

Modeling Box-Jenkins Methodology on Retail Prices of Rice in Nigeria

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-----ABSTRACT-----

This research work mainly focuses on the application of principles of George Box and Gwilym Jenkins to estimate the appropriate models that can be used for forecasting retail prices of imported and local rice in Nigeria. The Dickey Fuller test was carried out to confirm if the series is stationary or non stationary. From the results obtained, conclusion was made that, series for both rice are non stationary, this will lead us to differencing of the data in estimating the Autoregressive Integrated Moving-Average (ARIMA) models. Usually, first order differencing is always recommended in order to obtain the appropriate ARIMA model. An attempt was made in identifying the respective models with the aids of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots, possible ARIMA models were estimated based on the description of the ACF and PACF plot. The model with the least Mean Square Error (MSE) value is chosen as the best model for both imported and local rice which is ARIMA (2,1,1). Models estimations for imported and local rice have the same number of parameters, this shows that prices of both rice exhibits similar pattern.

KEYWORDS: Stationary, Autocorrelation, Autoregressive, Moving-Average, and Forecasting

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I. INTRODUCTION

Rice is a major staple food for about 2.6 billion people in the world (Spore, 2005 and De Datta, 1981) and it is the fastest growing commodity in Nigeria's food basket. The production of rice rose from 2.5 million tonnes in 1990 to about 4 million tonnes in 2008, representing about 37 percent rise in domestic production (FOSTAT (2010), FAO (2007)). However, despite the numerous government policies and programmes on rice and rise in domestic production, the demand and consumption of this commodity exceeds the local production resulting in rice importation. In the past three decades, rice has become one of the Nigeria's most important foods Alam (1991), Mijindadi and Njoku (1985), Singh et al. (1997). The Nigerian rice sector has seen some remarkable developments over the last quarter-century. Both rice production and consumption in Nigeria have vastly increased during the aforementioned period. Notwithstanding, the production increase was insufficient to match the consumption increase with rice imports making up the shortfall. With rice now being a structural component of the Nigerian diet and rice imports making up an important share of Nigerian agricultural imports, there is considerable political interest in increasing the consumption of local rice. This has made rice a highly political commodity in Nigeria. However, past policies have not been successful in securing the market share for local rice, Adeniyi (1978).

The present study tries to address this information gap through a survey of imported and local rice retailers. Amongst the stakeholders consulted, it is generally agreed that one of the major constraints that affect the development of Nigerian rice sector is the inability of the local rice to match the quality of imports. Consumers are the ultimate and foremost deciders when it comes to select between different types of goods. The quality differential between local and imported rice thereby seems an important consideration in the decision making process. Price is of course also an important determinant, but it is only one factor among a wider range of attributes that characterize the product. Indeed, imported rice consumption in Nigeria is still increasing rapidly in spite of a heavy custom duty implying a higher price on the market compared to local rice Akinsola (1985). In order to know the pattern in which the retail prices of imported and local rice are being sold in Nigeria, some techniques of time series analysis were employed. The analysis of time series data is based on the assumption that the successive values in the data file represent consecutive measurement taken at equally spaced time intervals.

For the purpose of this research work, data on monthly retail prices for imported and local rice in Nigeria were collected over a period of six years which made up of seventy-two (72) months. This research paper is mainly concerned with the application of time series techniques (Box Jenkins methodology) to estimate the appropriate models that can be used for forecasting retail prices of imported and local rice in Nigeria based on the past observations.

II. METHODOLOGY

In time series analysis, the **Box-Jenkins** methodology, named after the statisticians George Box and Gwilym Jenkins, applies Autoregressive Moving Average (ARMA) or Autoregressive Integrated Moving Average ARIMA models to find the best fit of a time series to past values of this time series, in order to make forecasts. Box-Jenkins represents a powerful methodology that addresses trend and seasonality well, see George et al. (1994). ARIMA models have a strong theoretical foundation and can closely approximate any stationary process. The process consists of model identification by using autocorrelation functions, evaluation by assessing the fit of the possible models and forecasting using the best model, see Chatfield (1984), Brockwell and Richard (1987), Hurvich and Tsai (1989).

The original method used an iterative three stage modeling approach, they are defined below:

1. **Model identification and model selection:** making sure that the variables are stationary, identifying seasonality in the dependent series (seasonally differencing it if necessary), and using plots of the ACF and PACF of the dependent time series to decide which (if any) autoregressive or moving average component should be used in the model.
2. **Parameter estimation:** using computation algorithms to arrive at coefficients which best fit the selected ARIMA model. The most common methods is Maximum likelihood estimation or non-linear least-squares estimation.
3. **Model checking:** by testing whether the estimated model conforms to the specifications of a stationary univariate process. In particular, the residuals should be independent of each other and constant in mean and variance over time. (Plotting the mean and variance of residuals over time and performing a “Ljung-Box test” or plotting autocorrelation and partial autocorrelation of the residuals are helpful to identify misspecification.) If the estimation is inadequate, we have to return to step one.

The first step in developing a Box-Jenkins model is to determine if the time series is stationary and if there is any significant seasonality that needs to be modeled. Stationarity can be detected from an autocorrelation plot. Specifically, non-stationarity is often indicated by an autocorrelation plot with very slow decay, see Delurgio (1998), and George et al. (1994). Also seasonality (periodicity) can usually be assessed from an autocorrelation plot. Box and Jenkins recommend the differencing approach to achieve stationarity. At the model identification stage, the goal is to detect seasonality, if it exists, and to identify the order for the seasonal autoregressive and seasonal moving average terms. For many series, the period is known and a single seasonality term is sufficient. For example, for monthly data one would typically include either a seasonal AR 12 term or a seasonal MA 12 term. For Box-Jenkins models, one does not explicitly remove seasonality before fitting the model. Instead, one includes the order of the seasonal terms in the model specification to the ARIMA estimation software. However, it may be helpful to apply a seasonal difference to the data and regenerate the autocorrelation and partial autocorrelation plots. This may help in the model identification of the non-seasonal component of the model, see Harris and Robert (2003).

Once stationarity and seasonality have been addressed, the next step is to identify the order p and q of the autoregressive and moving average terms. The primary tools for doing this are the autocorrelation plot and the partial autocorrelation plot. The sample autocorrelation plot and the sample partial autocorrelation plot are compared to the theoretical behavior of these plots when the order is known, see Delurgio (1998), and George et al. (1994). The followings are the guidelines for choosing order p and q :

1. The ACF has spikes at lags 1, 2, ..., r and cuts off after lag r and also the PACF dies down; use $q=r$ and $p=0$.
2. The ACF dies down and the PACF has spikes at lags 1, 2, ..., r and cuts off after lag r ; use $q=0$ and $p=r$.
3. The ACF has spikes at lag 1, 2, ..., r and cuts off after lag r and also the PACF has spikes at lag 1, 2, ..., s and cuts off after lag s ; use $q=r$ and $p=s$.

4. The ACF contains small autocorrelations at all lags and the PACF contains small autocorrelations at all lags; use $q=0$ and $p=0$.
5. The ACF dies down and the PACF dies down; use $p=1$ and $q=1$.

Estimating the parameters for the Box–Jenkins models is a quite complicated non-linear estimation problem. For this reason, the parameter estimation should be left to a high quality software program that fits Box–Jenkins models. The main approaches to fitting Box–Jenkins models are non-linear least squares and maximum likelihood estimation (MLE). The MLE is generally the preferred technique, see George et al. (1994). Model diagnostics for Box–Jenkins models is similar to model validation for non-linear least squares fitting. That is, the error term A_t is assumed to follow the assumptions for a stationary univariate process. The residuals should be white noise drawings from a fixed distribution with a constant mean and variance. If the Box–Jenkins model is a good model for the data, the residuals should satisfy these assumptions. If these assumptions are not satisfied, one needs to fit a more appropriate model. That is, go back to the model identification step and try to develop a better model. One way to assess if the residuals from the Box–Jenkins model follow the assumptions is to generate statistical graphics (autocorrelation plots) of the residuals, see George et al. (1994) and Harris and Robert (2003).

Empirical Illustration Using Imported and Local Rice Data

To analyze the data for imported rice, the plot that displayed the features (pattern) in which the retail prices of imported rice occurs in Nigeria is shown below, the prices are expressed in naira (#) per one kilogram.

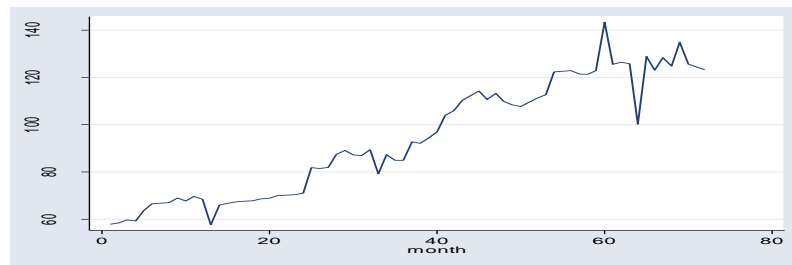


Figure 1: Time plot for the retail price of imported rice

From the time plot, we could easily observe that, the retail prices of imported rice rises gradually from January, 2001 and then dropped in the month of October of the same year. We also observe a sudden rise in the month of December, 2005 and there was a sudden drop in the month of April, 2006. The plot also notified the presence of upward trend in the series. Therefore, differencing will be necessary so as to obtain stationarity.

Test for Autocorrelation

In order to check if the errors are autoregressive in nature, the Durbin Watson test for autocorrelation was performed. The hypothesis is stated as: H_0 : no serial correlation vs H_1 : there is serial correlation.

Table 1: Regression table

Source	SS	df	MS	Number of obs = 72		
Model	28771.8523	1	28771.8523	F(1, 70) = 865.82	Prob > F = 0.0000	
Residual	2326.14773	70	33.2306818	R-squared = 0.9252	Adj R-squared = 0.9241	
Total	31098	71	438	Root MSE = 5.7646		
month	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
imported	.8214094	.0279155	29.42	0.000	.7657337	.8770851
_cons	-40.87836	2.716032	-15.05	0.000	-46.29532	-35.46141

Durbin-Watson d-statistic (2, 72) = 0.9515258

Decision: d_{cal} (0.9515) is less than d_{tab} (1.57), since d is substantially less than 2, we reject H_0 and conclude that there is an evidence of positive autocorrelation.

Test for stationarity

The Dickey Fuller test is used to check for stationarity in series for retail prices of imported rice. The hypothesis statement is stated below;

$$H_0 : \phi_1 = 1$$

$$H_A : \phi_1 < 1$$

Decision Rule: If the test statistics Z(t) is less than the critical value (usually 10% CV), we reject Ho and conclude that the series is stationary.

Table 2 Dickey-Fuller Test

Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-1.352	-3.551	-2.592

Decision: Since -1.352 > -2.592, we do not reject the hypothesis and we then conclude that there is a unit root in the series.

In order to fit the appropriate models, one does not explicitly remove seasonality and trend before fitting the model. Instead, one includes the order of the seasonal terms in the model specification to the ARIMA estimation software. Box and Jenkins recommend the differencing approach to achieve stationarity. The first order differencing ($d=1$) will be preferred when fitting the ARIMA models.

Estimation of ARIMA Models

Before we can estimate ARIMA models, we first identify the order of the models with the aids of autocorrelation function (ACF) and partial autocorrelation functions (PACF). The ACF and PACF plots are displayed below;

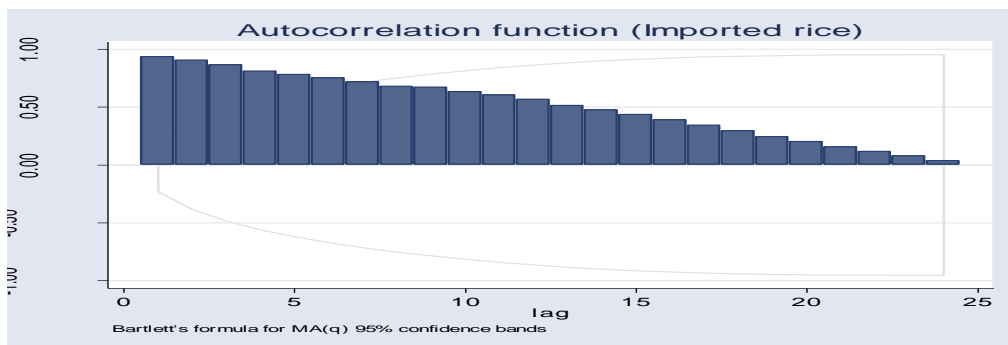


Figure 2 Autocorrelation function plot

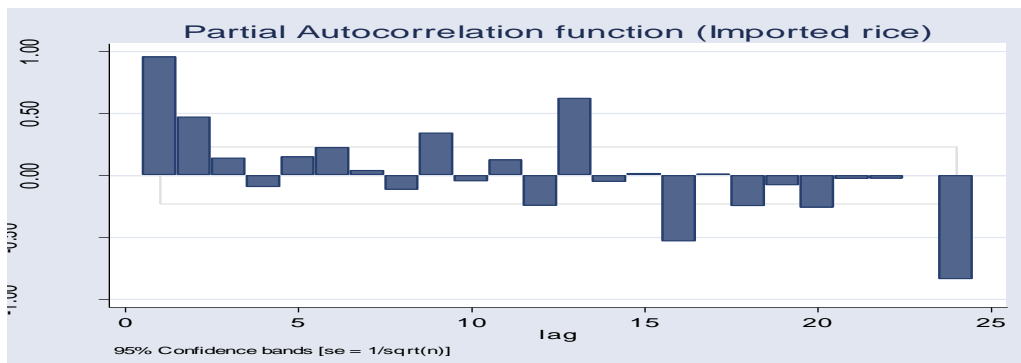


Figure 3 Partial Autocorrelation function plot

From figure 2, we observe that the autocorrelation function plot exhibit exponential decay. This indicates that the order of Autoregressive model can be identified by using the Partial Autocorrelation function plot. From figure 3, the PACF has significant spikes at lag 1 and lag 2, this suggests fitting of AR (2) and alternative model of MA (1). Also, from the time plot (figure 1), we noticed an upward trend in the series, this call for fitting of mixed model with differencing of order one, ARIMA (2,1,1)

Table 3 Estimation of AR (2) Model

Number of obs = 72 Log likelihood = -128.3527

month	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
imported	.1145121	.0278579	4.11	0.000	.0599116	.1691126
_cons	54.79285	23.61363	0.75	0.451	48.48901	66.07472
AR:	L1 .8620397	.0849272	10.15	0.000	.6955854	1.028494
	L2 .1366306	.0851152	1.61	0.108	-.0301921	.3034534

The fitted autoregressive model of order 2 is given as:

$$\hat{X}_t = 54.7929 + 0.8620X_{t-1} + 0.1366X_{t-2}$$

Table 4: Estimation of MA (1) model

Number of obs = 72 Log likelihood = -220.8093

month	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
imported	.7857546	.0312593	25.14	0.000	.7244874	.8470218
_cons	47.31309	3.396736	-10.98	0.000	43.97057	63.65561
MA L1	.3692167	.1038579	3.56	0.000	.165659	.5727745

The fitted MA (1) model can be estimated as:

$$\hat{X}_t = 47.3131 + 0.3692a_{t-1}$$

TABLE 5: Estimation of ARMA (2,1) model

Number of obs = 72 Log likelihood = -158.2870

month	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
imported	.2931712	.0461779	6.35	0.000	.2026641	.3836783
_cons	59.543362	22.18396	0.43	0.667	33.9364	73.02312
AR:	L1 .6109667	.2496642	2.45	0.014	.1216338	1.1003
	L2 .3834428	.2452636	1.56	0.118	-.0972649	.8641506
MA:	L1 -.1130025	.2568081	-0.44	0.660	-.616337	.3903321

The fitted ARMA (2,1) model can be given as:

$$\hat{X}_t = 59.5434 + 0.6110X_{t-1} + 0.3834X_{t-2} - 0.1130025a_{t-1}$$

Table 6: Estimation of ARIMA (2,1,1) model

Number of obs = 71 Log likelihood = -225.4229

D.imported	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
_cons	.8786367	.4729777	1.86	0.063	-.0483826	1.805656
AR:	L1 -1.474065	.1839555	-8.01	0.000	-1.834611	-1.113518
	L2 -.5787382	.0902008	-6.42	0.000	-.7555285	-.401948
MA:	L1 1.032651	.2629669	3.93	0.000	.5172454	1.548057

The fitted ARIMA (2,1,1) model can be estimated as:

$$\hat{X}_t = 0.8786 - 1.4741X_{t-1} - 0.5787X_{t-2} + 1.0327a_{t-1}$$

In order to choose the best model, the Mean Square Error (MSE) Criterion for model selection is adopted. The preferred model is the one with the minimum MSE value. This can be simply expressed as: $MSE = SSE/DF$ Where SSE is the sum of square errors and DF means the degree of freedom. DF can be expressed as: $n-p$. Where n is the number of observations and p is the number of parameters in the model. The comparison of the results is displayed in the table below:

Table 7: Comparison of the Estimated Models

Model	MSE
AR (2)	65.62
MA (1)	264.7
ARMA (2,1)	43.11
ARIMA (2,1,1)	36.01

From the table above, we observe that, ARIMA (2,1,1) has the least MSE value, this model is chosen as the best.

Analysis for the Prices of Local Rice

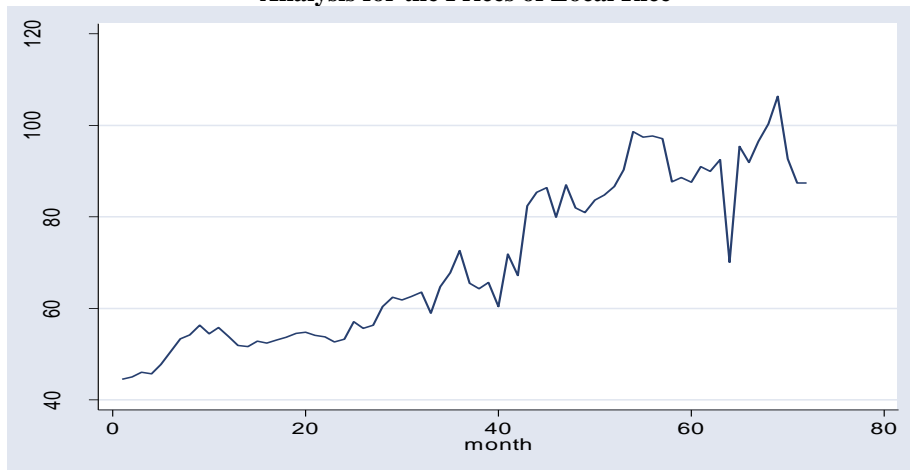


Figure 4: Time plot for the retail price of local rice

From the time plot, we can easily observe that, the retail prices of local rice rises and drops with a gradual process from the first month. The plot also notified the presence of upward trend in the series.

4.7 TEST FOR AUTOCORRELATION

Table 8: Regression table on Local Rice

Source	SS	df	MS	Number of obs = 72		
-----				F(1, 70) = 473.96		
Model	27096.1233	1	27096.1233	Prob > F = 0.0000		
Residual	4001.87668	70	57.1696669	R-squared = 0.8713		
-----				Adj R-squared = 0.8695		
Total	31098	71	438	Root MSE = 7.5611		

month	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	

local	1.102371	.0506357	21.77	0.000	1.001381	1.203361
_cons	-41.12178	3.675101	-11.19	0.000	-48.45154	-33.79202

Durbin-Watson d-statistic (2, 72) = 0.7265975

Decision: Since d (0.7266) is less than the critical value ($d_{L,\alpha} = 1.57$), we reject H_0 and conclude that the error terms are autoregressive in nature.

Test for Stationarity

Table 9 Dickey-Fuller Test

Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-3.551	-2.913	-2.592

Decision: Since $-1.622 > -2.592$, we do not reject the hypothesis and we then conclude that there is a unit root in the series.

Estimation of ARIMA Models for Local Rice

The plots for the calculated ACF and PACF values are displayed below for the purpose of model identification.

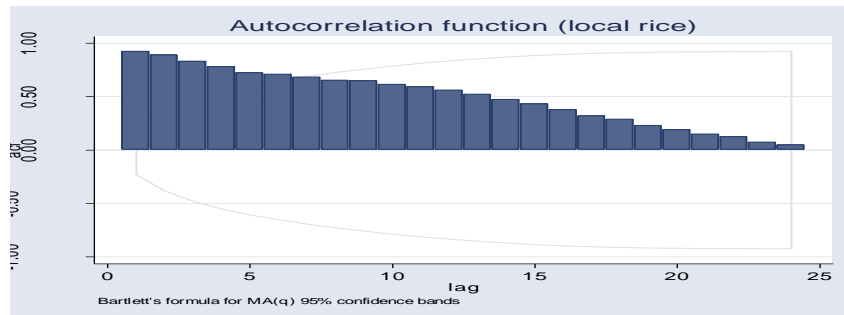


Figure 5: Autocorrelation function

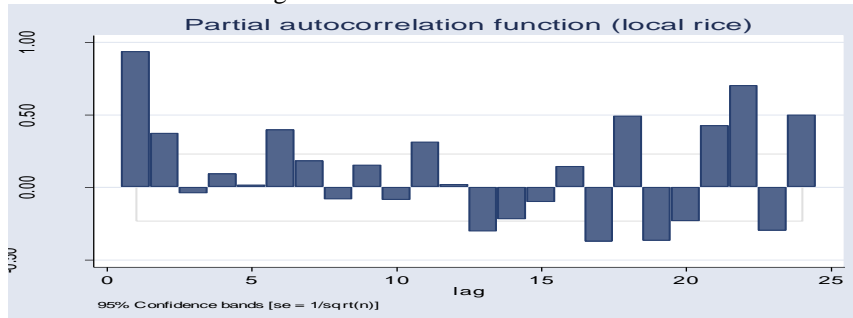


Figure 6: Partial Autocorrelation function

The ACF plot in figure 4.8.1, exhibits exponential decay. The order of Autoregressive model can be identified by the plot Partial Autocorrelation function. Both figures suggest fitting of AR (2), MA (1) and

ARIMA (2,1).

Table 10: Estimation of AR (2) model

Number of obs	=	72	Log likelihood	=	-132.2155		
month		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
local		.1736396	.0414548	4.19	0.000	.0923896 .2548895	
_cons		48.132846	14.42854	0.56	0.573	20.14657 56.41226	
AR	L1		.8220907	.1351067	6.08	0.000	.5572865 1.086895
	L2		.174355	.1406584	1.24	0.215	-.1013303 .4500403

The fitted AR (2) model is given as:

$$\hat{X}_t = 48.1328 + 0.8221X_{t-1} + 0.1744X_{t-2}$$

TABLE 11: Estimation of MA (1) model

Number of obs = 72 Log likelihood = -236.8481

month		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
local		1.023605	.0655703	15.61	0.000	.8950894	1.15212
_cons		35.33241	4.900818	-7.21	0.000	25.72698	48.75634
MA	L1	.4193208	.1013462	4.14	0.000	.2206858	.6179558

The fitted MA (1) model can be estimated as:

$$\hat{X}_t = 35.3324 + 0.4193a_{t-1}$$

TABLE 12: Estimation of ARMA (2,1) model

Number of obs = 72 Log likelihood = -172.428

month		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
local		.4189884	.0593169	7.06	0.000	.3027294	.5352473
_cons		38.02211	295.0786	0.23	0.818	21.3214	84.3656
AR	L1	.4357787	.5478283	0.80	0.426	-.6379451	1.509502
	L2	.5599945	.5496435	1.02	0.308	-.5172868	1.637276
MA	L1	.2019025	.5906723	0.34	0.732	-.9557939	1.359599
	sigma	2.866552	.3713929	7.72	0.000	2.138635	3.594468

The fitted ARMA (2,1) model can be estimated as:

$$\hat{X}_t = 38.0221 + 0.4358 X_{t-1} + 0.5560 X_{t-2} + 0.2019 a_{t-1}$$

TABLE 13: Estimation of ARIMA (2,1,1) model

Number of obs = 71 Log likelihood = -219.3419

D.local		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
_cons		.6161748	.5599209	1.10	0.271	-.4812499	1.7136
AR	L1	-1.361776	.3555533	-3.83	0.000	-2.058648	-.6649046
	L2	-.3706954	.2269425	-1.63	0.102	-.8154945	.0741037
MA	L1	.9471462	.3566943	2.66	0.008	.2480382	1.646254

$$\hat{X}_t = 0.6162 - 1.3618 X_{t-1} - 0.3707 X_{t-2} + 0.9471 a_{t-1}$$

From Table (14) below, we observe that, ARIMA (2,1,1) has the least MSE value. Therefore, this model is chosen as the best for forecasting retail prices of imported rice in Nigeria.

Table 14: Comparison Of The Estimated Models

Model	MSE
AR (2)	70.0
MA (1)	148.5
ARMA (2,1)	44.18
ARIMA (2,1,1)	35.14

III. CONCLUSION

Based on the outcome of the results of the analysis, conclusion can be made, since the aim of this work is to know the pattern of feature that is/are present in the retail prices of rice in Nigeria and also to estimate the

best model for the commodities. From the analysis of local rice, we observed from the time plot, the presence of upward trend before fitting the suggested models, with the use of MSE as the criterion for model selection. We then concluded that ARIMA (2,1,1)

$$\hat{X}_t = 0.8786 - 1.4741X_{t-1} - 0.5787X_{t-2} + 1.0327a_{t-1}$$

is the best model that can be used for forecasting retail prices of local rice in Nigeria.

Also, from the analysis of imported rice, we also observed from the time plot, the presence of upward trend but not as obvious as that of imported rice. Choosing the minimum value of MSE, conclusion is made that, ARIMA (2,1,1)

$$\hat{X}_t = 0.6162 - 1.3618X_{t-1} - 0.3707X_{t-2} + 0.9471a_{t-1}$$

is the best model that can be used for forecasting retail prices of imported rice in Nigeria.

In general, our finding shows that more of imported rice is consumed than the local rice in Nigeria, the reason is that, the upward trend in imported rice is higher compared to that of local rice.

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