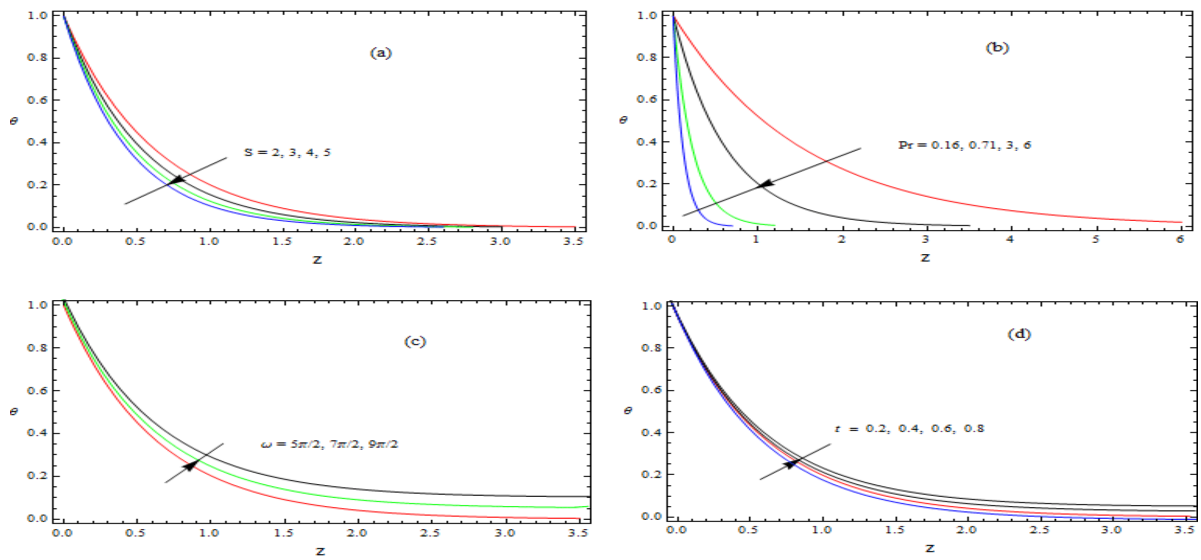


Fig. 4. The temperature profiles for with $A = 0.05$ ; $\omega = 5\pi/2$ ; $\varepsilon = 0.001$ , $t = 0.2$

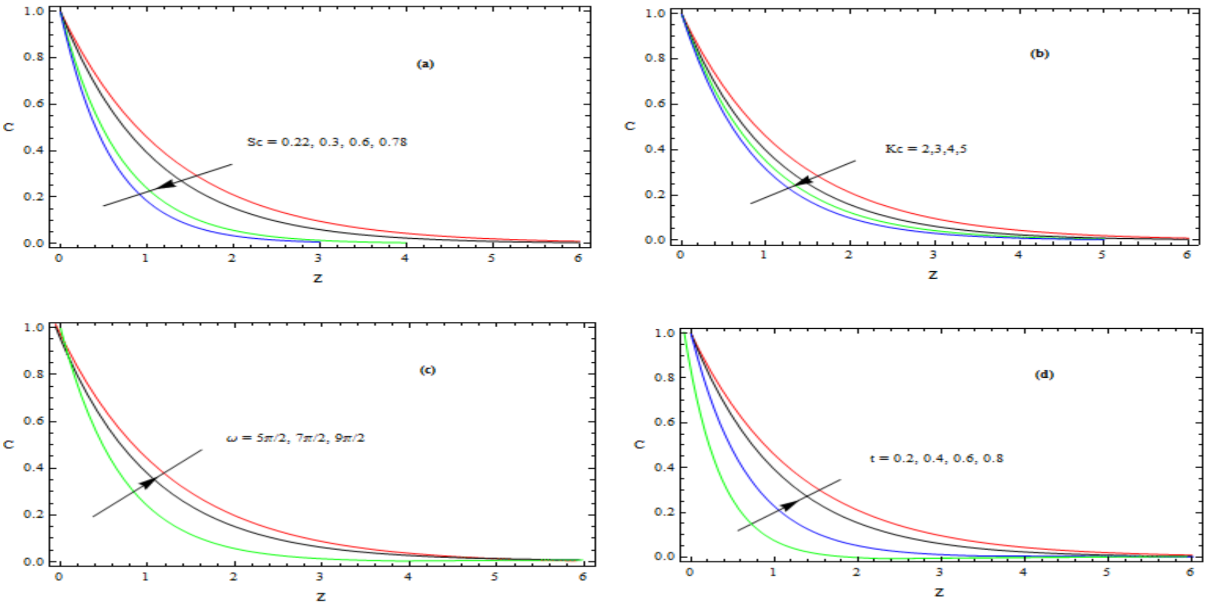


Fig. 5. The Concentration profiles for with $A = 0.05$ ; $\omega = 5\pi / 2$ ; $\varepsilon = 0.001$ , $t = 0.2$

Table. 1. Skin Friction

| M | $\alpha$ | K | R | S | Gr | Gm | Sc | Kc | Pr | $\tau_{xz}$ | $\tau_{yz}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | **1** | **2** | **1.2** | **2** | **5** | **10** | **0.22** | **2** | **0.71** | 6.01449 | -1.79578 |
| **3** | 1 | 2 | 1.2 | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 5.61228 | -1.33598 |
| **4** | 1 | 2 | 1.2 | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 5.24833 | -1.03404 |
| 2 | **2** | 2 | 1.2 | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 6.01413 | -1.79617 |
| 2 | **3** | 2 | 1.2 | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 6.01393 | -1.79633 |
| 2 | 1 | **3** | 1.2 | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 6.08210 | -1.89415 |
| 2 | 1 | **4** | 1.2 | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 6.11556 | -1.94614 |
| 2 | 1 | 2 | **1.4** | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 5.78945 | -1.93967 |
| 2 | 1 | 2 | **1.8** | 2 | 5 | 10 | 0.22 | 2 | 0.71 | 5.36688 | -2.13303 |
| 2 | 1 | 2 | 1.2 | **3** | 5 | 10 | 0.22 | 2 | 0.71 | 5.89153 | -1.73822 |
| 2 | 1 | 2 | 1.2 | **4** | 5 | 10 | 0.22 | 2 | 0.71 | 5.79982 | -1.69880 |
| 2 | 1 | 2 | 1.2 | 2 | **6** | 10 | 0.22 | 2 | 0.71 | 6.34324 | -1.87197 |
| 2 | 1 | 2 | 1.2 | 2 | **7** | 10 | 0.22 | 2 | 0.71 | 6.67199 | -1.94816 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | **5** | 0.22 | 2 | 0.71 | 3.82912 | -1.08836 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | **8** | 0.22 | 2 | 0.71 | 5.14034 | -1.51281 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | 10 | **0.3** | 2 | 0.71 | 5.76037 | -1.61967 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | 10 | **0.6** | 2 | 0.71 | 5.10555 | -1.23180 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | 10 | 0.22 | **4** | 0.71 | 5.58892 | -1.50959 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | 10 | 0.22 | **7** | 0.71 | 5.19728 | -1.28158 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | 10 | 0.22 | 2 | **3** | 5.24164 | -1.51814 |
| 2 | 1 | 2 | 1.2 | 2 | 5 | 10 | 0.22 | 2 | 7 | 4.87287 | -1.44886 |

Table. 2. Nusselt Number

| S | Pr | $\omega$ | $t$ | Nu |
|---|---|---|---|---|
| 2 | **0.71** | $5\pi/2$ | **0.2** | -1.59653 |
| **3** | 0.71 | $5\pi/2$ | 0.2 | -1.85503 |
| **4** | 0.71 | $5\pi/2$ | 0.2 | -2.07512 |
| 2 | **3** | $5\pi/2$ | 0.2 | -4.36861 |
| 2 | **7** | $5\pi/2$ | 0.2 | -8.61827 |
| 2 | 0.71 | $7\pi/2$ | 0.2 | -1.59538 |
| 2 | 0.71 | $9\pi/2$ | 0.2 | -1.59431 |
| 2 | 0.71 | $5\pi/2$ | **0.4** | -1.59854 |
| 2 | 0.71 | $5\pi/2$ | **0.6** | -1.60026 |

Table. 3. Sherwood Number

| Sc | Kc | $\omega$ | $t$ | Sh |
|---|---|---|---|---|
| 2 | **0.22** | $5\pi/2$ | **0.2** | -0.781334 |
| **3** | 0.22 | $5\pi/2$ | 0.2 | -0.928700 |
| **4** | 0.22 | $5\pi/2$ | 0.2 | -1.053333 |
| 2 | **0.3** | $5\pi/2$ | 0.2 | -0.937762 |
| 2 | **0.6** | $5\pi/2$ | 0.2 | -1.434060 |
| 2 | 0.22 | $7\pi/2$ | 0.2 | -0.780754 |
| 2 | 0.22 | $9\pi/2$ | 0.2 | -0.778487 |
| 2 | 0.22 | $5\pi/2$ | **0.4** | -0.782446 |
| 2 | 0.22 | $5\pi/2$ | **0.6** | -0.783434 |

## III. Conclusions

The resultant velocity increasing with second grade fluid parameter $(\alpha)$, permeability of porous medium (*K*) and reduces with increasing Hartmann number (*M*). Lower the permeability of porous medium lesser the fluid speed in the entire fluid region. The resultant velocity increases with increasing rotation parameter (*R*) or Grashof number (Gr) and mass Grashof number (Gm) and reduces with increasing source parameter (*S*). The resultant velocity reduces with increasing chemical reaction parameter (*Kc*) or Schmidt number (*Sc*) and increases with increasing Prandtl number (Pr) and time (*t*). The heat source parameter (*S*) and Prandtl number (Pr) reduces the temperature in all layers. The temperature increases with increasing the frequency of oscillation ($\omega$) and time. The Schmidt number and the chemical reaction parameter reduce the concentration in all layers. The concentration increases with increasing the frequency of oscillation ($\omega$) and time *t*. The skin friction components $\tau_{xz}$ and $\tau_{yz}$ increase with increase in permeability parameter (*K*), thermal Grashof number (Gr) and mass Grashof number (Gm), and where as it reduces with increase in Hartmann number (*M*), second grade fluid parameter ($\alpha$), heat source parameter (*S*), Schmidt number (*Sc*), chemical reaction parameter (*Kc*) anf Prandtl number (Pr). The rotation parameter (*R*) enhances skin friction component $\tau_{xz}$ and reduces skin friction component $\tau_{yz}$. The heat transfer coefficient increases with increasing heat source parameter (*S*) and Prandtl number (Pr) or time span (*t*), and it reduces with the frequency of oscillation ($\omega$). The Sherwood number enhances for increasing the parameters Schmidt number (*Sc*) and chemical reaction parameter (*Kc*) or time span (*t*) and reduces with increasing the frequency of oscillation ($\omega$).

## References:

[1]. Anand Rao. G., "Hydro magnetic flow and heat transfer in a saturated porous medium between two parallel porous wall in a rotating system," Proceedings of the Eighth National heat and mass transfer conference, Assam, India, 1985.

[2]. Ariel. P.D., "On exact solution of flow problems of a second grade fluid between two parallel porous wall," Int J Engg Sci, Vol. 40 (2002), pp. 913-941.

[3]. Bég.O.A., Takhar.H.S., Singh. AK. "Multi-parameter perturbation analysis of unsteady, oscillatory magnetoconvection in porous media with heat source effects," Int J Fluid Mech Res, Vol. 32(5)(2005), pp. 635-661.

[4]. Dunn. J.E, Fosdick. R.L., "Thermodynamics, stability and boundedness of fluids of complexity 2 and fluids of second grade," Arch Rat Mech Anal, Vol. 56 (1974), p. 191.

[5]. El-Kabeir. S.M.M., Rashad. A.M, Gorla. R.S.R., "Unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux," Math Comput Modell., Vol. 46(3,4) (2007), pp. 384-397.

[6]. Franchi. F, Straughan. B., "Convention stability and uniqueness for a fluid of third grade," Int J Non-Linear Mech, Vol. 34 (1999), Vol. 895-899.

[7]. Hayat. T, Asghar. S, Siddiqui. A.M., "On the moment of a plane disk in a non-Newtonian fluid," Acta Mech, Vol. 136 (1999), pp. 125-131.

[8]. Krishna. D.V, Prasada Rao. D.R.V, Ramachandra Murthy. A.S., "Hydro magnetic convection flow through a porous medium in a rotating channel," Inzhenerno-Fizicheskii Zhurnal, Vol. 75(2)(2002), pp. 12-21.

[9]. Rajagopal. K.R., "Longitudinal and torsional oscillations of a rod in a non-Newtonian fluid" Acta Mech, Vol. 49 (1983), pp. 281-285.

[10]. Ram. P.C., "Unsteady MHD free-convective flow through a porous medium with Hall currents," Astrophysics Space Science, Vol. 149(1)( 1988), pp. 171-174.

[11]. Ram. P.C., "Hall effects on the hydro magnetic free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux," Int. J. Energy Res. Vol. 12(2)( 1988), pp. 227-231.

[12]. Veera Krishna.M and S.G. Malashetty, "Unsteady flow of an incompressible electrically conducting second grade fluid through a composite medium in a rotating parallel plate channel bounded by a porous bed," International Journal of Applied Mathematics and Mechanics, Vol. 8(5) (2012), pp. 61-84.

[13]. Veera Krishna.M and S.G. Malashetty, "Hall effects on unsteady flow of an incompressible electrically conducting second grade fluid through a composite medium in a rotating parallel plate channel bounded by a porous bed," International Journal of Advances in Science and Technology, Vol. 2(2) (2011), pp. 87-102.

[14]. Zakaria. M., "Magneto hydro dynamic unsteady free convection flow of a couple stress fluid with one relaxation time through a porous medium," Appl Math Comput., Vol. 146(2-3)(2003), pp. 469-494.