

Coordinate Transformation Of Birnin Kebbi

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I. INTRODUCTION

The set of parameters that define the shape and size, the orientation and position of a particular local ellipsoid relative to the real earth (the geoid) is called a **geodetic datum**). The individual (national) geodetic datums are often referred to as local datums as they only apply to a region of the earth's surface. More than 150 local datums have been used by different countries of the world, and an example of the local geodetic datums is the Minna datum of Nigeria. There exist quite a number of methods for transforming geodetic data in the form of coordinates from one datum to another (Babatunde, 1986).In the past, the local ellipsoid served as reference datum for the horizontal components (latitude and longitude) of the geodetic coordinates while the geoid served as reference datum for the vertical controls (the height).The advent of space geodetic positioning systems has brought about determination of global datum which is associated with global ellipsoid. These have been used (and still in use) to accurately map the entire Earth, without separation of horizontal datum from the vertical datum

The coordinates (latitude, longitude and height) of a point are dependent on the datum being used; the coordinates of a point referred to the local datum will certainly be different from its coordinates defined on the global datum. Coordinates can thus be transformed from one datum to another if the relationship between the two datums, that is, if a set of transformation parameters which can be used to transform the coordinates in one datum to the other are known. The process of converting coordinates from one datum to the other is known as **datum transformation**. The GPS derived coordinates and local datum coordinates of common points may be processed together using an appropriate **transformation model**, several of which have been developed by Geodesists. The models differ from each other in several ways including the type of coordinates used and the interpretation of results (Hofmann-Wellenhof & Mortiz, 2005). No conclusion has been drawn as to which is the preferred approach and no doubt this is an area where discussion will continue for some time to come (Smith, 1997). The outcome of the processing would be a set of quantities known as **transformations parameters**; these are later substituted into the formulae for transforming coordinates from one datum to the other.

The 7-parameter transformation model defines relationship between two consistent sets of Cartesian coordinates through estimation of seven parameters by relating the two coordinate sets with three translations, three rotations and a scale change (Krakiwisky & Thomson, 1974; Singh, 1994 & 2002).

In the differential projective transformation, if the local system is aligned to the global datum using Laplace equation, that is, if Laplace equation is fulfilled at least at the initial point P_0 (origin of the network)

then, there may be no need to introduce the rotations in the model because the enforcement of Laplace equation guarantees the parallelism of the axes of the global astronomic and ellipsoidal systems and hence ensure that the spatial computations take place in ϕ , λ , h system whose axes are parallel to the global system X, Y, Z(Heiskanen & Moritz, 1967; Vanicek & Krakiwisky, 1986, Hofmann-Wellenhof & Mortiz, 2005). However in practice, the parallelism may not be achieved exactly and that is why a small rotation of the geodetic system around ellipsoidal normal with respect to the global system is sometimes introduced (Vanicek & Krakiwisky, 1986).

The coordinate reference system used in Nigeria as the basis for most mapping and engineering projects refers to the Minna Datum, which is based on Clarke 1880 ellipsoid. The origin point (L40) of the geodetic network of Nigeria is located at the northern end of Minna base, in Niger State. The geodetic coordinates adopted for this origin were arrived at by taking the mean of astronomic coordinates propagated through four arms of the network, and the astronomic coordinates of L40. Like most local geodetic datums, the Nigerian horizontal geodetic datum is separated from the vertical datum (Nwilo et al., 2006). The centre of the reference ellipsoid does not coincide with the centre of mass of the Earth.

In order to optimize the benefit of high precision GPS network in Nigeria, any combination of the GPS results with coordinate information related to the Minna datum must ensure that the transformation parameters are determined. Efforts have been made to determine transformation parameters for the Nigerian geodetic network (Fubara, 1995 & Nwilo et al, 2006) using various methods. There exist large discrepancies between the results; this may be due to the differences in the data and methods used by the investigators. This research generate transformation parameters for Birnin Kebbi, the state capital of Kebbi state by datum transformation from Birnin Kebbi local datum to Global datum (WGS 84), on which the GPS is based. This research compares the results of three types of transformation procedures between the Nigerian Geodetic Datum (Minna Datum) and the global datum, the World Geodetic system (WGS 84), on which the GPS is based.

II. PROBLEM DEFINITION

Most data in the Nigerian Geodetic Network are based on the local (Minna) datum and some other minor local datum while some are established on the global datum using the Doppler and GPS as tools for geodetic positioning. Different GPS manufacturers are using different transformation parameters, therefore a position obtained using particular GPS equipment will be different from the one obtained using another type of GPS equipment after transformation. It was observed from the review of the previous research that no appropriate transformation parameters for the Birnin Kebbi Geodetic datum have been determined to relate the GPS results to a local datum.

This inherent problem therefore implies a need for appropriate transformation model to transform the Birnin Kebbi datum coordinates to WGS84, and this is what this study has tried to achieve.

Aim

The aim of this research is to generate transformation parameters for Birnin Kebbi, the state capital of Kebbi state.

Objectives of the Research

The objectives of this research are:

- To obtain local coordinate of points within the study area
- Observation to some of the points within the boundary of the study area
- To determine an appropriate functional model for the estimation of transformation parameters.
- To write computer program to transform(using visual basic)

Limitation

Coordinate transformation is wide in context and as a research topic; this project will be limited to Birnin Kebbi Local Government which may be extended to other parts by other researcher. The results of this project are therefore based on the limited number of available data from the existing geodetic controls in Birnin Kebbi local Government as at the period of this research.

Study area

Kebbi state is a city located in northwestern Nigeria geopolitical zone with its headquarters at Birnin Kebbi, an ancient town dating back to the 14th century. The state is divided into four emirate councils (Gwandu, Argungu, Yauri and Zuru) and twenty one local government areas which (include Alieru, Arewa, Argungu, Augie, Bagudo, Bunza, Dandi Kamba, Danko/Wasagu, Fakai, Gwandu, Jega, Kalgo, Koko-Besse, Maiyama, Ngaski, Sakaba, Shanga, Suru, Yauri and Zuru. (Gazetteer" 2007) Birnin Kebbi is a city located on latitude 12^0 07 37'' - $12^{\circ}27'13''$ & longitude 04^0 07' 56'' - $4^{\circ}12'01''$ (WGS84) northwestern Nigeria. It is located along Sokoto River and is connected by road to Argungu (45 km northeast), Jega (35 km southeast), and Bunza (45

km southwest). As of 2007 the city has an estimated population of 115,547 people which consists of farmer, civil servants, Businessmen and Students. See (tageo 2012`).

Methodology

This aspect of the project deals with the various steps involved in the transformation of the coordinates as explained below:

Data Acquisition

The data used for this project were obtained first from the Office of the Surveyor General of the state (OSGOS) and secondly the use of Differential GPS (Promark 3). These coordinates consist of the geodetic (curvilinear) coordinates coincident points on both Birnin Kebbi datum and WGS84.

	WGS84 Datum coordinates				
points	Northing (m)	Easting (m)			
Csk 14s	1376637.225	633762.753			
Csk 15s	1376442.954	633300.362			
Csk 23s	1374219.395	630027.475			
Css 749p	1378436.352	630966.666			
Css 1502s	1378820.628	631392.697			
Css 1507s	1378458.025	632688.977			

Table 1: The data obtained using Differential GPS

 Table 2 below shows some of the data obtained from the (OSGOKS)

Birnin Kebbi datum coordinates			
point	Northing (m)	Easting (m)	
Css749p	15240.000	15240.000	
Css750p	14966.832	15264.384	
Css759p	15312.323	13069.129	
Css1471s	14323.233	15022.233	
Css1472s	13910.296	14906.237	
Css1499s	16349.360	14981.609	
Css1500s	15870.202	14994.424	
Css1501s	15698.756	15158.820	
Css1502s	15617.677	15669.322	
Css1503s	15781.688	15983.678	
Css1504s	15806.456	16403.239	
Csk006s	15079.326	20980.901	
Csk007s	14665.604	21112.399	
Csk011s	13972.531	19396.270	
Csk014s	13412.217	18004.271	
Csk015s	13224.355	17540.956	
Csk016s	12883.193	17170.276	
Csk021s	11621.294	15131.260	
Csk023s	11048.965	14251.500	

From the office of the Surveyor General of the state.

Instrument use and Method of data collection

The instrument used in this Research work for data collection was differential GPS (Promark 3). The base station was set on a known point and leveled, the instrument heights was measured and inputted into the instrument. Observation was made on static mode allowing the instrument at least 45 minutes to track down enough satellite. The data collected was downloaded in Rinex form into the GNSS solution software and processed to obtain the most probable value of the coordinates in question.

Data Analysis

The 2D affine equations are:

E = A1* X + A2* Y + A0N = B1* X + B2* Y + B0 The affine parameters are:

A0 = tx - translation in the east-west direction of the local coordinate system

B0 = ty -- translation in the north-south direction of the local coordinate system

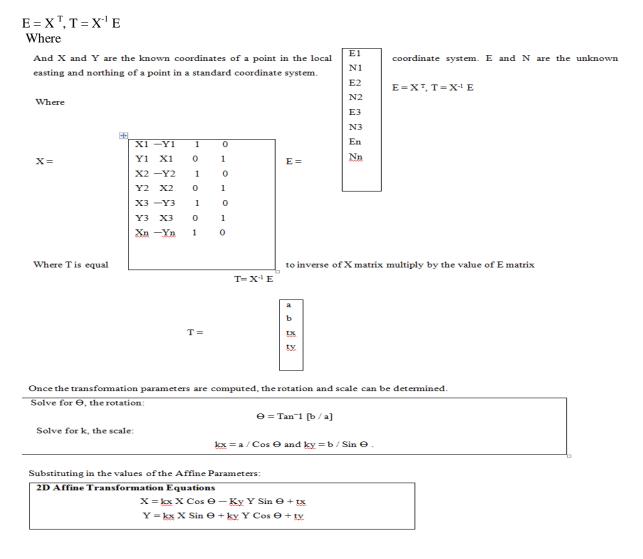
A1 = a = kx Cos Θ -- where kE is the scale in the east-west direction.

 $A2 = -b = ky Sin \Theta$ -- where kN is the scale in the north-south direction

B1 = b =kx Sin Θ –where Θ is the rotation about the origin of the local coordinate system to make it parallel with the standard system.

B2 = a = ky Cos Θ (Heiskanen & Moritz, 1967; Hofmann-Wellenhof & Moritz, 2005).

And X and Y are the known coordinates of a point in the local coordinate system. E and N are the unknown easting and northing of a point in a standard coordinate system.



Then computer programmed can be written to transform any local coordinate in this datum within that area to UTM coordinate in WGS84 datum using the transformation equation above. (James R Smith, 1997)

Table 3	J			
Location	Birnin Kebbi datum coordinates		WGS84 datum co	ordinates
POINT	Northing (m)	Easting	Northing (m)	Easting (m)
CSK 14s	13412.217	18004.271	1376637.225	633762.753
CSS 1507s	15240.177	16956.761	1378458.025	632688.977
CSS 749p	15240.000	15240.000	1378436.352	630966.666
CSS 1502s	15669.322	15617.677	1378820.628	631392.697
CSK 23s	11048.965	14251.500	1374219.395	630027.475

Data Use for Project Analysis

T =	(X ⁻ 1))*(E)
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	33762.753 376637.224 32688.977 378458.025
$\mathbf{X} = \begin{bmatrix} 15240.177 & 16956.761 & 0 & 1 \end{bmatrix} \begin{bmatrix} - & - & - & - & - & - & - & - & - & -$	30966.666
	378436.352
15240.000 15240.000 0 1 6	31392.697
15669.332 -15617.677 1 0 13	378820.628
15617.677 15669.322 0 1 6	30027.475
So 14251.500 -11048.965 1 0 13	374219.395
11048.965 14251.500 0 1	

a = 1.00323
a = 1.00323 b = 0.01253
tE = 615868.428 tN = 1362956.129
tN = 1362956.129

Solve for Θ , the rotation: $\Theta = \text{Tan}^{-1}$ [b/a $\Theta = \text{Tan}^{-1}$ (0.01253/1.00323) $\Theta = 00^{\circ}42^{\circ}56.04^{\circ\circ}$ Solve for k, the scale: kx = a / Cos Θ and ky = b / Sin Θ . K = 1.00323/ Cos 00° 42 ' 56.04" K = 1.003308244

T =

Table 4

Points	Local Coordin	ates	UTM Coordinates'		Transformation Coordinates		Residual	
							DN(m)	DE
Location	Northing	Easting	Northing	Easting	Northing	Easting		(m)
Csk 14s	13412.217	18004.271	1376637.225	633762.753	1376637.261	633762.798	± 0.036	±0.04
								5
Csk 15s	13224.355	17540.956	1376442.965	633300.362	1376442.987	633300.340	±0.022	$\begin{array}{c}\pm0.02\\2\end{array}$
Css 1507s	15240.177	16956.761	1378458.025	632688.977	1378458.000	632688.999	±0.025	$\frac{\pm 0.02}{2}$
<u>Css</u> 749p	15240.000	15240.000	1378436.352	630966.666	1378436.312	630966.696	± 0.040	±0.03 0
Css 1502s	15617.677	15669.322	1378820.628	631392.697	1378820.588	631392.672	± 0.040	±0.02 5
Csk 23s	11048.965	14251.500	1374219.395	630027.475	1374219.353	630027.517	±0.042	$\frac{\pm 0.04}{2}$

Table 4 shown the data used to generate the transformation parameters (both local and UTM coordinates) and the corresponding transformed coordinates with their residual

 Table 5 Shows some of the Birnin Kebbi Local Datum Coordinates Transformed to WGS84 Datum Coordinates

POINTS	NORTHING	EASTING	
Css 748p	1378270.136	630583.582	
Css 749p	1378436.311	630966.696	
Css 750p	1378162.567	630994.582	
Css 1501s	1378895.532	630879.506	
Css 1502s	1378820.588	631392.674	
Css 1503s	1378989.067	631705.989	
Css 1504s	1379019.172	632126.595	
Css1505s	1378957.670	632401.095	
Css 1506s	1378875.964	632720.976	
Css 1507s	1378458.000	632688.000	

Csk 13s	1376830.310	634225.688
Csk 14s	1376637.261	633762.798
Csk 23s	1374219.353	630027.517
Csk 26s	1364324.692	628561.371

Statistical Analysis

Compute the Standard Deviations for the Transformation Parameters

(1)- Compute the easting and northing residuals for the five pairs of points.

R =(ET-E). Where

E = The obtained five controls coordinates using GPS Promark (processes Data)

ET = Is the corresponding Transformed coordinate using the Transformation par

R = is the differences between the value of ET and E (Residual)

parameters and

ET =	633762.798	7	633762.753		0.045
	1376637.261	-	1376637.225		0.036
	633300.340	-	633300.362		-0.022
	1376442.987	-	1376442.965	-	0.022
	632688.999	-	632688.977		0.022
	1378458.000	E =	1378458.025	R =	-0.025
	630966.696	-	630966.666	_	0.030
	1378436.312	-	1378436.352		-0.040
	631392.672		631392.697		-0.025
	1378820.588	-	1378820.628		-0.040

 $\mathbf{R}^{\mathrm{T}} =$

Where

" \underline{r} " = the redundancy: the number of (equation minus unknown). In this case of five pair of points, there would be ten equations

and four unknown transformation parameters.

S0 "r" =(10-4) "r" = 06 So² = 0.01/06(3)- Compute the covariance matrix (C^T): $So^2 = 0.0016666666$ $CT = So^2 (X^{T*} X)^{-1}$

X matrix =

	18004.271 -13412.217 1	
	13412.217 18004.217 0	1
$(X^{T*} X)^{-1} =$	16956.761 -15240.177 1	
	15240.177 16956.761 0	1
	15240.000 -15240.000 1)
	15240.000 15240.000 0	1
	15669.332 -15617.677 1)
	15617.677 15669.322 0	1
	14251.500 -11048.965 1	D
	11048.965 14251.500 0	1
$(X^{T*} X)^{-1} =$	0.000 0.000 -0.001 -0.001	
	0.000 0.000 0.001 -0.001	
	-0.001 0.001 17.904 0.000	
	-0.001 -0.001 0.000 17.904	
$C^{T} = So^{2} (X^{T*} X)^{-1}$		
	•	
	0.000 0.000 -0.001 -0.001	
C^{T}	0.000 0.000 0.001 -0.001	X= 0.001666666
	-0.001 0.001 17.904 0.000	
	-0.001 -0.001 0.000 17.904	
	n.	

 $Sa^2 = 0.000$, and $Sb^2 = 0.000$

StE² =0.029839988, and StN² =0.029839988

So

 $Sa^2 = Sb^2 = 0.000m$ and $StE^2 = StN^2 = 0.029839988m$ $StE^2 = StN^2 = 0.0979ft^2$ Where Sa = standard deviation of "a"

Sb = standard deviation of "b" StE = standard deviation of "tE"

StN = standard deviation of "tN"

«»- Compute the standard deviation of the east and north translation:

 $StE = StN = \sqrt{0.0979 ft^2}$

 $StE = StN = 0.312889756ft^2$

StE = StN = 0.09537m

«»-Compute the standard deviation of the rotation:

- $S\Theta^2 = Sa^2 / (a^2 + b^2) (radians)^2$
- $S\Theta^2 = (1.00323)^2 + (0.01253)^2$
- $S\Theta^2 = 0.000 (radians)^2$
- $S\Theta = \sqrt{S\Theta^2}$ radians
- $S\Theta = \sqrt{0.000}$

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S\Theta = 0.000 radian
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S $\Theta = 180/\pi (\sqrt{S}\Theta^2)$ (degrees) S $\Theta = 180/\pi (\sqrt{0.000})$ S $\Theta = 0.000^{\circ}$ (degrees)

 $S\Theta = 10800/\pi (\sqrt{S\Theta})$ minutes

- $S\Theta = 10800/\pi (\sqrt{0.000})$
- $S\Theta = 0.000'$ (minutes)

 $S\Theta = 648000/\pi (\sqrt{S\Theta})$ second $S\Theta = 648000/\pi (\sqrt{0.000})$ $S\Theta = 0.000^{\circ\circ}$ second «»-Compute the standard deviation of the scale "Sk"

"K" = $\sqrt{a^2+b^2} = \sqrt{(1.00323)^2+(0.01253)^2}$ "K" = 1.00662743 Because Sa² = Sb²: Sk² = Sa² = Sb² Sk = $\sqrt{0.000}$ Sk = 0.000m

Compute the standard deviations for the transformed points

From the error propagation of the transformation parameters, compute the standard deviation of the transformed point locations:

 $SX = SY = \sqrt{Sa^2(E^2+N^2)} + StE^2$ Standard deviation for point (cks 14s) $SX = SY(csk14s) = \sqrt{0.000} x ((1376637.261)^2 + (633762.798)^2) + 0.029839988$ $SX = SY(csk14s) = \pm 0.172742548$ Standard deviation for point (cks 15s): $SX = SY(csk15s) = \sqrt{0.000} x ((1376442.987)^2 + (633300.340)^2) + 0.029839988$ $SX = SY = \pm 0.172742548$ Compute standard deviation for point (css1507s): $SX = SY(css1507s) = \sqrt{0.000} x ((1378458.000)^2 + (632688999)^2) + 0.029839988$ $SX = SY(css1507s) = \pm 0.172742548$ Compute standard deviation for point (css749ps): $SX = SY(css749p) = \sqrt{0.000} \times ((1378436.312)^2 + (630966.696)^2) + 0.029839988$ $SX = SY(css749p) = \pm 0.172742548$ Compute standard deviation for point (css1502s): $SX = SY(css1502s) = \sqrt{0.000} x ((1378820.588)^2 + (631392.672)^2) + 0.029839988$ $SX = SY(css1502s) = \pm 0.172742548$ Compute standard deviation for point (css749ps): $SX = SY(csk23s) = \sqrt{0.000} \times ((1374219.353)^2 + (630027.517)^2) + 0.029839988$ $SX = SY(csk23s) = \pm 0.172742548$

Summary

Local Geodetic datum has been developed in the past, to satisfy the surveying and mapping requirement of countries all over the earth. The broad used of GPS observations as part of the surveying routine, has shifted the interest from the local to the world Geodetic systems. The transformation of coordinates between geodetic systems as always been of interest but the new needs have it more important. Out all the method of transformation the two dimensional (2D) affine transformation has been used for this project to generate transformation parameters from Birnin kebbi local datum to WGS84 datum with the used of five parameters. The result obtained and analyses are as shown in chapter four, section4.3 above.

Conclusion

There are a number of ways to mathematically transform coordinate from one datum to another. The approach used in this Research is affine transformation procedure using mathematical models described by Heiskanen & Moritz in 1967. The observation equation method of least squares was applied to estimate the transformation parameters using the available common points in WGS84 datum and Birnin Kebbi datum. The estimates derived for each set of transformation parameters were later used to transform geodetic coordinates referred to Birnin Kebbi datum into coordinates referred to WGS84 ellipsoid. The results obtained from the transformation were compared with that obtained from the GPS Promark (Processes data) and the results were as presented in table4 above.

0	0	-0.000001666	-0.000001666
0	0	0.000001666	-0.000001666
-0.000001666	0.000001666	0.029839988	0
-0.000001666	-0.000001666	0	0.029839988

Recommendation

There is need for additional GPS observation points, well distributed over the Birnin Kebbi geodetic network to ensure a reasonable coverage of the geodetic network with coincident points. This will be needed for further studies in order to arrive at a unique choice of appropriate sets of transformation parameters for the Birnin Kebbi geodetic network. It is also recommended that another method of transformation be employed and result compared with that obtained from the affine transformation.

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