

## Development of Decentralized Controller Based On Gain and Phase Margin for Interacting Processes

Ria Maria Zacharia ,<sup>2</sup>Dr. Sanjeevi Gandhi A

<sup>1</sup>PG Student Department of Electronics and Instrumentation Engineering  
 Karunya University, Coimbatore

<sup>2</sup>Assistant Professor Department of Electronics and Instrumentation Engineering  
 Karunya University, Coimbatore

### -----ABSTRACT-----

The system under investigation is a two tank interacting process. The basic control principle of the two tank interacting system is to maintain a constant level of the liquid in the tank. In interacting systems a change in single parameters will affect all other loops. This paper aims at eliminating the interaction between TITO processes based on gain and phase margin specifications. In order to minimize the interaction between the loops decouplers are designed for the system. Then for each decoupled subsystem a first order plus dead time model is achieved based on frequency response fitting. For each reduced order decoupled subsystems an independent PI controller is designed so as to obtain the desired gain and phase margins.

**KEYWORDS:** decentralized controller, model order reduction, interaction process, decoupled system.

Date of Submission: 15 February 2014



Date of Acceptance: 05 March 2014

### I. INTRODUCTION

The dynamics employed in many of the industries is MIMO dynamics along with interaction among input and output variables. The use of decentralized controllers has gained popularity because of its simple design, ease to tune, implement and maintain. In this paper SISO PI controller is used for controlling MIMO systems with interaction, which is mainly due to its simple structure and its ability to meet the control objectives. The presence of interaction makes the design and tuning of centralized multi loop controllers difficult as compared to the single loop controllers. Since the controllers interact with each other in centralized multi loop controllers, tuning of one loop cannot be done independently. There are many methods to eliminate or reduce the effects of loop interactions such as decoupling method, relay - auto tuning method, sequential loop closing method and independent loop method. All the above mentioned methods have many advantages but has certain inherent disadvantages thereby prompting us to make use of a better method.

### II. SYSTEM DESCRIPTION:

#### 2.1 Two tank interacting system:

Two tank interacting process as shown in Figure 1 is considered. It consists of two cylindrical tanks (Tank 1 and Tank 2), two independent pumps that delivers the liquid flow ( $m_1$  and  $m_2$ ) to Tank 1 and Tank 2. The two tanks; Tank 1 and Tank 2 are interconnected through a manually controlled valve at the bottom.

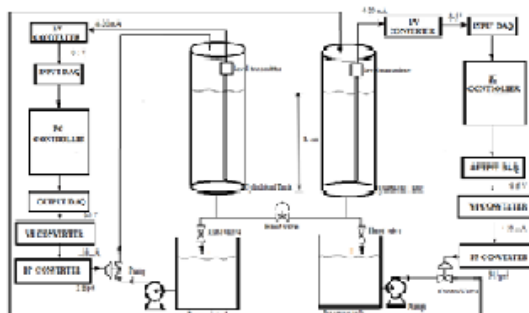


Figure 1: Experimental Setup of a Two Tank interacting process

Here the two tank interacting process is considered as a two input two output (TITO) process in which level H1 in Tank1 and level H2 in Tank2 are considered as output variables and m1 and m2 are considered as the respective manipulated variables. A change in a single parameter affects, in general all other loops as well.

**2.2 Block diagram:**

Decentralized controllers can reduce the interaction to a great extent. Hence “n×n” Multivariable system requires “n” multi-loop controllers.

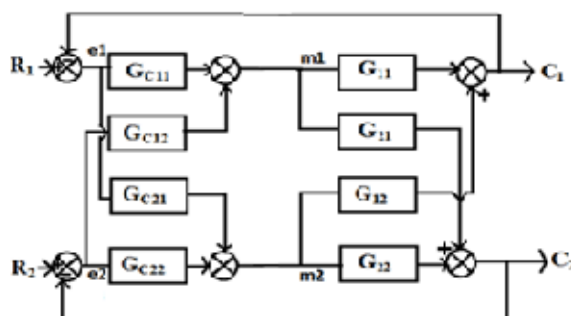


Figure 2: Block diagram of the proposed method

The basic closed loop block diagram of 2 X 2 system is shown in Figure 2. In an interacting system, G12 and G21 are not zero, but we can manipulate the controller signal so that the system appears (mathematically) to be decoupled. From the block diagram we can write,

$$H_1 = G_{11}m_1 + G_{12}m_2$$

$$H_2 = G_{21}m_1 + G_{22}m_2$$

where G<sub>11</sub>, G<sub>12</sub>, G<sub>21</sub>, G<sub>22</sub> are the process transfer functions, m<sub>1</sub> and m<sub>2</sub> are the inputs, H<sub>1</sub> and H<sub>2</sub> are the outputs.

**III. MATHEMATICAL MODELING:**

Consider two interacting tanks as shown in Figure 3. The interaction is caused by a manually controlled valve. When the valve is closed the two tanks do not interact with each other. But when it is open they interact with each other thereby affecting the performance of both the tanks.

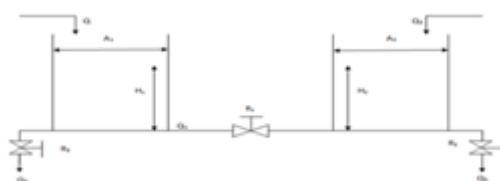


Figure 3: Two Tank Interacting System

Where Q and Q<sub>3</sub> denotes the inflow to the tanks 1 and 2 in l/hour, Q<sub>2</sub> and Q<sub>4</sub> denotes the flow in l/hour, A<sub>1</sub> and A<sub>2</sub> denotes the area of the tanks in cm<sup>2</sup>, H<sub>1</sub> and H<sub>2</sub> denotes the level of liquid in the tank in cm, R<sub>1</sub>, R<sub>2</sub> and R<sub>3</sub> are the resistances to the flow of liquid.

Using the law of conservation of mass,  
Rate of accumulation = mass flow in – mass flow out.

We can derive the equation for tank 1 when there is no interaction as follows:

$$\frac{H_1(s)}{Q_1(s)} = \frac{R_3}{1 + \tau_1(s)}$$

Similarly, we obtain the equation for tank 2 as:

$$\frac{H_2(s)}{Q_2(s)} = \frac{R_2}{1 + \tau_2(s)}$$

The equation showing the interaction from tank 1 to tank 2 is as follows:

$$\frac{H_2(s)}{Q(s)} = \frac{R_1 R_2}{A_1 R_2 s + 1 + \tau_2 s + \tau_1 s + \tau_2 s^2 \tau_1}$$

Similarly the equation showing the interaction from tank 2 to tank 1 can be derived as given below:

$$\frac{H_1(s)}{Q_3(s)} = \frac{R_3}{1 + s(\tau_4 - A_2 R_3 - \tau_3) + \tau_3 s^2 \tau_4}$$

#### IV. CONTROLLER DESIGN:

##### 4.1 Gain and phase margin method [1]:

The gain increment is required at the frequency where phase is  $180^\circ$  and the phase increment is required at the frequency where amplitude is 0 dB. The frequency where phase crosses  $180^\circ$  is called the phase crossover and the frequency where the amplitude crosses the 0dB point is called the gain crossover. Gain and phase margins are measures of stability for a feedback system, though often only phase margin is used rather than both. Based on the magnitude response of the loop gain,  $|A\beta|$ , gain margin is the difference between unity and  $|A\beta|(\omega 180^\circ)$  where  $180^\circ$  is the frequency at which the loop gain phase,  $A\beta$ , is  $-180^\circ$ . Phase margin is the phase difference between  $A\beta(\omega 0\text{dB})$  and  $-180^\circ$  where  $\omega 0\text{dB}$  is the frequency at which  $|A\beta|$  is unity. A target phase margin of  $60^\circ$  is highly desirable in feedback amplifier design as a trade off between loop stability and settling time in the transient response. Typically, the minimum acceptable phase margin is  $45^\circ$ .

##### 4.2 Design of decoupler [2]:

Most control systems are complex and multi-variable, i.e. they consist of several measurement signals and control signals, and there are often complicated couplings between the different signals. In interacting systems, any change in any of the input affects the all outputs. And the amount of interaction depends on the process parameters. This interaction cannot be reduced using single control loop. To completely eliminate the interaction between the outputs and set points, cross- controllers (decoupler) is used. This decoupler will reduce the interaction in MIMO systems to zero. This can be done by choosing the following for decouplers.

$$G(s) = \begin{pmatrix} g_{11}(s)e^{-\tau_{11}(s)} & g_{12}(s)e^{-\tau_{12}(s)} \\ g_{21}(s)e^{-\tau_{21}(s)} & g_{22}(s)e^{-\tau_{22}(s)} \end{pmatrix}$$

$$D(s) = \begin{pmatrix} v_1(s) & d_{12}(s)v_2(s) \\ d_{21}(s)v_1(s) & v_2(s) \end{pmatrix}$$

$$v_1(s) = 1; \tau_{21} \geq \tau_{22}$$

$$= e^{(\tau_{21}-\tau_{22})s}; \tau_{21} < \tau_{22}$$

$$v_2(s) = 1; \tau_{12} \geq \tau_{11}$$

$$= e^{(\tau_{12}-\tau_{11})s}; \tau_{12} < \tau_{11}$$

$$d_{12}(s) = \frac{g_{12}(s)}{g_{11}(s)} \bullet e^{-(\tau_{12}-\tau_{11})s}$$

$$d_{21}(s) = \frac{g_{21}(s)}{g_{22}(s)} \bullet e^{-(\tau_{21}-\tau_{22})s}$$

$$H(s) = G(s)D(s)$$

##### 4.3 Model reduction [4]:

A FOPDT model often reasonably describes the process gain, overall time constant and effective dead time of higher order processes. Here a FOPDT model  $I_{ii}(s)$  of each element  $h_{ii}(s)$  of  $H(s)$  is obtained as below

$$I_{ii}(s) = \frac{k_{ii}(s)e^{-L_{ii}(s)}}{T_{ii}(s) + 1}$$

Where  $i= 1, 2$ . Three unknown parameters are to be determined to find the FOPDT model. Here we use frequency response fitting at two points  $\omega=0$  and  $\omega=\omega_{cii}$ , where  $\omega_{cii}$  is the phase cross over frequency. As a result the parameters can be calculated using the following equations:

$$K_{ii}(s) = h_{ii}(0)$$

$$T_{ii}(s) = \sqrt{\frac{K_{ii}(s) - |h_{ii}(j\omega_{cii})|^2}{|h_{ii}(j\omega_{cii})|^2 \omega_{cii}^2}}$$

$$L_{ii}(s) = \frac{\pi + \tan^{-1}(-\omega_{cii} T_{ii})}{\omega_{cii} T_{ii}}$$

The reduced models obtained are as follows:

$$I_{11}(s) = \frac{41.9e^{-4.15s}}{.3663s + 1}$$

$$I_{22}(s) = \frac{37.6e^{-15.06s}}{.2929s + 1}$$

**4.4 Design of proportional integral controller [3], [6]:**

The gain and phase margins (GPM) are typical loop specifications associated with the frequency response. The GPMs have always served as important measures of robustness. It is known from classical control that phase margin is related to the damping of the system, and can therefore also serve as a performance measure. The controller design methods to satisfy the criteria are not new and therefore they are used widely. Here simple formulas are used to design the PI controller to meet user defined GPM specification.

$$I_{ii}(s) K_{ii}(s) = \frac{K_{pii} K_{ii} (sT_{lii} + 1)}{sT_{lii} (sT_{ii} + 1)} e^{-L_{ii}}(s)$$

Where,

$$K_{pii} = \frac{\omega_{pii} T_{ii}}{A_{mii} K_{ii}}$$

$$T_{lii} = (2\omega_{pii} - \frac{4\omega_{pii}^2 L_{ii}}{\pi} + \frac{1}{T_{ii}})^{-1}$$

$$\omega_{pii} = \frac{A_{mii} \phi_{mii} + \frac{1}{2} \pi (A_{mii} - 1)}{(A_{mii} - 1) L_{ii}}$$

**V. RESULTS AND DISCUSSION:**

This section presents the results obtained using a decentralized PI controller based on gain and phase margin specification for the TITO process. First, a decoupler is designed for the original system and the reduced model is obtained for the decoupled system. The controller parameters are estimated using gain and phase margin specifications, and we obtain well tuned parameters. When compared with the conventional method, the gain and phase margin specifications are met with minimal errors. The implementation of the interacting system, the decoupled system using conventional method and the decoupled system using GPM method are done in MATLAB Simulink and the various responses are obtained. When two interacting systems interact with one another the input to one system affects the output of the other as shown below in figure 4.1.1. It shows the response of the interacting systems, that is even though the input to the second system is 0, from the graph it is clear that the output of the system is not 0. From the graph it is observed that there is an output, which is due to the interaction between the two systems.

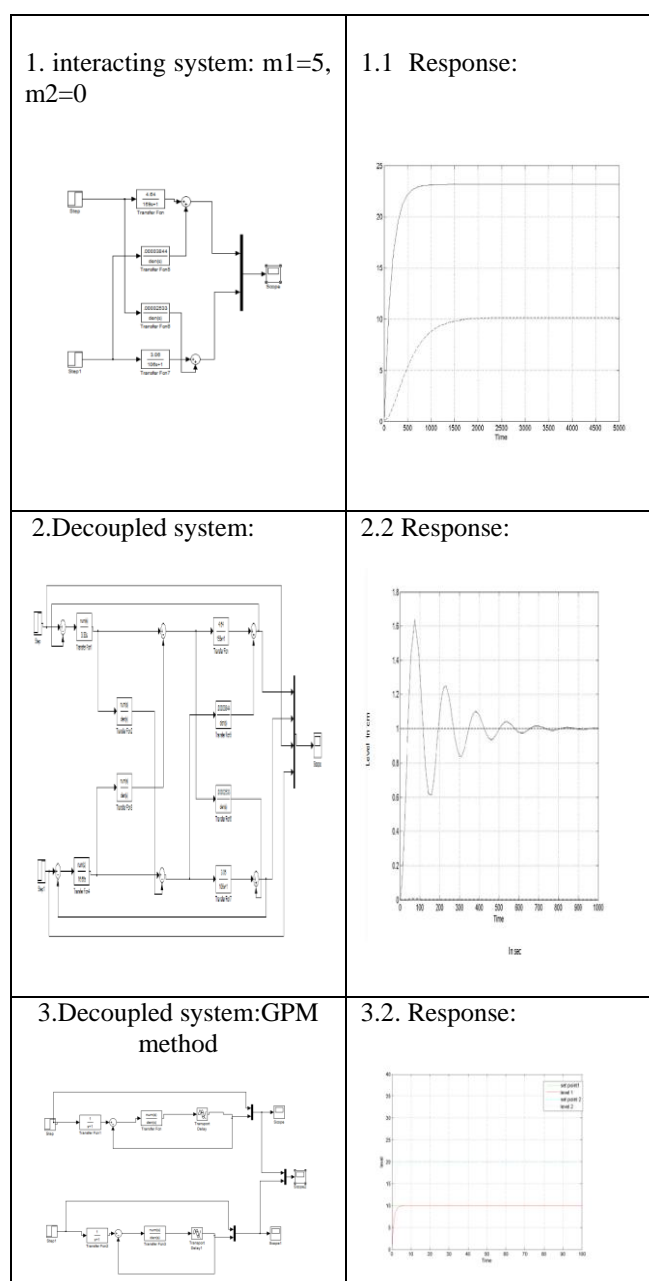


Figure 4:Block diagrams and Responses of interacting systems in MATLAB Simulink

The figure 4.2 given above shows the block diagram of the decoupled system. Here the controller is designed using a conventional method. The method employed here is the Zeigler Nichol's method.

The tuning method employed here is Zeigler Nichol's tuning method. The controller tuned using this method has some inherent defects such as it will cause undershoots and overshoots in the response. Another disadvantage is that the settling time required is more. The figure 4.2.1 shows the response of the decoupled system using conventional method. It is shown in the figure that the interaction between the two systems has been eliminated and the objective has been achieved. The figure 4.3 shown is the block diagram of the PI controller designed using gain and phase margin method. Here the interaction is completely eliminated, so is the undershoot and overshoot. Thus it is clear that this method is far better than the conventional method. Figure 4.3.2 shows the response of the system without interaction. Here the input to the system 1 is 10 and the input to system 2 is 20. Thus it is clear that interaction is completely eliminated and that there is no undershoot and

overshoot compared to other methods. Hence it can be said that tuning using gain and phase margin specifications reduces or eliminates the prevalent disadvantages of the conventional methods.

## **VI. CONCLUSION AND FUTURE WORK:**

A decentralized PI controller design method is proposed for two interacting processes. First a decoupler is designed for the original system and the reduced model is obtained for the decoupled systems. The gain and phase margin specifications are utilized to get well tuned parameters of the PI controller. The system considered is a TITO process in which the level is coupled with the flow. The transfer function for the system is determined and the decoupler is also designed. The decoupler is designed to reduce the interaction. From the obtained results it is clear that the proposed method is far better than the other prevailing methods as it reduces the settling time, overshoot, undershoot and the errors to a great extent.

## **REFERENCES:**

- [1] D.K. Maghade, B.M Patre. "Decentralized PI/PID controllers based on gain and phase margin specifications for TITO processes" *ISA Transactions* 51 (2012) 550-558.
- [2] Tavakoli S, Griffin I, Fleming PJ. "Tuning of decentralised PI (PID) controllers for TITO processes". *Control Engineering Practice* 2006; 14:1069–80.
- [3] Kaya I. Tuning PI controllers for stable processes with specifications on gain and phase margins. *ISA Transactions* 2004; 43:297–304.
- [4] Dougherty D, Cooper D. "A practical multiple model adaptive strategy for single-loop MPC." *Control Engineering Practice* 2003; 11(4):141–59.
- [5] De Paor AnnraoiM, O'Malley Mark. "Controllers of Ziegler–Nichols type for unstable processes." *Internat J Control* 1989; 49:1273–84.
- [6] Franklin GF, Powell JD, Naeini AE. "Feedback control of dynamics systems." *Workingham (UK): Addison-Wesley; 1986.*