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Optimal Designs For Random- Effects Model With Five Variance Components

Nuga O.A

Department of Physical Sciences, Bells University of Technology, Nigeria

ABSTRACT

Based on a completely random effect model, optimal designs were constructed for the estimation of five variance components in a model that has both crossed factors and nested factors usually called nested factorial. We considered a scenario where the same balanced two stage hierarchical nested design is nested within the treatment combinations of a two way crossed classification. Groups of design with the same total sample size were generated and for a particular configuration of the variance components, generated designs were compared for A-optimality and D-optimality of the information matrix of the maximum likelihood estimators.

KEYWORDS: *Variance Components, nested factor and crossed factors, information matrix, random effects, maximum likelihood estimator,*

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I. INTRODUCTION

We are interested in designing experiments for the following sampling scenario. Consider two crossed factors A and B, were a levels are sampled from a large population of levels of A and b levels are sampled from a large population of levels of B, if within each ab cells there is another factor C with c levels sampled from a large population of levels of C and m observations are made at each level of factor C giving a total of abcm observations. For a practical example, an experiment was conducted to determine the variation in the porosity of flour across batches of production. Flour is made from different varieties of cassava and wheat and the experimenter is also interested in variability across varieties of wheat and cassava. Knowing which variation source is largest could help to focus quality improvements effort. The resultant model for this experimental scenario will have two random crossed factor effect with and interaction effect and a nested factor effect within each treatment combination of the crossed factors, together with the random error term making five variance component

Aviles and Pinheiro (2001) provided experimental design scenario for the estimation of fixed effects associated with crossed factors and two variance components associated with a nested factor and the random error term. The assumption of fixed effect for the crossed factor has been very common with model of such from the few published work. Smith and Beverly (1981) introduced the idea of nested factorial, an experimental design were some factors appear in factorial relationship others in nested relationship. Split factorials design was introduced by Ankenman, Liu, Karr, and Picka (2001). Split factorial is an experimental design which split a factorial design into sub experiments, a different nested design is used for each sub experiment but within a sub-experiment all design points/treatment combination have the same nested design are some of the few published work with an assumption of fixed effect for the crossed factors. In general a linear mixed effect model is used for this entire experimental scenario

Random effect model also known has ANOVA model II has been used to describe several experimental situations in literature. Experiments where the primary interest lies in making inferences about the variance of the random effect has been demonstrated with the one way model, two way crossed model, nested model depending on the setting of the experiment. However whenever the setting of an experiment involves both crossed and nested factor it is assumed that the effect due to the crossed factor are fixed and the effect due to the nested factor are random.

We will assume a linear model like that of the assembled design of Aviles and Pinhero (2001) but with a random effect for both the crossed factor and the nested factor resulting in a completely random model.

II. LITERATURE REVIEW

Most of the published work in designs for variance components estimation dates back to the 60's and 70's and have been restricted to specific models namely, one-way random model, the two-way crossed classification random model and the two way nested model. R.L Anderson and many of his co-workers are the main contributors to the design area during that period (Anderson 1975, 1981). . For the one way model Hammersly (1949), Crump (1954), Anderson& Crump (1967) were some of the earliest authors. Hammersly (1949) showed that for a fixed N, the variance $Var(\sigma_\alpha^2)$ is minimized by allocating an equal number n, of observation to each class where $n = \dfrac{N\rho + N + 1}{N\rho + 2}$, since this formula may not yield an integer value, it was suggested that the closest integer value for n be chosen. Crump (1954) and Anderson & Crump (1967) showed that for fixed K and N , $Var(\sigma_\alpha^2)$ is minimized when $n_i = n = N/a$ for all i. The optimal value for a in this case is given as $a_1 = \dfrac{N(N\rho+2)}{N(\rho+1)+1} = \dfrac{N}{n}$

Other authors are Kussmaul & Anderson (1967), Thompson and Anderson (1975), Herrendofer (1979), Murkerjue &Huda (1988), Giovagnoli & Sebastiani (1989), Norell (2006). Norell (2006) studied design effect for the one way random model using the information matrix of the maximum likelihood estimators.

The construction of optimal design for the two way crossed models seems to have been considered first by Gaylor (1960). He considered the problem of optimal designs to estimate variance components using the fitting constant method of estimation of variance comxponents for the unbalanced data. Bush (1962) and Bush and Anderson (1963), HIrotsu(1966), Mostafa (1967) are some of the other contributors to the designing experiment using the two way random model.

Some pioneering articles that address the problem of estimating variance components in a nested classification are Bainbridge (1965) Prairie (1962), Prairie and Anderson (1962), Bainbridge (1965), they proposed designs that systematically spread the information in the experiment more equally among the variance components. Goldsmith and Gaylor (1970) carried out extensive investigation on optimal designs for estimating variance components in a completely random nested classification. Delgado (1999) defined a class of unbalanced design for estimating variance component in the three stage nested classification using the ANOVA method of estimation.

For the crossed nested model no assumption of a complete random model has been made, such work that design experiment for variance component estimation are based on the linear mixed effect model .Beverly (1981) , Ankenman, Liu, Karr, and Picka (2001) and Aviles and Pinheiro (2001) are authors that have published work.

III. DESCRIPTION /CHARACTERIZATION OF THE DESIGN

The class of crossed factor designs with an HND placed at each treatment combination is large and contains many designs that are too complex for practical use. This research work will focus on a two way crossed factor design with the same balanced hierarchical nested design (HND) placed at each treatment combination. The work also assumes that the crossed factors and the nested factors are random resulting into a completely random model.

The design is described as follows $a =$ number of levels sampled from a large population of levels of A, $b =$ number of levels sampled from a large population of levels of B, c = number of levels of the nested factor C within each treatment combination (ab), T = the number of treatment combination (a×b), m = number of observation in each level of C, n =the number of observations in each treatment combination (m×c), N= abcm Figure 1 is an example of a design with two levels of factor A, two levels of factor B, two levels of nested factor C and two observations at each level of C. the resultant design has four treatment combinations and a total of four observation at each treatment combination.

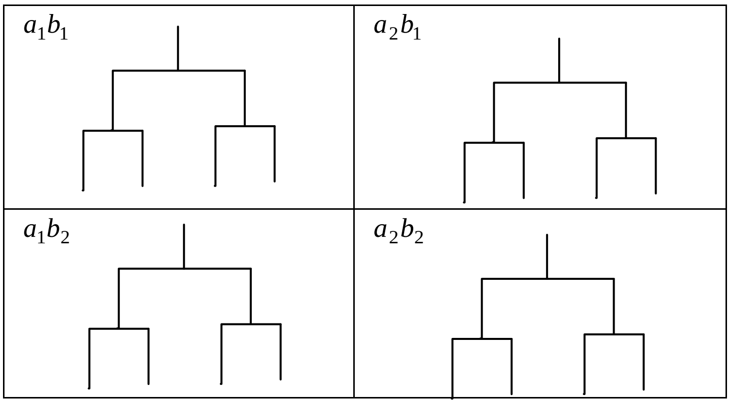

Figure 1: (a =2, b=2, c=2, m=2)

3.1 Analysis of Design (Model and Variance Structure)

The linear random effect model with interaction of the crossed factor used to represent the response in the design is written in vector form:

$$y = \mu 1 + Z_1 w_1 + Z_2 w_2 + Z_3 w_3 + Z_4 w_4 + Z_0 w_0 \tag{1}$$

is a vector of abcm observations, μ is the overall mean, Z_i is an indicator matrix associated with the ith variance component, w_i is a vector of normally distributed random effects associated with the ith variance component such that $w_i \approx N(0, \sigma_i^{\,2} I)$. The T×n variance covariance matrix of the observations is

$$V = Var(y) = \sigma_1^2 Z_1 Z_1^{'} + \sigma_2^2 Z_2 Z_2^{'} + \sigma_3^2 Z_3 Z_3^{'} + \sigma_4^2 Z_4 Z_4^{'} + \sigma_0^2 Z_0 Z_0^{'} \tag{2}$$

Z_1 and Z_2 are indicator matrices associated with the variance component of factor A and factor B respectively. Z_1 and Z_2 have as many rows as the total number of observations (abcm) and as many column as the number of levels of factor A and B respectively. Z_3 is an indicator matrix associated with the variance components of the interaction. Z_3 has as many rows as the number of observation(abcm) and as many column as the number of treatment combination T=ab. Z_4 is an indicator matrix associated with the variance components of the nested factor C.

$$Z_4 = \bigoplus_{t=1}^{T} Z_{4t} \tag{3}$$

Z_{4t} has cm rows and as many columns as the number of levels of factor C used in treatment combination t. Since the same structure of nested design is used in each treatment combination, Z_4 has as many rows as the total number of observation (abcm) and as many columns as the total number of levels of factor C used in each treatment combination multiply by the total number of treatment combinations used (Č =cT). z_0 is an identity matrix of order abcm.

We define the Z's has the following kronecker product

$$Z_0 = \mathrm{I}_a \otimes \mathrm{I}_b \otimes \mathrm{I}_c \otimes \mathrm{I}_m \qquad Z_0 Z_0^{'} = \mathrm{I}_a \otimes \mathrm{I}_b \otimes \mathrm{I}_c \otimes \mathrm{I}_m$$

$$Z_1 = \mathrm{I}_a \otimes 1_b \otimes 1_c \otimes 1_m \qquad Z_1 Z_1^{'} = \mathrm{I}_a \otimes J_b \otimes J_c \otimes J_m$$

$Z_2 = 1_a \otimes I_b \otimes 1_c \otimes 1_m \qquad Z_2 Z_2' = J_a \otimes I_b \otimes J_c \otimes J_m$

$Z_3 = I_a \otimes I_b \otimes 1_c \otimes 1_m \qquad Z_3 Z_3' = I_a \otimes I_b \otimes J_c \otimes J_m$

$Z_4 = I_a \otimes I_b \otimes I_c \otimes 1_m \qquad Z_4 Z_4' = I_a \otimes I_b \otimes I_c \otimes J_m$ (4)

3.2 Method of Estimation

Several estimation methods have been developed for the estimation of variance components in random and mixed effect model for both the balanced and the unbalanced data experiments. This work will use the maximum likelihood method to estimate the parameters of the linear random effects model. A maximum likelihood (ML) estimator is a point on the parameter space where the likelihood function assumes absolute maximum. They are generally derived as the solutions of the likelihood equations obtained by equating to zero the partial derivatives of the log likelihood function with respect to the parameters of the distribution. Unfortunately for linear random effect model, explicit solutions of the ML estimators cannot be obtained. The exact sampling variances cannot also be obtained. We can however obtained an asymptotic large sample variances and co-variances of the ML estimates of the parameters of our model as the inverse of the Fisher information determined by the negative matrix of the second order partial derivative of the log likelihood. The information matrix is given as

$$In\begin{bmatrix}\beta \\ \sigma^2\end{bmatrix} = \begin{bmatrix} X'V^{-1}X & 0 \\ 0 & \frac{1}{2}tr(V^{-1}Z_iZ_i'V^{-1}Z_jZ_j') \end{bmatrix} i, j = 0.......4 \qquad \text{(M1)}$$

Where β represents the fixed effects occurring in the model, and since the only fixed effects in the model is the general mean (μ), the information matrix concerns only the random effects in the model.

$$In[\sigma^2] = \frac{1}{2}tr(V^{-1}Z_iZ_i'V^{-1}Z_jZ_j') \;\; i, j = 0.......4$$

The inverse of V is obtained using the results of Henderson and Searle (1979) :

$$V^{-1} = \theta_0^{-1}(I_a \otimes I_b \otimes I_c \otimes C_m) + \theta_1^{-1}(I_a \otimes I_b \otimes C_c \otimes \bar{J}_m) + \theta_2^{-1}(C_a \otimes C_b \otimes \bar{J}_c \otimes \bar{J}_m) + \theta_3^{-1}(\bar{J}_a \otimes C_b \otimes \bar{J}_c \otimes \bar{J}_m)$$
$$+ \theta_4^{-1}(C_a \otimes \bar{J}_b \otimes \bar{J}_c \otimes \bar{J}_m) + \theta_5^{-1}(\bar{J}_a \otimes \bar{J}_b \otimes \bar{J}_c \otimes \bar{J}_m)$$

Where:

$\theta_0 = \sigma_e^2$, $\theta_1 = (\sigma_e^2 + m\sigma_\gamma^2)$, $\theta_2 = (\sigma_e^2 + m\sigma_\gamma^2 + cm\sigma_{\alpha\beta}^2)$, $\theta_3 = (\sigma_e^2 + m\sigma_\gamma^2 + cm\sigma_{\alpha\beta}^2 + acm\sigma_\beta^2)$,

$\theta_4 = (\sigma_e^2 + m\sigma_\gamma^2 + cm\sigma_{\alpha\beta}^2 + bcm\sigma_\alpha^2)$, $\theta_5 = (\sigma_e^2 + m\sigma_\gamma^2 + cm\sigma_{\alpha\beta}^2 + bcm\sigma_\alpha^2 + acm\sigma_\beta^2)$

A useful result for computing the information matrix is given by

$$tr(V^{-1}Z_iZ_i'V^{-1}Z_jZ_j') = sesq(Z_i'V^{-1}Z_j) i, j = 0....4.$$

Where **sesq** represents the sum of squares of all elements in the matrix (see Searle et al Pg. 247)

The information matrix then is

$$\begin{bmatrix} \sigma_e^2 \\ \sigma_\gamma^2 \\ \sigma_{\alpha\beta}^2 \\ \sigma_\beta^2 \\ \sigma_\alpha \end{bmatrix} = \begin{bmatrix} t_{ee} & t_{\gamma\gamma}/m & t_{(\alpha\beta)^2}/cm & t_{\beta\beta}/acm & t_{\alpha\alpha}/bcm \\ & t_{\gamma\gamma} & t_{(\alpha\beta)^2}/c & t_{\beta\beta}/ac & t_{\alpha\alpha}/bc \\ & & t_{(\alpha\beta)^2} & t_{\beta\beta}/a & t_{\alpha\alpha}/b \\ & symmetric & & t_{\beta\beta} & ab(cm)^2/\theta_5 \\ & & & & t_{\alpha\alpha} \end{bmatrix} \quad \text{(M2)}$$

For

$$t_{ee} = \frac{v_e}{\theta_0^2} + \frac{t_{\gamma\gamma}}{m^2} \qquad ,t_{\alpha\alpha} = b^2(cm)^2\left(\frac{v_\alpha}{\theta_4^2} + \frac{1}{\theta_5^2}\right) \qquad ,t_{\beta\beta} = a^2(cm)^2\left(\frac{v_\beta}{\theta_3^2} + \frac{1}{\theta_5^2}\right)$$

$$,t_{(\alpha\beta)^2} = (cm)^2\left(\frac{v_{\alpha\beta}}{\theta_2^2} + \frac{v_\beta}{\theta_3^2} + \frac{v_\alpha}{\theta_4^2} + \frac{1}{\theta_5^2}\right) \qquad ,t_{\gamma\gamma} = m^2\left(\frac{v_\gamma}{\theta_1^2} + \frac{v_{\alpha\beta}}{\theta_2^2} + \frac{v_\beta}{\theta_3^2} + \frac{v_\alpha}{\theta_4^2} + \frac{1}{\theta_5^2}\right)$$

Where

$$v_e = abc(m-1), v_\gamma = ab(c-1), v_{\alpha\beta} = (a-1)(b-1), v_\beta = b-1, v_\alpha = a-1$$

IV. DESIGN ENUMERATION AND GENERATION

Groups of Designs with a fixed total sample size will be generated and enumerated for comparisons. The total sample size (N) was systematically chosen in such a way that the structure of the design generated is balanced. By balanced design, we mean that the same balanced two stage hierarchical nested design is placed at each treatment combination of the crossed factors. i.e. c and m are equal for all treatment combinations T. these group of designs by our definition constitute the design space. In other to obtain these balanced designs for a fixed total sample size N, the following restrictions are placed.

(1) c≥2.i.e. designs where c=1 are not sufficient (the degree of freedom of c =0)

(2) m≥2. To be able to obtain separate estimates of ${\sigma_\gamma}^2$ and ${\sigma_e}^2$

The two conditions above invariably mean that cm≥4 and infact for a balance design, the least sample size that can be obtained is 16. Figure…1…, is the simplest balanced design. We give a set of rules to obtain groups of design for a fixed total sample size N.

(1) Choose N such that $N = T \times n$, for all non-prime numbers of T (number of treatment combinations) and n (number of observations in each treatment combination)
(2) For each distinct $N = T \times n$ from (1) split $T = a \times b$ $, n = c \times m$, such that $a, b \geq 2$ and $c, m \geq 2$
(3) From (2), $N = a \times b \times c \times m$ (all sets of four possible factors that generated the sample size)
(4) Obtain all possible permutations of distinct a, b, c, m from (3) resulting into the total number of groups of design for a fixed total sample size.

For the balanced design, based on the above restrictions the smallest sample feasible is 16. As an illustration

- For $N = 24$, 4 and 6 are factors of 24, also 3 and 8 are also of 24, but it is only the pair 4 and 6 that satisfy the condition for non-prime numbers and so satisfy rule (1) above. 3 and 8 are also factors of 24, but 3 is a prime number, therefore the pair 3 and 8 are not included as possible values of T and N
- $24 = 4 \times 6 = 2 \times 2, \times 2 \times 3,$
- There are four possible permutations for a, b, c, m generated above

- $a = 2, b = 2, c = 2, m = 3$
- $a = 2, b = 2, c = 3, m = 2$
- $a = 2, b = 3, c = 2, m = 2$
- $a = 3, b = 2, c = 2, m = 2$

Resulting into four group of designs for $N = 24$

- For $N = 64,$

8×8, 4×16 are the only set of factors that satisfy rule 1

- $N = 8\times 8 = 2\times 4\times 2\times 4$ and $N = 4\times 16 = 2\times 2\times 4\times 4$ $N = 4\times 16 = 2\times 2\times 2\times 8$
- (a,b,c,m) generated from 8×8 and 4×16 have two sets of distinct values.
- $N = 64 = 2\times 2\times 4\times 4$ $N = 64 = 2\times 2\times 2\times 8$
- There are six possible permutations of $(2,2,4,4)$ and four possible permutations of $(2,2,2,8)$ generated above
  - $a = 2, b = 2, c = 4, m = 4$
  - $a = 2, b = 4, c = 2, m = 4$
  - $a = 2, b = 4, c = 4, m = 2$
  - $a = 4, b = 2, c = 2, m = 4$
  - $a = 4, b = 2, c = 4, m = 2$
  - $a = 4, b = 4, c = 2, m = 2$
  - $a = 2, b = 2, c = 2, m = 8$
  - $a = 2, b = 2, c = 8, m = 2$
  - $a = 2, b = 8, c = 2, m = 2$
  - $a = 8, b = 2, c = 2, m = 2$

Resulting into ten groups of designs $N = 64,$ the above steps of condition can be used to obtain possible sample sizes and generate corresponding groups of designs. Corresponding groups of design for possible sample sizes between 0 and 100(0≤N≤100) are found on table at the end of the manuscript.

V. OPTIMALITY

Designs enumerated in the last section will be compared in terms of their ability to accurately estimate the five variance components (based on the sample sizes). Since no closed form analytical expression is available for the variance covariance matrix in this linear random effect model, we examine optimality using the asymptotic variance covariance matrix. A design from a group of designs with equal sample size is said to be optimal if it minimizes an optimality criterion related to the variance- covariance matrix of the parameter estimates. Equivalently we seek the design that maximizes an optimality criterion related to the information matrix of the five variance components. Optimal design in a linear random effects model depends on the relative size of the true values of the variance components, and we will not be able investigate optimality unless an assumption is made on the true values of the variance components. Since optimality for such models is similar to that of nonlinear models, we borrow an idea from optimization on theory of nonlinear models and use the local optimality. A MATHLAB code was written in the context of the information matrix of section 3.2; in such a way that enumerated design for a fixed total sample size can be compared based on any configuration of the true values of the variance components. In this paper we present a general result based on a particular configuration of variance components after empirically comparing for several values of these components in accordance with the configuration. The D-optimal criterion and A-optimal criterion were used.

5.1 Results Discussion

For the local optimum designs, the parameter space is

$$\sigma^2 = \left[\sigma_e^2, \quad \sigma_\gamma^2, \quad \sigma_{\alpha\beta}^2, \quad \sigma_\beta^2, \quad \sigma_\alpha^2\right] \qquad \sigma_r^2 \in \left[0 \quad 1\right] \quad r = e, \quad \gamma, \quad \alpha\beta, \quad \beta, \quad \alpha$$

σ_r^2 represents the proportion of individual variance components

In many industrial experiments where this model can be useful, the variance components of the crossed factors is expected to be at least as large as that of the nested factors. This work investigated optimal designs in possible sample sizes between 0 and 100 for a configuration of the variance components. The configuration of variance components used in this work is such that $\sigma_\gamma{}^2 = \left[\sigma_e^2 < \quad \sigma_\gamma^2 < \quad \sigma_{\alpha\beta}^2 < \quad \sigma_\beta^2 < \quad \sigma_\alpha^2\right]$

We empirically compared for the different proportions of variance components based on the configuration above. The A- Optimal and D- optimal designs for only one proportional comparison of the five variance components are presented in the table below.

$$\sigma^2 = \left[0.12, \quad 0.13, \quad 0.24, \quad 0.25, \quad 0.26\right]$$

Sample Sizes(N)	TPD	D-Optimal	A-Optimal
24	(4)	3	1
32	(4)	3	1
36	(6)	2	1
40	(4)	4	1
48	(16)	12	13
54	(4)	4	1
56	(4)	4	1
60	(12)	12	9
64	(10)	6	7
72	(28)	28	13
80	(16)	15	1
84	(12)	12	1
88	(4)	4	1
90	(12)	7	1
96	(40)	16	1
100	(6)	2	1

The table above shows the D-optimal designs and the A-Optimal design for different sample sizes. The second column indicates the total number of designs generated for each sample size, categorization of design is based on the table at the end of the manuscript, for example, N=48 has a total of sixteen (16) candidates designs generated and based on the categorization at the end of the table, Design Twelve (D12) is the D-Optimal and Design thirteen (D13) is A-Optimal. Likewise N= 72 has a total of twenty eight (28) candidates designs and Design Twenty Eight (D28) is D-Optimal and Design Thirteen (D13) is A –Optimal.

From the above results and from other empirical comparisons made for different proportional value of the variance components based on the same configuration, the following general statements can be made for the choice of A-Optimal and D-Optimal Design.

Determinant Criterion

(1) From the candidates designs generated for a particular sample size, select designs in which the product of the levels of the two crossed factors (axb) are largest.

(2) Reduce the designs selected in (1) to those ones in which the degree of freedom for the interaction factor is largest

(3) From the remaining designs in (2), since $\sigma_{\alpha}^{2} > \sigma_{\beta}^{2}$, then in most cases, designs with the number of levels of the crossed factor A that is at least as large as the number of levels of the crossed factor B is selected. i.e. a≥b (except for some smaller sample sizes)

(4) If there are still more than one design from (3), then in most cases the design with the larger number of levels for the crossed factor C is the D-Optimal design

Trace Criterion

(1) Select all designs in which the number of observation within each level of the crossed factor C is largest. i.e. m is largest

(2) If there are more than one design in (1), select the one in which the levels of the nested factor C is largest. i.e. c is largest.

(3) If there are still more than one design from (2), the remaining designs should be empirically investigated for optimality has the A-Optimality in this case varies with values of the variance components and sample sizes.

VI. CONCLUSION

We have constructed optimal experimental designs in a linear random effect model with five variance components. The model has random nested factors nested within the treatment combination of the crossed factors. We systematically generated groups of designs for a fixed total sample size in such a way that designs generated are balanced. Local A-Optimal and D-Optimal designs for maximum likelihood estimators were obtained for a particular configuration of the variance components after empirically comparing designs for several proportional values of the components. We also presented a general result from the comparisons made. Although we have only generated designs for sample sizes between 0 and 100, the procedure stated in this paper can be used to generate and compare larger sample sizes, the procedure can also be used to compare different configurations of the variance component. Overall for the linear random effect model the D-Optimal design is preferred since samples are concentrated at the level where the true values of the variance components are larger.

Table of Sample sizes with total number of generated designs

N=24					N=36						N=32				N=40			
D	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
A	2	2	2	3	2	3	2	2	3	3	2	2	2	4	2	2	2	5
b	2	2	3	2	2	3	3	3	2	2	2	2	4	2	2	2	5	2
c	2	3	2	2	3	2	2	3	2	3	2	4	2	2	2	5	2	2
m	3	2	2	2	3	2	3	2	3	2	4	2	2	2	5	2	2	2

N = 48																	
D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	
A	2	2	3	3	2	2	4	4	2	2	3	4	2	2	2	6	
b	3	3	2	2	4	4	2	2	2	2	4	3	2	2	6	2	
c	2	4	2	4	2	3	2	3	3	4	2	2	2	6	2	2	
m	4	2	4	2	3	2	3	2	4	3	2	2	6	2	2	2	

N=54					N=60												N=56			
D	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**	**4**
A	2	3	3	3	2	2	3	3	2	2	5	5	2	2	3	5	2	2	2	7
b	3	2	3	3	3	3	2	2	5	5	2	2	2	2	5	3	2	2	7	2
c	3	3	2	3	2	5	2	5	2	3	2	3	3	5	2	2	2	7	2	2
m	3	3	3	2	5	2	5	2	3	2	3	2	5	3	2	2	7	2	2	2

N = 72																		
D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
A	3	3	2	2	3	3	4	4	3	3	2	4	2	2	2	9	2	2
b	2	2	3	3	4	4	3	3	3	3	4	2	2	2	9	2	2	2
c	3	4	3	4	3	2	3	2	2	4	3	3	2	9	2	2	3	6
m	4	3	4	3	2	3	2	3	4	2	3	3	9	2	2	2	6	3

N=72 (Contd.)											**N=64**									
D	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	2	2	3	3	2	2	6	6	3	6	2	2	4	4	2	4	2	2	2	8
b	3	3	2	2	6	6	2	2	6	3	4	4	2	2	2	4	2	2	8	2
c	2	6	2	6	2	3	2	3	2	2	2	4	2	4	4	2	2	8	2	2
m	6	2	6	2	3	2	3	2	2	2	4	2	4	2	4	2	8	2	2	2

N=88					**N=80**											
D	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
A	2	2	2	11	2	2	2	10	2	2	5	5	2	2	4	4
b	2	2	11	2	2	2	10	2	5	5	2	2	4	4	2	2
c	2	11	2	2	2	10	2	2	2	4	2	4	2	5	2	5
m	11	2	2	2	10	2	2	2	4	2	4	2	5	2	5	2
N=80 Contd.					**N=90**											
D	**13**	**14**	**15**	**16**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
A	2	2	5	4	2	2	3	3	3	3	5	5	3	3	2	5
b	2	2	4	5	3	3	2	2	5	5	3	3	3	3	5	2
c	5	4	2	2	3	5	3	5	2	3	2	3	2	5	3	3
m	4	5	2	2	5	3	5	3	3	2	3	2	5	2	3	3

N=96																				
D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	2	2	2	12	2	2	4	4	2	2	6	6	2	2	4	6	4	4	2	2
b	2	2	12	2	4	4	2	2	6	6	2	2	2	2	6	4	2	2	4	4
c	2	12	2	2	2	6	2	6	2	4	2	4	4	6	2	2	4	3	4	3
m	12	2	2	2	6	2	6	2	4	2	4	2	6	4	2	2	3	4	3	4
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A	4	4	3	3	4	4	2	3	2	2	3	3	2	2	8	8	2	2	3	8
b	3	3	4	4	4	4	3	2	3	3	2	2	8	8	2	2	2	2	8	3
c	4	2	4	2	3	2	4	4	2	8	2	8	2	3	2	3	3	8	2	2
m	2	4	2	4	2	3	4	4	8	2	8	2	3	2	3	2	8	3	2	2

N=84													**N=100**					
D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**	**4**	**5**	**6**
A	2	2	2	2	3	3	2	2	7	7	3	7	2	5	5	5	2	2
b	2	2	3	3	2	2	7	7	2	2	7	3	2	5	2	2	5	5
c	3	7	2	7	2	7	2	3	2	3	2	2	5	2	5	2	5	2
m	7	3	7	2	7	2	3	2	3	2	2	2	5	2	2	5	2	5

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