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# Single Machine Power Network Load Frequency Control Using T-S Fuzzy Based Robust Control With Relax LMI Condition

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**ABSTRACT**
*The stabilizing and robust control performance condition formulated using polynomial objective constraint with Relax Linear Matrix Inequality (RLMI) is presented in the paper. To achieve the objective performance, a two part fuzzy controller consisting of a stabilizing and robust controllers is developed, Linear Matrix Inequality constraint relaxed using a scaling parameter is formulated, the developed controller and the RLMI conditions were then applied for the control of load frequency deviation in a single area power system network. Simulation results obtained, showed that the RLMI has some merit over the so called strict LMI in providing better transient performance.*

***KEYWORDS:*** *Robust performance, Relax Linear Matrix Inequality*

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## I. INTRODUCTION

Demand for efficient performance of the current industrial systems is growing. Control of these systems therefore requires that, they be formulated for achieving robust or global optimal performance. One optimal concept employed for multi-objective performance is characterized by a polynomial objective and constraints. Robust stabilization to parameter uncertainties, static output feedback optimization through pole assignment are some of the typical examples of the polynomial objective constraint formulation. The major problem associated with these formulations is that, in general, they are highly non-convex and solving them requires additional computational cost. Simplification of the non-convexity to convexity through the use of Linear Matrix Inequality (LMI) is currently in used. However, simplification to convex polynomial objective function would leads to strict or conservative constraint that is difficult to solve. The strict LMI stability conditions have been reported in the context of controller designs for Lurie control systems and descriptor linear systems using pole placement technique in Jin-Feng et al., ( Jin-Feng et al.,2008) and Bai et al., (Bai et al., 2012). Recently, new methodologies that allow transformation of the non-convex formulations into a convex polynomial time constraint have been developed that can easily be solved using the highly efficient interior point algorithm. Works on this area have been reported (Henrion and Tarbouriech, 2002; Henrion and Lasserre, 2003).Fuzzy systems have also been used for implementing robust and optimal control of nonlinear systems with high degree of success (Taylor et al. 2006;Tanaka,Ikeda and Wang, 1998). Example, Wang and Sun (Wang and Sun, 2002) proposed a relaxed LMI for discrete Takagi-Sugeno (T-S) fuzzy system by setting up rules boundaries that help reduce the number of fuzzy rules inequality solutions. Sala and Arino (Salana and Arino, 2008), Narimani and Lam (Narimani and Lam, 2009) employed a membership dependent polynomials that would lead to a relax LMI conditions. Maximum fuzzy membership overlap regions were also used as the basis for reducing the number of rules required, thereby simplifying the determination of a feasible Lyuapunov P matrix (Song-Teo, 2010).

In this article, a two part T-S fuzzy based control law consisting of a fuzzy state feedback stabilizing and robust controllers is developed. LMI relaxation method that involved parameter scaling factor, selected from a prior knowledge of the number of fired rules would be used to formulate the polynomial objective constraint for determining a state feedback gain matrix. Single area load frequency power network model would use as an example to test the performance of the developed controller.

**Takagi-Sugeno Fuzzy Model**

We consider a nonlinear system of the form

$$\dot{x}(t) = f(x(t), u(t), w(t)) \tag{1}$$

$$y(t) = f(x(t), u(t), v(t)) \tag{2}$$

Where ) and are nonlinear but smooth system and input functions, is 1 by *n* system state vector, is 1 by *m* input control vector, is 1 by *p* external disturbance vector, *y(t)* is output, is measurement noise. Nonlinear systems in practice are complex and difficult to explicitly describe mathematically. For an n-th order nonlinear system, a T-S fuzzy model based on a qualitative expert knowledge of the system can be constructed. In T-S fuzzy modeling method, a local model for a giving fuzzy input space is defined, and so for different fuzzy input spaces, we have a corresponding local models. Nonlinear interpolation between the local models produce what is called a global fuzzy model of the system. Some in depth discussion on the application of the T-S fuzzy paradigm can be found in (Abdelkarim et al., 2010;Machidi at al., 2008; Lee,Park and Joo, (2006). Let for the purpose of this paper, proposed an i-th T-S fuzzy rule for the system in (1) and (2), as follows:

$R^i$: IF $x_1$ is $\mu_1^i(x_1)$ AND ....... AND $x_n$ is $\mu_j^i(x_j)$ THEN $\dot{x}_1 = A^i x + B^i u + w(t)$ (3)

$$y_1 = C_i x(t) + v(t), \quad i = 1, 2, \ldots. \text{R},$$

Where R is number of rules, $\mu_j^i(x_j)$ is i-th rule, j-th fuzzy linguistic value . Assuming singleton fuzzifier, product inference and centre average defuzzification), different methods can be adopted for obtaining the defuzzified fuzzy model (Tanaka and Wang, 2001).

**Fuzzy Control Policies :** In order to realize a stable and robust control policy, we assume that control policy consists of two parts, written as

$$\mathbf{u} = \mathbf{u}_s + \mathbf{u}_r \tag{4}$$

Where is the stabilizing control, is robust control law. Following the works in (Tanaka and Sugeno, 1999; Li et al., 2000) we adopt the stabilizing control law as

$$\mathbf{u}_s = -\sum_{j=1}^{R} \sigma_i \mathbf{K}_j \mathbf{x} \tag{5}$$

Where , with being the index for number of T-S fuzzy rules, is the index for the number of fuzzy control laws, is strength of a T-S fuzzy rule and can easily computed for the well known relation (Tanaka and Sugeno, 1999), $\mathbf{K}_j$ is state feedback gain matrix, can also be determine from the fuzzy state feedback control method known as the *Parallel Distributed Control* (PDC). In similar manner, we adopt a fuzzy robust control law proposed in Stanislaw, (2003), here with slight modification as follows

$$\mathbf{u}_r = \sigma_i \gamma \boldsymbol{B}^T \boldsymbol{P} \tag{6}$$

Where is newly introduced gain constant, **B** is input matrix, **P** is symmetric positive definite matrix to be determined. For the number of *R* T-S fuzzy rules, we can write (6) as,

$$\mathbf{u}_r = [u_1, \ldots\ldots. u_R] = \sum_{i=1}^{R} \gamma \sigma_i \mathbf{B}_i^T \mathbf{P} \quad \ldots \tag{7}$$

It can be shown that if the control policy of (4) is plunged into a centroid-based defuzzified fuzzy model of (3), the closed loop fuzzy stable and robust fuzzy control system can be written as,

$$\dot{\mathbf{x}} = \sum_{i=1}^{R} \sum_{j=1}^{q} h_i h_j (\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j)\mathbf{x} - \sum_{i=1}^{R} h_i \mathbf{B}_i \vartheta \,\mathrm{sign}( u_r ) - h_i^2) \quad \ldots \quad (8)$$

Where q is less than or equals to R, The closed loop system in (13) should ensure not only stable performance, but robust to bounded uncertain conditions. Here we intend to determine the state feedback matrix **K** by solving a relax LMI criterion.

**Remark 1**: If (12) is observed and the relax LMI condition being feasible, then the proof of the asymptotic stability of the PDC based system given in Stanislaw, 1999 also applied to (13).

**Relax LMI Formulation :** To formulate the relax LMI condition, we write (13) considering only the first term on the left as,

$$\dot{\mathbf{x}} = \sum_{i=1}^{R} \sum_{j=1}^{R} h_i h_j G_{ij} \mathbf{x} \quad (9)$$

Where

$$\mathbf{G}_{ij} = (A_i - B_i K_j)$$

Based on (9) , Tanaka and Wang (Tanaka and Wang, 2001) proposed the following theorem:
**Theorem:** (Tanaka and Wang, 2001): The equilibrium of a fuzzy control system described by (8) is globally asymptotically stable if there exist a common positive definite matrix P such that

$$\mathbf{G}_{ii}^T \mathrm{P} + \mathrm{P}\mathbf{G}_{ii} < 0 \quad (10)$$

$$\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij} + G_{ji}}{2}\right) \leq 0 \quad (11)$$

$$i < j$$

Where $G_{ji} = A_j - B_j K_i$.

Condition (11) is also true only when i-th and j-th rules overlapped. Of course (10) and (11) applies to strict LMI stability conditions. If we consider that some rule weights are inactive (not fired), and take number of rules that fire as s ( 1< s R ), then the result of the relax LMI criterion can be written as

$$\mathbf{G}_{ii}^T \mathrm{P} + \mathrm{P}\mathbf{G}_{ii} + (s-1)\mathrm{Q} < 0 \quad (12)$$

$$\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij} + G_{ji}}{2}\right) - \mathrm{Q} \leq 0 \quad (13)$$

$$i < j$$

Where **P** is feasible positive definite matrix to be determined, **Q** is matrix to be chosen. Following the process proposed in (Tanaka and Wang, 2001), the basic results of the relax LMI are as follows:

$$\begin{bmatrix} X-(s-1)Y & XA^T - M_i^T B_i^T \\ A_i X - B_i M_i & X \end{bmatrix} > \mathbf{0} \tag{14}$$

$$\begin{bmatrix} X+2Y & 0.5\left[XA_i^T + XA_j^T - M_j^T B_i^T - M_i^T B_j^T\right] \\ 0.5\left[A_i X + A_j X - B_i M_j - B_j M_i\right] & X \end{bmatrix} \geq \mathbf{0} \tag{15}$$

$$\boldsymbol{X} > \mathbf{0} \tag{16}$$

$$\mathbf{Y} \geq \mathbf{0} \tag{17}$$

The relax LMIs in (14)-(17) are to be solved for **X,** and **Y.** So that the feasible matrix **P** and state feedback matrix $\mathbf{K_i}$ can be calculated as follows:

$$\mathbf{P} = \boldsymbol{X}^{-1} \tag{18}$$

$$\mathbf{K}_i = \mathbf{M}_i \boldsymbol{X}^{-1}. \tag{19}$$

**Application Example**

For purpose of verifying the validity of the remark1 above, we consider a single area power system load frequency model, here represented by its nominal linear model (Sadat, 1999) as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta P_L \quad \ldots \tag{20}$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \ldots \tag{21}$$

Where $x_1$, $x_2$ are the states of the system representing rotor angle □ and the frequency, respectively, $\Delta P_L$ is per unit input load change , $\omega_n$ and $\zeta$ are natural frequency and damping constants expressed as,

$$\omega_n = \sqrt{\frac{\pi f_o P_s}{H}} \quad \ldots \tag{22}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_o}{HP_s}} \quad \ldots \tag{23}$$

Where $D$ is the damping constant, $H$ is the per unit inertia constant, $Ps$ is the synchronizing power coefficient, $f_o$ is the operating frequency in Hz. We next, develop the system T-S fuzzy model.

## II. T-S FUZZY MODEL OF THE SYSTEM

For each input state, we assign two fuzzy membership values as,

$$x_1 = [\mu_1^1, \ \mu_2^1]$$

$$x_2 = [\mu_1^2, \ \mu_2^2]$$

We propose a four rules T-S fuzzy system as follows:

$$\begin{array}{l} \text{IF } x_1 \text{ is } \mu_1^1 \text{ AND } x_2 \text{ is } \mu_2^1 \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_1 x + \mathbf{B}_1 u \\ \text{IF } x_1 \text{ is } \mu_1^1 \text{ AND } x_2 \text{ is } \mu_2^2 \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_2 x + \mathbf{B}_2 u \\ \text{IF } x_1 \text{ is } \mu_1^2 \text{ AND } x_2 \text{ is } \mu_1^2 \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_3 x + \mathbf{B}_3 u \\ \text{IF } x_1 \text{ is } \mu_1^2 \text{ AND } x_2 \text{ is } \mu_2^2 \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_4 x + \mathbf{B}_4 u \end{array} \quad \ldots \quad (24)$$

**Remark 1:** The fuzzy membership function to be use is center type triangular mathematically expressed as,

$$\mu(x_j) = \max\left(0,\ 1 + \frac{|x_j + c_j|}{0.5 w_j}\right), \quad j = 1, 2, \ldots, q \qquad (25)$$

Where $c$ and $w$ are the center and width of the triangular membership function.

**Simulation**

To specify fuzzy system parameters, let the machine parameters be assigned as shown in Table 1.

**Table 1: System Parameters**

| Electrical | Mechanical |
|---|---|
| $V$ = 1.0 p.u. | $f$o = 50Hz |
| $X$ = 0.65 p.u. | $H$ = 8.89MJ/MVA |
| $E_o$ = 1.35 p.u. (max. gen. emf) | $D$ = 0.138 |
| cos□ = 0.8 | □$_o$ = 0.29304rad |

Giving the following relations (Sadat, 1999):

$$P_s = \frac{E_o \angle \delta_o \times V \cos(\delta_o)}{X} \qquad (26)$$

$$\Delta u = \frac{\pi f_o}{H} \Delta P_L \qquad (27)$$

Using (22) and (23), $P_s$ = 1.9884, $\omega_n$ = 5.93rad/s and $\zeta$ = 13.25 respectively. At load deviations ($\Delta P_L$) of 0.8 and 0.87 p.u. we have = 5.0 and = 15.0. Using a numerical simulation on (25) and (26) for transient step responses the universal range of the states are obtained as = .

Having the two elements as $\Delta x_1\,(\Delta\delta) = [0.1346 \quad 0.3590]$ and $\Delta x_2\ (\Delta\omega_n) = [0.0036 \quad 0.0096]$

The triangular membership function of Fig.1 parameters and the selected operating points for the system are shown in Table 2

**Table 2: Fuzzy System Parameters**

| States | Centers of triangular membership functions | Operating Points | | | |
|---|---|---|---|---|---|
| $x_1$ = [0.1 0.36] | $\mu_1^1$ = [0.0 0.125 0.25] <br> $\mu_2^1$ = [0.125 0.25 0.5] | $x_1^1$ = 0.11 | $x_1^2$ = 0.25 | $x_1^3$ = 0.45 | $x_1^4$ = 0.5 |
| $x_2$ = [0.0036 0.009] = 0.05, s = 1 | $\mu_1^2$ = [0 2.5 □ 10$^3$ 5 □ 10$^3$] <br> $\mu_2^2$ = [2 □ 10$^3$ 5 □ 10$^3$ 0.01] | $x_2^1$ = 0.001 | $x_2^2$ = 0.005 | $x_2^3$ = 0.007 | $x_2^4$ = 0.01 |

Where $x_f^i$ is selected operating point ($f = [1,2]$). The following simulation stages are to be carried out:

[1] Formulate the power network T-S fuzzy model in line with (9)

[2] Formulating the relax LMI conditions in line with (14)-(17) and solving for (18) –(19) in Matlab LMI toolbox.

[3] Finally using Matlab control toolbox, solving (9) for step dynamic response of the robust fuzzy controlled system.

**Simulation Cases**

The robust performance test on the closed loop power network will be carry out under the following cases:

CASE 1: Under the action of conventional optimal PID controller

CASE 2: Under a strict LMI conditions as reported in Shehu and Dan-Isa (Shehu and Dan-Isa, 2011)

CASE 3: Under the relax LMI conditions

All simulations are to be performed under varying system parameters shown in Table 3.

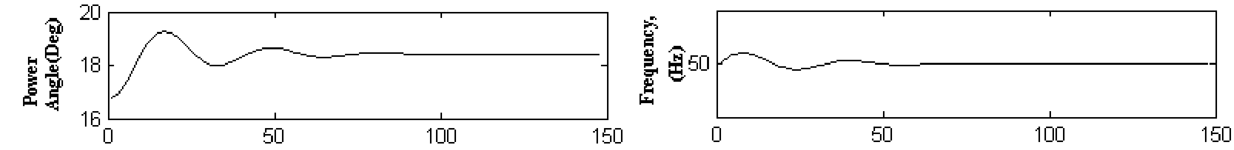
**Table 3: Generator Parameter Adjustments**

| SN | Damping Constant $D$ | Synchronous Power Coeff. $P_s$ |
|---|---|---|
| 1. | 0.1380 | 1.9884 |
| 2 | 0.0138 | 1.9884 |
| 3 | 0.0138 | -1.9884 |

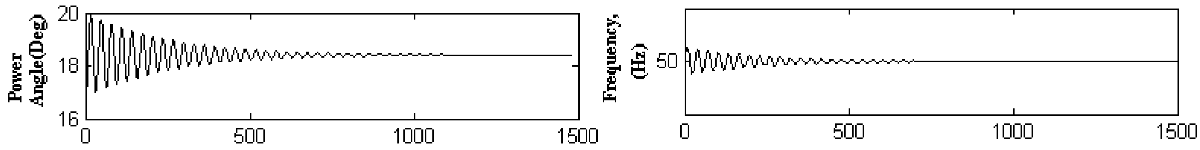
## III. RESULTS

With optimal PID controller, rotor angle deviation from 17.0 p.u step load change and load frequency deviation from 50Hz operating frequency are obtained as shown in Fig.1.

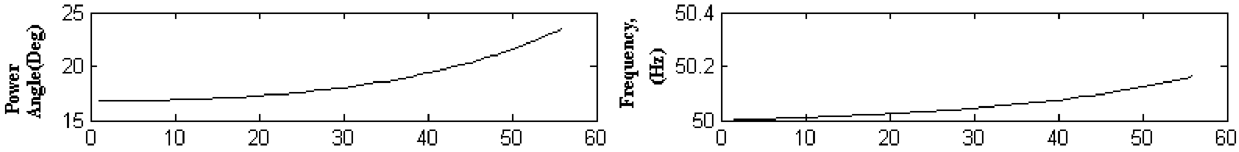
**CASE 1**: Results




(a)



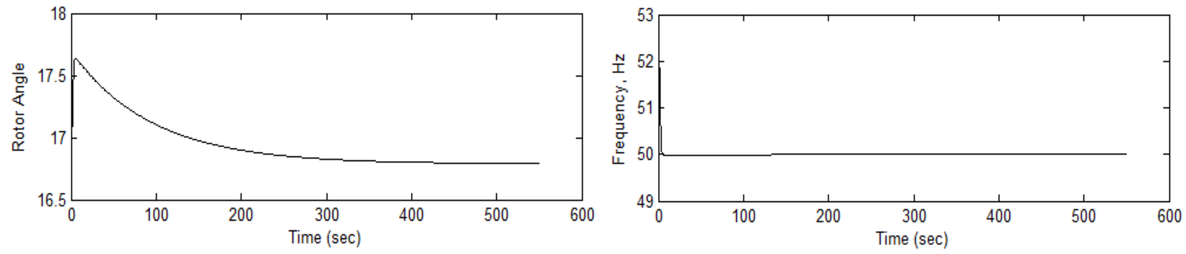

(b)



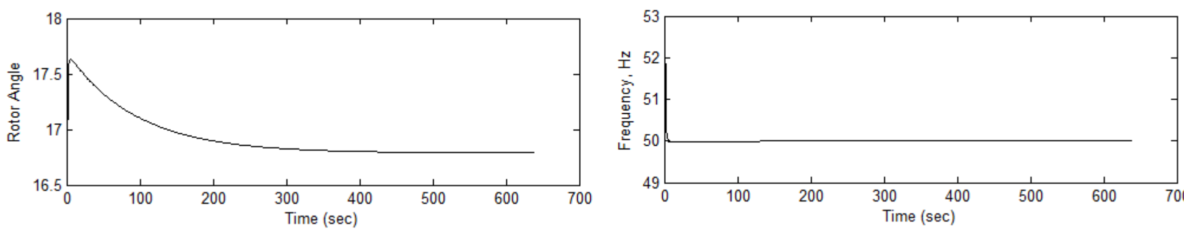

(c)

Fig. 1: Optimal PID Control (a) Angle and Frequency Deviations at $D$ = 0.138 and $P_s$ =1.9884 (b) At $D$ = 0.0138, $P_s$ = 1.9884 (c) At D = 0.138 and $P_s$ = -1.9884

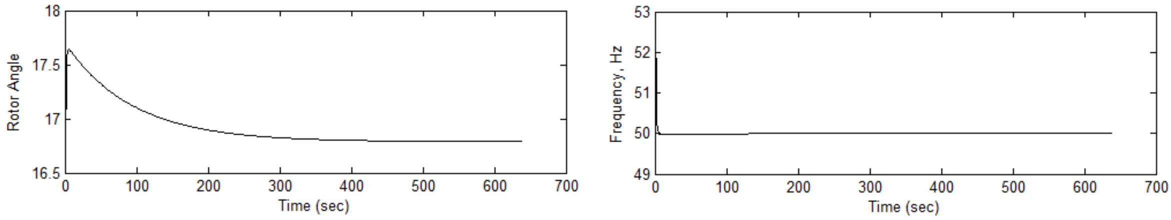
**CASE 2** Results: Strict LMI




(a)



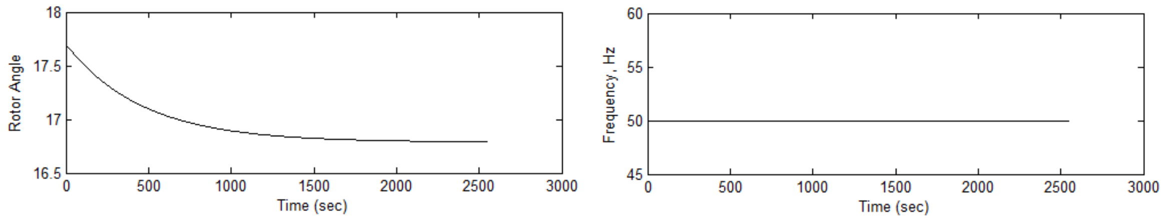
(b)



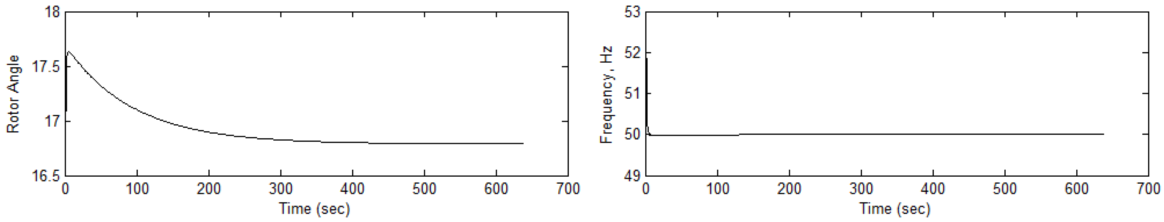

(c)

Fig. 2: LMI Based Robust Fuzzy Control (a) Angle and Frequency Deviations at $D =$ 0.13 and $P_s =$ 1.9884 (b) At $D$ = 0.0138 and $P_s$ = 1.9884 (c) At $D$ = 0.138 and $P_s$ = -1.9884

**CASE 3** Results: Strict and relax LMI at D = 0.00138, Ps = -1.9884




(a)




(b)

Fig.3: Rotor Angle and Frequency Responses Under Strict and Relax LMI Stability Robust Fuzzy Control at D = 0.00138, Ps = -1.9884 (a) Responses Under Strict Condition (b) Under Relax LMI Condition

## IV. DISCUSSION OF RESULTS

With optimal conventional PID controller, stable and robust performances were shown to be achieved only at nominal parameter condition, as demonstrated in Fig.1(a). It is sensitive to mild parameter changes (shown in Fig..12(b)) and unstable at worst case parameter conditions as shown in Fig.1(c). With fuzzy robust control, employing strict LMI criterion, stable and robust performances were achieved even at worst case system's parameters variation as obtained in responses shown in Fig.2(a)-(c). Fig.3 is a comparison result, of the robust fuzzy control performance with strict and relax LMI conditions, at even further reduced damping constant (D = 0.00138, Ps = -1.9884). Though, in both situations, stable and robust performances were achieved, better transient performance in terms of speed of response was achieved when under relax LMI condition, as shown in Fig.3(b). Realizing a preset rotor angle value at 450 seconds simulation time compared with about 2000 seconds with strict LMI.

## V. CONCLUSION

In the paper, a method for maintaining robust stability of a parameter disturbed load frequency single machine power system network was proposed. The method is a fuzzy state feedback control conditioned using an LMI relaxation constraint that was easily implemented using available numerical tools. One of the good function of a relax LMI stability constraint is allowing easy determination of a common **P** and hence, **K** gain matrices for realizing the PDC scheme, when large number of T-S fuzzy rules are involved. In the paper, the relax LMI based robust fuzzy controller was designed. First conventional optimal PID control was shown to give good performance only under nominal parameter perturbations. Robust fuzzy control with strict and relax LMIs were shown to give stable and robust performances. Further improved performance in terms of speed of response was shown to be realized when relax LMI conditions are involved. The result of the relax LMI constraint obtained here, demonstrated that, not only reduced number fuzzy rules can be achieved by adopting the method as reported by most of works reviewed in the paper, but good transient performance can also be achieved.

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