The International Journal Of Engineering And Science (IJES)
||Volume||2 ||Issue|| 7 ||Pages|| 01-06||2013||
ISSN(e): 2319 – 1813 ISSN(p): 2319 – 1805

T-Homomorphism of Semispaces in Ternary Semigroups

[1,]Ch. Manikya Rao , [2,] D. Madhusudhana Rao , [3,]P. Koteswara Rao
[1,]*Department of Mathematics, Bapatla Arts & Sciences College,*
Bapatla, Guntur (Dt), A. P. India.
[2,]*Department of Mathematics, V. S. R & N. V. R. College, Tenali, A. P. India.*
[3,]*Acharya Nagarjuna University, Nagarjuna Nagar, Guntur, A.P. India.*

ABSTRACT

In this paper we introduce semispaces and characterize idempotents of rank 1 and exhibit a class of primitive idempotents of rank 1 in the ternary semigroup of all T-homomorphisms on a semispace. We obtain a characterization of minimal (one sided) ideals in ternary semigroups of T-homomorphisms of a semispace containing all T-homomorphisms of rank 1 and obtain equivalent conditions for the ternary semigroup of T-homomorphisms on a semispace, to be a ternary group.

Mathematical subject classification (2010) *: 20M07; 20M11; 20M12.*

KEY WORDS : *Ternary semigroup, T-system, fixed element, transitive, irreducible, T-homomorphism, semispace.*

Date of Submission: 23 July. 2013 Date of Publication: 07.Aug 2013

I. INTRODUCTION

The theory of ternary algebraic systems was introduced by LEHMER [4] in 1932,but earlier such structures was studied by KASNER [2] who give the idea of n-ary algebras. LEHMER[4] investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. Ternary semi-groups are universal algebras with one associative ternary operation. It is well known that if Li, i = 1, 2 are the rings of linear transformations of finite dimensional vector spaces Mi over division ring Di, then f is an isomorphism of L1 into L2 if and only if there exists a one to one semi linear transformation S of M1 into M2 such that Af = S-1AS holds for all A ∈ L1. This theorem was extended to near rings by RAMAKOTAIAH[5] by introducing the notion of a semi space (M, S) where M is a group and S, a set of endomorphism of M containing the zero endomorphism O of M satisfying (1) S\{O} (the complement of O in S) is a group of auto-orphisms' on M and (2) ms = mt for some 0 ≠ m ∈ M and s, t ∈ S implies s = t. ANJANEYULU[1] introduce S-semi spaces and obtain an isomorphism theorem of semi-group of S-homomorphism on semi spaces and deduce the well known LJAPIN's[3] theorem on the semi-group of transformations over a set. In this paper we introduce T-semi spaces and exhibit a class of primitive idempotents of rank 1 in the ternary semigroup of all T-homomorphisms on a semispace. We obtain a characterization of minimal (one sided) ideals in ternary semigroups of T-homomorphisms of a semispace containing all T-homomorphisms of rank 1 and obtain equivalent conditions for the ternary semigroup of T-homomorphisms on a semispace, to be a ternary group.

II. PRELIMINARIES :

DEFINITION 2.1 : Let T be a non-empty set. Then T is said to be a ***Ternary semigroup*** if there exist a mapping from T×T×T to T which maps $(x_1, x_2, x_3) \rightarrow [x_1 x_2 x_3]$ satisfying the condition : $[(x_1x_2x_3)x_4x_5] = [x_1(x_2x_3x_4)x_5] = [x_1x_2(x_3x_4x_5)] \ \forall \ x_i \in$ T, $1 \le i \le 5$.

NOTE 2.2 : For the convenience we write $x_1x_2x_3$ instead of $[x_1x_2x_3]$

NOTE 2.3 : Let T be a ternary semigroup. If A,B and C are three subsets of S , we shall denote the set ABC = $\{abc : a \in A, b \in B, c \in C\}$.

DEFINITION 2.4 : An element *a* of ternary semigroup T is said to be ***left identity*** of T provided *aat* = *t* for all *t*∈ T.

NOTE 2.5 : Left identity element a of a ternary semigroup T is also called as ***left unital element.***

DEFINITION 2.6 : An element a of a ternary semigroup T is said to be a ***lateral identity*** of T provided $ata = t$ for all $t \in$ T.

NOTE 2.7 : Lateral identity element a of a ternary semigroup T is also called as ***lateral unital element.***

DEFINITION 2.8: An element a of a ternary semigroup T is said to be a ***right identity*** of T provided $taa = t$ $\forall$ $t \in$ T.

NOTE 2.9 : Right identity element a of a ternary semigroup T is also called as ***right unital element.***

DEFINITION 2.10 : An element a of a ternary semigroup T is said to be a ***two sided identity*** of T provided $aat = taa = t \; \forall \; t \in$ T.

NOTE 2.11 : Two-sided identity element of a ternary semigroup T is also called as ***bi-unital element***.

DEFINITION 2.1: An element a of a ternary semigroup T is said to be an ***identity*** provided $aat = taa = ata = t \; \forall \; t \in$ T.

NOTE 2.13 : An identity element of a ternary semigroup T is also called as ***unital element.***

NOTE 2.14 : An element a of a ternary semi group T is an ***identity*** of T iff a is left identity , lateral identity and right identity of T.

DEFINITION 2.15 : An element a of a ternary semi group T is said to be an ***idempotent*** element provided $a^3 = a$.

DEFINITION 2.16 : An idempotent is said to be ***primitive idempotent*** if it is non-zero and is minimal in the set of non-zero idempotents.

NOTE 2.17 : A non-zero idempotent element e of a ternary semigroup T is primitive if for any non-zero idempotent f of T, the relation $eff = fef = ffe = f$ implies $e = f$.

THEOREM 2.18 : An idempotent element *e* is an identity of a ternary semi group T then it is unique.

DEFINITION 2.19 : A nonempty subset A of a ternary semigroup T is said to be ***left ternary ideal*** or ***left ideal*** of T if $b, c \in$ T, $a \in$ A implies $bca \in$ A.

NOTE 2.20 : A nonempty subset A of a ternary semigroup T is a left ideal of T if and only if TTA $\subseteq$ A.

DEFINITION 2.21 : A nonempty subset of a ternary semigroup T is said to be a ***lateral ternary ideal*** or simply ***lateral ideal*** of T if $b, c \in$ T , $a \in$ A implies $bac \in$ A.

NOTE 2.22 : A nonempty subset of A of a ternary semigroup T is a lateral ideal of T if and only if TAT $\subseteq$ A.

DEFINITION 2.23 : A nonempty subset A of a ternary semigroup T is a ***right ternary ideal*** or simply ***right ideal*** of T if $b, c \in$ T , $a \in$ A implies $abc \in$ A

NOTE 2.24 : A nonempty subset A of a ternary semigroup T is a right ideal of T if and only if ATT $\subseteq$ A.

DEFINITION 2.25 : A nonempty subset A of a ternary semi group T is said to be ***ternary ideal*** or simply an ***ideal*** of T if $b, c \in$ T , $a \in$ A implies $bca \in$ A, $bac \in$ A, $abc \in$ A.

DEFINITION 2.26 : An ideal A of a ternary semi group T is said to be a ***minimal ideal*** provided A is a proper ideal of T and is properly contained in any ideal of T.

DEFINITION 2.27 : A ternary semigroup T is said to be ***left cancellative*** if $abx = aby \Rightarrow x = y$ for all $a, b, x, y \in$ T.

DEFINITION 2.28 : A ternary semigroup T is said to be ***lateral cancellative*** if $axb = ayb \Rightarrow x = y$ for all $a, b, x, y \in$ T.

DEFINITION 2.29 : A ternary semigroup T is said to be ***right cancellative*** if $xab = yab \Rightarrow x = y$ for all $a, b, x, y \in$ T.

DEFINITION 2.30 : A ternary semigroup T is said to be ***cancellative*** if T is ***left, right*** and ***lateral cancellative***.

DEFINITION 2.31 : An element a of a ternary semigroup T is said to be ***invertible*** in T if there exists an element b in T such that $abx = bax = xab = xba = x$ for all $x \in$ T.

DEFINITION 2.32 : A ternary semigroup T is said to be a ***ternary group*** if for $a, b, c \in$ T, the equations $abx = c$, $axb =$ c and $xab = c$ have solutions in T.

3. SEMISPACES :

DEFINITION 3.1 : Let T be a ternary semigroup. A non empty set M is called a ***right operand or right*** **T-*system*** or simply an **T-*system*,** provided there exists a mapping $(m, n, s) \rightarrow mns$ of M × M × T → M such that $mn(stu) = m(nst)u = (mns)tu$ for all $m, n \in$ M and $s, t, u \in$ T. We denote a right T-*system* M by M_T.

DEFINITION 3.2 : Let T be a ternary semigroup. A non empty set M is called a ***lateral*** **T-*system*** provided there exists a mapping $(m, s, n) \rightarrow msn$ of M × T × M → M such that $m(stu)n = (mns)tu = mn(stu)$ for all $m, n \in$ M and $s, t, u \in$ T. We denote a lateral T-*system* M by $\underset{T}{M}$.

DEFINITION 3.3 : Let T be a ternary semigroup. A non empty set M is called a ***left*** **T-*system*** provided there exists a mapping $(m, n, s) \rightarrow mns$ of T × M × M → M such that $(stu)mn = s(tum)n = st(umn)$ for all $m, n \in$ M and $s, t, u \in$ T. We denote a left T-*system* M by ${}_TM$.

DEFINITION 3.4 : Let M_T be a right T-*system*. Then an element $x \in$ M is called a ***fixed element*** of M_T provided $mmt = m$ for all $t \in$ T.

NOTE 3.5 : If M_T is a right T-*system*. Then we denote the set FM = { $m \in$ M : $mmt = m$ for all $t \in$ T} and FM is read as the set of fixed(= invariant) elements of an operand M_T over a ternary semigroup T.

DEFINITION 3.6 : Let M_T be a right T-*system*. Then a nonempty subset N of M is called an T-*subsystem* of M_T provided NNT ⊆ N, that is for all $m, n \in$ N and $t \in$ T, $mnt \in$ N.

DEFINITION 3.7 : A right T-*System* M_T is said to be ***unital*** provided T contains 1 and $m.1.1 = m$ for all $m \in$M.

DEFINITION 3.8 : A right T-*System* M_T is said to be ***transitive*** provided for any $m, n, p \in$ M, there exists an $t \in$ T such that $mnt = p$.

DEFINITION 3.9 : A right T-*System* M_T is said to be ***irreducible*** provided MMS ⊄ FM and the only subsystem of M of cardinality greater than one is M itself.

THEOREM 3.10 : Let M_T be a right T-*System* with FM = ∅, that, M_T has no fixed elements. Then M_S is a transitive T-*System* if and only if M_T is an irreducible

III. T-SYSTEM.

Proof **:** Let M_T is a transitive T-*System*. Suppose if possible M_T is not irreducible. Then MMS ⊆ FM ⇒ for all $m \in$ M, $t \in$ T, $mmt = m$ and hence M_T is not transitive. We have the contradiction. Therefore MMS ⊄ FM implies that M_T is an irreducible.

Conversely suppose that M_T is an irreducible T-*System*. That is MMS ⊄ FM ⇒ for $m \in$ M, $mmt \neq m$ for all $t \in$ T ⇒ $m, n, p \in$ M, there exists an $t \in$ T such that $mnt = p$. Therefore M_T is a transitive T-*System*.

DEFINITION 3.11 : Let M_T and N_T be two right T-*Systems*. A mapping f : M → N is called an ***T-homomorphism*** from M_T into N_T provided $f(mnt) = f(m)nt$ for all $m \in$ M and $n, t \in$ T.

NOTE 3.12 : We denote the set of all T-homomorphisms form M_T into N_T by H_T (M, N) and the set of all T-homomorphisms form M_T into itself by $H_T(M)$ or simply H.

DEFINITION 3.13 : Let f : $M_T \rightarrow N_T$ be a T-homomorphism from the right T-*system* M_T into the right T-*system* N_T. Then we say that f is an ***T-monomorphism*** provided f is one one.

DEFINITION 3.14 : Let f : $M_T \rightarrow N_T$ be a T-homomorphism from the right T-*system* M_T into the right T-*system* N_T. Then we say that f is an ***T-epimorphism*** provided f is onto.

DEFINITION 3.15 : Let f : $M_T \rightarrow N_T$ be a T-homomorphism from the right T-*system* M_T into the right T-*system* N_T. Then we say that f is an ***T-isomorphism*** provided f is bijection.

DEFINITION 3.16 : An unital T-*system* M_T is said to be an ***T-semispace*** or simply a ***semispace*** provided T is a ternary group such that $mns = mnt$ for some $m, n \in$ M and $s, t \in$ T implies that $s = t$. We call T, a centralizer of M.

NOTE 3.17 : Let M_T be any semi-space. Then the transitive relation on M_T is an equivalence relation, which we call T-equivalence relation and the corresponding equivalence classes as T- equivalence classes. Also each equivalence class is a transitive T-*system* and hence an irreducible T-*system.*

Let $\{C_\alpha\}_{\alpha \in \Delta}$ be the family of T-equivalence classes. By axiom of choice, there exists $\{W_\alpha\}_{\alpha \in \Delta}$ such that $W_\alpha \in C_\alpha$. In what follows we fix the family of elements $\{W_\alpha\}_{\alpha \in \Delta}$ and for simplicity, we write α instead of W_α for each $\alpha \in \Delta$, that is, we consider Δ, as subset of M.

Let $\alpha \in \Delta$. We define a mapping S_α on M as follows. Let $m \in$ M. Then $m = \beta st$ for some $\beta \in \Delta$ and $s,t \in$ T. Write for $r \in$ T, $mS_\alpha = (\beta st)S_\alpha = \alpha st$. Now clearly S_α is an T-homomorphism.

THEOREM 3.18 : For every X ∈ H, range of X is a union of T-equivalence classes.

***Proof* :** Let $n \in$ range of X. Then there exists an element $m \in$ M such that mX = n. If $n \in C_\alpha$ then $n = \alpha pt$ for some $p \in$ M, $t \in$ T. Let $q \in C_\alpha$. Then $q = \alpha ps$ for some $s \in$ T.
Consider $(mt^{-1}s)X = (mX)t^{-1}s = nt^{-1}s = \alpha ptt^{-1}s = \alpha ps = q$. So $q \in$ range of X.
Thus range of X is a union of T-equivalence classes.

DEFINITION 3.19 : Let X ∈ H. The cardinality of the set of all T-equivalence classes if the range of X is called the ***rank of X.***

NOTE 3.20 : It is clear that rank of X is greater than or equal to 1 for all X ∈ H and for each $\alpha \in \Delta$, S_α has rank 1. We denote the set of all T-homomorphism of rank 1 by U. We note that U does not depend on Δ. Write V = $\{S \in U : \alpha SS = \alpha$ for some $\alpha \in \Delta\}$.

We now characterize the idempotent of rank 1 in H

THEOREM 3.21 : V is the set of all idempotents of rank 1 in H.

Proof : Let S ∈ V. So $\alpha S = \alpha$ for some $\alpha \in \Delta$. Since S has rank 1, the range of S is C_α. Let $m, s \in$ M.
Then $m = \beta st$ for some $\beta \in \Delta$ and $t \in$ T. Assume $\beta S = \alpha pq$ for some $p, q \in$ T.
Now $mS^3 = (\beta st)S^3 = (\beta S)stS^2 = (\alpha pq)stS^2 = (\alpha S)pqstS = \alpha pqstS = (\beta S)stS = (\alpha pq)stS = (\alpha S)pqst = \alpha pqst = (\beta S)st = (\beta st)S = mS$. Since this is true for all $m \in$ M, S is an idempotent. Conversely suppose that S ∈ U is an idempotent. Suppose range of S is C_α.
If $\alpha S = \alpha st$ for some $s, t \in$ T, then $\alpha st = \alpha S = \alpha S^3 = (\alpha st)SS = (\alpha S)stS = (\alpha S)stS = (\alpha st)stS = (\alpha S)stst = \alpha s^3t^3$. So $s = t = e$, Where e is the identity of T. Hence $\alpha S = \alpha$ for some $\alpha \in \Delta$. Therefore S ∈ V.

In the following theorem exhibit a class of primitive idempotents of rank 1 in H.

THEOREM 3.22 : For each $\boldsymbol{\alpha \in \Delta}$, the T-homomorphism $\boldsymbol{S_\alpha}$ is a primitive idempotent in H.

***Proof* :** Let $\alpha \in \Delta$. Clearly S_α is an idempotent in H. Suppose S is an idempotent in H such that $SS_\alpha = S_\alpha S = S$. Let $m, s \in$ M. Then $m = \beta st$ for some $\beta \in \Delta$ and $t \in$ T. Now since mS ∈ M and range of S = range of $S_\alpha = C_\alpha$, we have $mS = \alpha pq$ for some $p, q \in$ T.

Now $(\alpha S)st = (\alpha st)S = (\beta st)S_{\alpha}S = (\beta st)S = (\beta st)S^{3} = (mS)SS = (\alpha pq)SS = (\alpha SS)pq = (\alpha S)pqS = (\alpha pq)S = (\alpha S)pq$. Since M_T is a semi-space, it follows that $s = p$, $t = q$. Therefore, $mS = \alpha pq = \alpha st = mS_{\alpha}$.
Since this is true for all $m \in M$, we have $S = S_{\alpha}$. Therefore S_{α} is a primitive idempotent.

IV. MINIMAL IDEALS:

In this section we study minimal (one sided) ideals in ternary semi groups of T-homomorphism on a semi space containing U. We start with the following.

THEOREM 4.1 : Let X, Z ∈ U. If range of X = range of Z, then there exists *a*Y ∈ U such that YX = Z.

Proof **:** Suppose range of X = range of Z = C_{α} for some $\alpha \in \Delta$. Let $\alpha X = \alpha uv$ and $\alpha Z = \alpha yz$ for some $u, v, y, z \in T$. Let $m,s \in M$. Then $m = \alpha st$ for some $\alpha \in \Delta$ and $t \in T$. Define a mapping Y on M as follows. $mY = (\alpha st)Y = \alpha u^{-1}v^{-1}yzst$. Clearly Y is an T-homomorphism with range C_{α} and hence $Y \in U$.
Now $m(YX) = (mY)X = (\alpha u^{-1}v^{-1}yzst\,)X = (\alpha X)\, u^{-1}v^{-1}yzst = \alpha uv\, u^{-1}v^{-1}yzst = \alpha yzst = (\alpha Z)st = mZ$.
Since this is true for all $m \in M$, we have YX = Z.

THEOREM 4.2: If Δ and Γ are two sets of representative elements of equivalence classes, then for any ternary semigroup G of T-homomorphism on the semispace M_T containing U, $GGS_{\alpha} = GGS_{\beta}$; $\alpha \in \Delta$, $\beta \in \Gamma$ if and only if α and β belong to the same equivalence class.

Proof **:** Suppose $GGS_{\alpha} = GGS_{\beta}$; $\alpha \in \Delta$ and $\beta \in \Gamma$. Let $X \in G$. Then there exists aY ∈ G such that $XXS_{\alpha} = YYS_{\beta}$. Since the range of XXS_{α} is C_{α} and YYS_{β} is C_{β}, we have $C_{\alpha} = C_{\beta}$. So α and β belong to the same equivalence class. Conversely suppose that α and β belong to the same equivalence class.
That is $C_{\alpha} = C_{\beta}$. Now by theorem 4.1, there exists Y, Y' $\in U \subseteq G$ such that $S_{\alpha} = YYS_{\beta}$ and $S_{\beta} = Y'Y'S_{\alpha}$. Therefore we have $S_{\alpha} \in GGS_{\beta}$ and $S_{\beta} \in GGS_{\alpha}$. Hence $GGS_{\alpha} = GGS_{\beta}$. This completes the proof of the theorem.

We now prove the following useful

THEOREM 4.3: Let M_T be a semispace and let *m, n* ∈ M. Then there exists *a*S ∈ U such that *m*S = *n*.

Proof**:** Since $m, n, s, u \in M$, $m = \alpha st$ and $n = \beta uv$ for some $\alpha, \beta \in \Delta$ and $t, v \in T$. Define a mapping S on M as follows. Let $p, q \in M$. Then $p = \delta qr$ for some $\delta \in \Delta$ and $r \in T$. Write $pS = (\delta qr)S = \beta uvs^{-1}t^{-1}qr$. Clearly S is an T-homomorphism of rank 1 and hence $S \in U$. Further $mS = (\alpha st)S = \beta uvs^{-1}t^{-1}st = \beta uv = n$.

NOTE 4.4: By theorem 4.3., we note that if G is a ternary semigroup of T-homomorphisms on a semispace M_T containing U, then M is a transitive G-System and hence an irreducible G-*System.* We now characterize minimal ideals in a ternary semi- group of T-homomorphisms on a semi-space, containing U.

THEOREM 4.5: If G is a ternary semigroup of T-homomorphisms on a semispace M, containing U, then

a) For every $\alpha \in \Delta$, GGS_{α} is a minimal left ideal of G, and G has no other minimal left ideals.

b) For every S ∈ V, SGG is a minimal right ideal of G which is G-isomorphic to M and G has no other minimal right ideals.

Proof**:** (a) Let $\alpha \in \Delta$. We know that GGS_{α} is a left ideal of G. Let L be any left ideal of G such that $L \subseteq GGS_{\alpha}$. Now we shall show that $S_{\alpha} \in L$. Let $X \in L$. So $X \in GGS_{\alpha}$ and hence X has rank 1 with range C_{α}. Since range of X = range of $S_{\alpha} = C_{\alpha}$, by theorem 4.1., there exists aZ ∈ U such that $ZZX = S_{\alpha}$. Therefore $S_{\alpha} \in L$. Hence $L = GGS_{\alpha}$. Thus GGS_{α} is a minimal left ideal.

Let L be any minimal left ideal of G. Let $X \in L$. Then the range of X contains an equivalence class, say C_{α}. So there is an equivalence class C_{δ} such that X maps C_{δ} into C_{α}. Now Now $\delta X = \alpha st$ for some $s, t \in T$. Define Z on M as follows. Let $m \in M$. Now $m = \beta uv$ for $u \in M$ and $v \in T$. Write $mZ = (\beta uv)Z = \delta s^{-1}t^{-1}uv$. Then $Z \in U$, with range C_{δ}. So $Z \in G$. Now $m(ZZX) = (mZ)ZX = ((\beta uv)Z)ZX = (\delta s^{-1}t^{-1}uv)ZX = (\delta X)s^{-1}t^{-1}uvZ = (\alpha sts^{-1}t^{-1}uv)Z = (\alpha uv)Z = (\beta uv)S_{\alpha} = mS_{\alpha}$. So $ZZX = S_{\alpha}$. Thus $S_{\alpha} = ZZX \in L$. So $GGS_{\alpha} \subseteq L$. Since L is a minimal left ideal, $GGS_{\alpha} = L$. Thus every minimal left ideal is of the form GGS_{α} for some $\alpha \in \Delta$.

(b) Let $S \in V$. Then SGG = K is a right ideal of G. Since $S \in V$, there exists an $\alpha \in \Delta$ with $\alpha S = \alpha$ such that range of $S = C_{\alpha}$. Since M is an irreducible G-*system*, $\alpha K = \alpha SGG = \alpha GG = M$. Now consider the mapping $k \rightarrow \alpha kk^{1}$ from K into M. Clearly this mapping is an G-epimorphism.

Suppose $\alpha k_1k_2 = \alpha k_3k_4$ for some $k_1, k_2, k_3, k_4 \in$ K. Now $k_i = SA_i$, i = 1, 2, 3, 4 for $A_i \in$ G. Since $\alpha k_1k_2 = \alpha k_3k_4$, we have $\alpha A_1A_2 = \alpha A_3A_4$. Let $m, s \in$ M. Then $m = \beta st$ for some $\beta \in \Delta$ and $t \in$ T.
Suppose $\beta S = \alpha pq$ for some $p, q \in$ T. Now $mk_1k_2 = (\beta st)SA_1SA_2 = (\beta S)SstA_1A_2 = (\alpha pq)SA_1A_2st = (\alpha A_1A_2)Spqst = (\alpha A_3A_4)Spqst = (\alpha pq)SA_3A_4st = (\beta S)SA_3A_4st = (\beta st)SA_3SA_4 = mk_3k_4$. Since this is true for all $m \in$ M, $k_1 = k_3$, $k_2 = k_4$. Therefore K is G-isomorphic to M. Since M is an irreducible G-*system*, K = SGG is a minimal right ideal of G.

Let K be any minimal right ideal of G. Then clearly K $\subseteq$ U. Let S $\in$ K. Now SKK is a right ideal of G contained in K. Since K is minimal, K = SKK. So there exists $S' \in$ K such that $S = SS'$. Suppose range of S is C_α and assume $\alpha S = \alpha st$ for some $s, t \in$ T. Then $\alpha SS' = \alpha S$ implies $\alpha st = \alpha S = \alpha SS' = (\alpha st)S' = (\alpha S')st$ So $\alpha = \alpha S'$. Hence $S' \in$ V and also since $S' \in$ K, we have S'GG = K. So every minimal right ideal of G is of the form SGG for some S $\in$ V.

THEOREM 4.6: The following are equivalent on a semispace M_T.
[1] H is a ternary group
[2] H is a simple ternary semigroup
[3] H = U.
[4] T is T-isomorphic to M.
[5] Every element of H is a T-isomorphism.

***Proof* :** Suppose H is a simple ternary semigroup. So T has no proper ideals. Since U is three sided ideal of H, we have H = U. If H = U, then the identity mapping I $\in$ H = U. So I has rank 1 and hence range of I = M is the only T-equivalence class. Let $\alpha \in$ M. Define θ: T $\rightarrow$ M as follows $s\theta = \alpha st$. Suppose $s\theta = u\theta$ for some $s, u \in$ T. This implies $\alpha st = \alpha ut$. Since M_T is a semispace, we have $s = t$. therefore θ is one-one. Since M is the only T-equivalence class, we have θ is onto. $(sut)\theta = \alpha(stu)t = (\alpha st)ut = (s\theta)ut$. Thus T is T-isomorphic to M.

Suppose T is T-isomorphic to M. Let θ be an T-isomorphism from T into M. Write $(e)\theta = \alpha \in$ M, where e is the identity of T. Now for any $t, s \in$ T, $s\theta = \alpha st$. Since θ is onto, M is the only T-equivalence class.
Let S $\in$ H. Write $\alpha S = \beta$. Suppose $mS = nS$ for some $m, n \in$ M. Now $m = \alpha st$ and $n = \alpha pq$ for some $s, t, p, q \in$ T. $(\beta st) = (\alpha S)st = (\alpha st)S = mS = nS = (\alpha pq)S = (\alpha S)pq = \beta pq$. Since M_T is a semispace, we have $s = p$ and t = q. Therefore $m = n$. Hence S is one – one. Let $n \in$ M. Now $n = \beta st$ for some $s, t \in$ T.
Put $m = \alpha st$. Now $mS = (\alpha st)S = (\alpha S)st = \beta st = n$. So T is onto.
Therefore every element of H is an T-isomorphim.

V. CONCLUSION

The theory of ternary semi groups can applied to many algebraic structures in pure mathematics like semi groups, Gamma semi groups, Partially ordered semi groups, Partially ordered ternary semi groups, near rings, semi rings and gamma semi rings ect,.

REFERENCES

[1] ANJANEYULU A., Structure and ideal theory of semigroups – Thesis, ANU (1980).
[2] Kasner. E., An extension of the group concept, Bull. Amer. Math. Society, 10(1904), 290-291.
[3] Lehmer. D. H., A ternary analogue of abelian group, Amer. J. Math., 39(1932) 329-338.
[4] Ljapin. E. S., Semigroups, American Math. Society, Providence, Rhode Island (1974).
[5] Ramakotaiah. D., Isomorphisms of near rings of transformations, Jour. London Math. Society, (2), 9(1974), 272-278.

Corresponding Author :

Dr. D. Madhusudhana Rao
M.Sc., M.Phil., Ph. D., D.O.M.
Lecture & Head,
Department of Mathematics,
VSR & NVR College,
Tenali-522201, A.P. India