

## Analysis of Detent Torque in Hybrid Stepping Motors

Jonathan U Agber

Department of Electrical and Electronics Engineering, University of Agriculture, Makurdi, Benue State, Nigeria.

### ABSTRACT

Detent or cogging torque plays an important role in the hybrid stepping motor. This torque is a useful feature in applications where the rotor position must be preserved during stoppage or power failure. It contributes to the static and dynamic characteristics of the hybrid stepping motor. Prediction of detent torque is, therefore, desirable, especially now that the hybrid stepping motors are widely used in office and industrial automation. This paper formulates a method for predicting detent torque of the  $1.8^\circ$  hybrid stepping motor using measured flux-linkage data. It assumes a linear magnetic circuit, where the magnetic stored energy and coenergy are numerically equal. Detent torque is calculated as the differential of the magnetic coenergy with respect to rotor angular position at constant rotor excitation. Validation of the detent torque equation is performed by comparing results obtained from the proposed method with laboratory test results. The predicted results show good agreement with the measured.

**Keywords** - Detent torque, Cogging torque, Stepping motor, Hybrid stepping motor, Incremental motion control

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### I. INTRODUCTION

The hybrid stepping motor (HSM) is the most commonly used motor since it has the advantage of higher efficiency and torque capability. The characteristics of the HSM can be analyzed using the equivalent circuit of the synchronous motor [1, 2]. Detent torque, a periodic oscillating torque, is due to energy variation within a permanent magnet (PM) HSM, with the tendency of the rotor field to align with stator poles even when the stator excitation is removed. It is called by various terms depending on how it works or what causes it, e.g. cogging torque, detent torque, salient-pole torque, reluctance torque, no-current torque, etc [3]. It is desirable when it is used for detenting [2, 3] and undesirable when it causes torque and speed ripples [4 - 8]. Research on detent torque has been mainly focused on methods of reducing the negative effect, i.e. alleviating the cogging torque effects. This shows the high relevance of the cogging torque problem in this class of motor. However, detent torque is equally important, especially in applications requiring the retention of rotor position when the stator windings are turned off. It is, therefore, essential to have a reliable model which allows the calculation of detent torque accurately in design and analysis of HSM. Work on prediction of detent/cogging torque has also been reported [1, 2, 9 - 15]. Chai [2] developed an analytical method that used the stator and rotor tooth geometry to calculate the air gap permeance at the aligned and unaligned positions, which was used to predict cogging torque. He noted that permeance varied in a cosinusoidal manner and that for the  $1.8^\circ$  HSM, the 4<sup>th</sup> harmonic component of permeance is associated with cogging torque. Stuebig and Ponick [12] used the flux-tube approach with 2D finite element analysis (FEA) to determine permeance and noted that the flux-tube approach was the best method for calculating permeance in all conditions except saturation-dependant air gap permeance. Kim et al [7], on minimization of cogging torque, developed the flux-density distribution method, which was used to analytically compute cogging torque without recourse to FEA. van Riesen et al [15] used FEA based on Maxwell's stress tensor method for simulation and claimed that the method is accurate and easy for the analysis of cogging torque and design process. Deodhar et al [11] proposed the flux-mmfm method and claimed to be a true universal technique for cogging torque prediction. Kaiyuan et al [9] simplified the work [11], by eliminating the flux-mmfm diagram and coming up with a function which was used with Maxwell's stress tensor and FEA to predict cogging torque. Most of the cases outlined above used one method or the other to predict flux or flux related parameters that were used for the calculation of detent/cogging torque.

The aim of this paper is not to study the negative effect of cogging torque but to develop a method for predicting detent torque/rotor angular position,  $T_{det}(\theta)$ , characteristics, using measured PM flux-linkage data. The flux-linkage data was measured from a conventional PM HSM in a laboratory set up.

## II. FORMULATION OF THE METHOD

For a conservative electromagnetic system, the total system energy change due to angular displacement of rotor can be expressed as:

$$\Delta W_{elect} = \Delta W_{fld} + \Delta W_{mech} \quad (1)$$

where  $\Delta W_{elect}$  is the electrical input energy supplied to the stator winding,  $\Delta W_{fld}$  is the variation of the stored magnetic energy due to the variation of the electrical energy and/or the mechanical work  $\Delta W_{mech}$  given by the product of the torque and the angular displacement  $\Delta\theta$ . For detent torque calculation, the electrical input power is zero, i.e.  $\Delta W_{elect} = 0$ ; hence the magnetic energy is equal to and opposite the mechanical work  $\Delta W_{fld} = -\Delta W_{mech}$ . For a specific position,  $\Delta\theta$ , this torque can be written as:

$$T_{cog} = - \frac{\Delta W_{fld}(\theta)}{\Delta\theta} \quad (2)$$

Kaiyuan et al [9] used the flux-MMF diagram to show that:

$$T_{cog} = - \frac{dW_{fld}(\theta)}{d\theta} = - \lim_{\Delta\theta \rightarrow 0} \frac{\Delta W_{fld}}{\Delta\theta} \quad (3)$$

Watterson [14] had shown that the calculation of the system energy related to cogging torque production can be expressed as:

$$W_{fld} = - \frac{1}{2} (H l_m) (B_r S_m) = - \frac{1}{2} F_m \Phi_r \quad (4)$$

where H is the magnetic field intensity,  $B_r$  is the component of remanent magnetic flux density,  $l_m$  is the magnet length along the magnetization direction,  $S_m$  is the magnet area perpendicular to the magnetization direction,  $F_m$  is the magnet's MMF and  $\Phi_r$  is the component of remanent flux inside the magnet that is perpendicular to the magnetization direction.

Substituting equation (4) into (3) and noting that only  $\Phi_r$  is a function of  $\theta$ , gives:

$$T_{det}(\theta) = - \frac{dW_{fld}(\theta)}{d\theta} = \frac{1}{2} F_m \frac{d\Phi_r(\theta)}{d\theta} \quad (5)$$

Equation (5) is the general equation for detent torque.

## III. DETERMINATION OF PM MMF

The use of the present method on the PM hybrid motors requires the estimation of the PM mmf. One way of doing this is to calculate the PM flux density  $B_m$  from the voltage,  $V$ , measured when the HSM was driven with the stator windings open-circuited.

$$B_m = \frac{\sqrt{2} V}{2 \pi f_r N S_m} \quad (6)$$

where  $N$  is the number of turns of stator winding per phase and  $f_r$  - frequency of the induced voltage.

Using the B-H characteristic of the magnetic material, ALCOMAX III, the magnetic field intensity was found, from which the PM mmf was calculated as:

$$F_m = H_m l_c \quad (7)$$

where  $l_c$  is length of the mean magnetic flux path.

#### IV. MEASUREMENT OF FLUX-LINKAGE DATA

The cross-sectional views of the 1.8° HSM under test are shown in Figure 1. The motor has four pairs of stator poles carrying bifilar windings A-A', B-B', C-C' and D-D'. For the purpose of this measurement, phase windings A-A' and B-B' were connected in series to form winding A-B'.

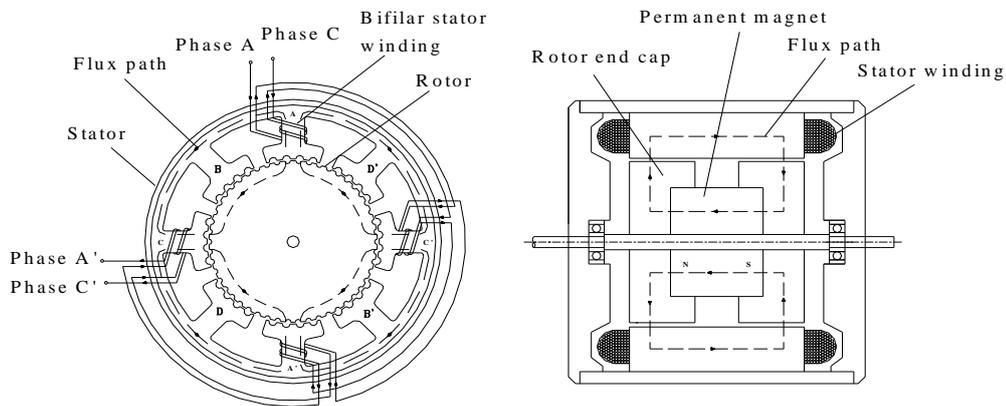


Figure 1. The cross-sectional views of the 1.8° hybrid stepping motor showing the PM flux path and phase winding connections.

A static rig comprising a displacement transducer connected to one end of the shaft of the HSM and on the other end, a torque transducer and a dynamometer, was used. An analogue flux meter was connected to the winding A-B' and torque meter was connected to the torque transducer. The displacement transducer was connected to the X-input of the X-Y plotter while both flux and torque meters were connected to the Y-input via a change-over switch.

The method for measuring the flux-linkage that produce cogging torque is described below.

By connecting the winding A-A' and B-B' in series, the winding A-B' had twice the number of turns of a phase winding and covered all the four pole-pairs. With this arrangement the variation of flux linking the four poles could be observed.

The flux measurement was done by displacing the rotor from the aligned position of pole-pair A-A' to misaligned position and back to the aligned position, covering a displacement of half a rotor tooth pitch or 3.6°. The forward and backward flux and torque curves were plotted one after the other. The curves were digitised and averaged to give the flux-linkage and cogging torque curves as shown in equations (8) and (9). The measured flux-linkage was halved to give the flux linking one phase winding and is shown in Figure 2.

$$\phi_{det}(\theta) = 0.5\{\phi_f(\theta) + \phi_b(\theta)\} \quad (8)$$

$$T_{det}(\theta) = 0.5\{T_f(\theta) + T_b(\theta)\} \quad (9)$$

where the subscripts  $f$  and  $b$  denote forward and backward curves.

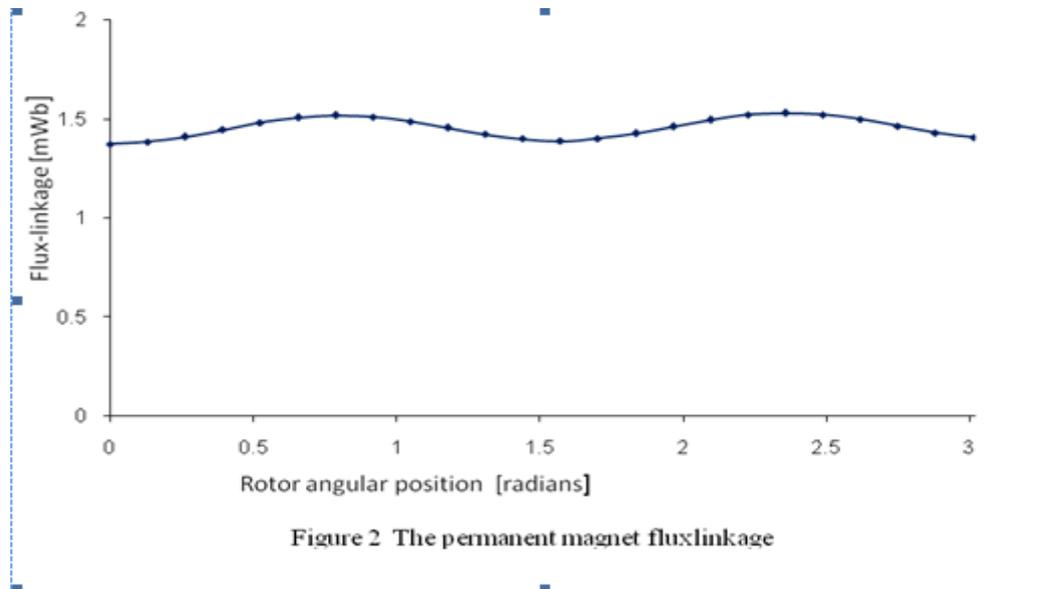


Figure 2 The permanent magnet fluxlinkage

### V. CALCULATION OF DETENT TORQUE

A Matlab program based on equation (5) was used to read the flux-linkage data, fit a cubic spline through it and calculate the slopes at each  $\theta$  point. These slopes were multiplied by one-half of the PM mmf to produce detent torque/rotor angular position  $T_{det}(\theta)$ , shown in Figure 3.

The maximum average error per experimental point is 0.0988Nm, which is better than 5.69% of the peak detent torque.

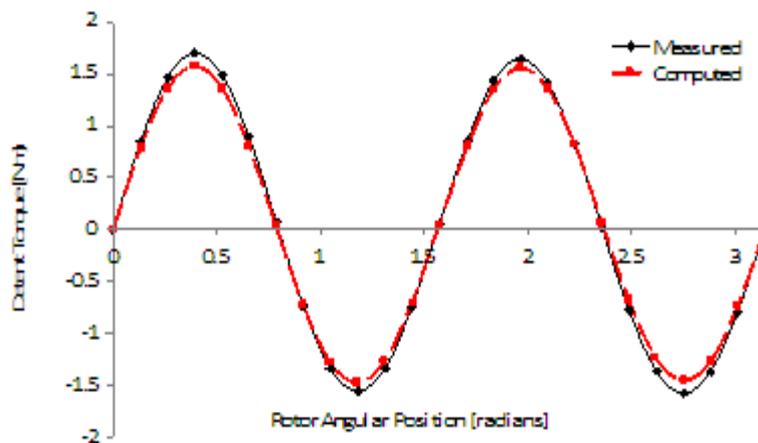


Figure 3 Measured and computed detent torque/rotor angular position

### VI. VALIDATION OF THE METHOD

To validate the method of measurement of the flux-linkage data and prediction of detent torque/rotor angular position, the PM in the HSM rotor was replaced with a wound soft-iron former shown in Figure 4. Tests similar to those described in section 4 were carried out at various rotor excitations, ranging from 1.0 – 3.5 Amps in steps of 0.5Amp, corresponding to 240 – 840At. The measured  $\Phi(F, \theta)$  curves are shown in Figure 5 and the measured and predicted  $T_{det}(F, \theta)$  curves are shown in Figure 6. The average error between the measured and computed  $T_{det}(F, \theta)$  is 1.4% at  $F = 240At$ , 3.8% at 600At and 5.2% at 840At.

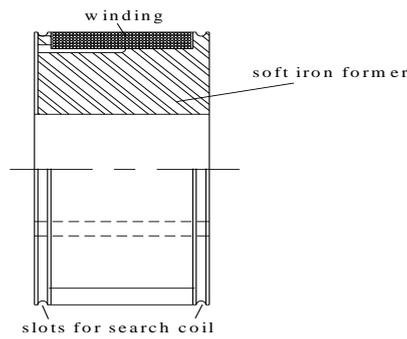
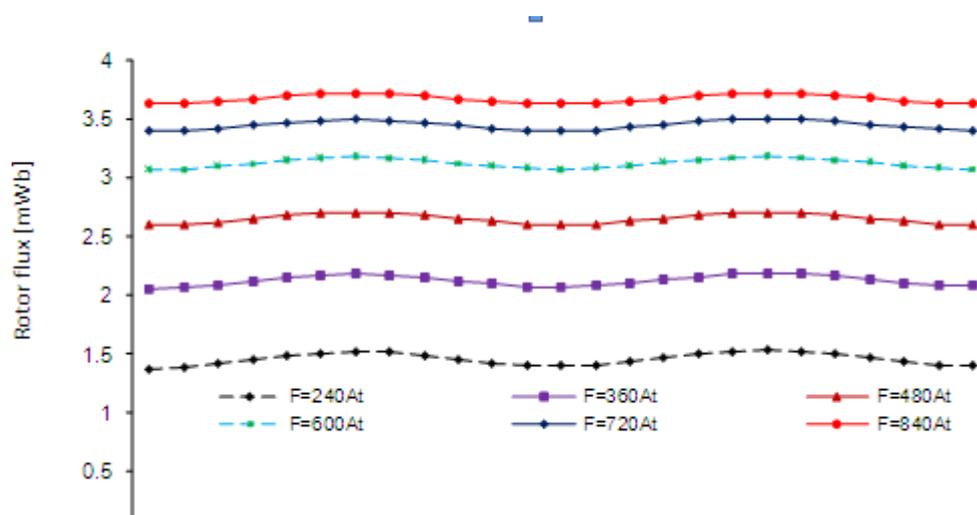


Figure 4. The soft-iron former with winding to replace the permanent magnet (Material = MAXIMAG)



### VII. DISCUSSION

The flux-linkage curves shown in Figures 2 and 5 have the same waveform, i.e. they resemble cosinusoidal waves while the detent torque curves of Figures 3 and 6 resemble sinusoidal waves. The average error between measured and predicted detent torque/rotor angular position for the wound-rotor HSM lies between 1.4% for  $F = 240\text{At}$  and 5.2% for  $F = 840\text{At}$ . This may be due to saturation at higher rotor mmf levels. For the PM HSM, the discrepancy between the measured and computed results may be due to the error in estimating the PM mmf from the B-H characteristic of the magnetic material. From the overall results obtained, it is evident that the proposed method of measuring the flux-linkage data that gives rise to detent/cogging torque is valid and can be used for the analysis of the characteristics of the HSM.

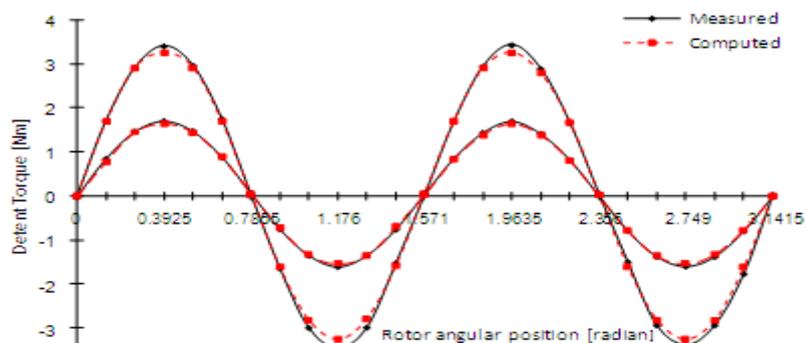


Figure 6 Measured and computed detent torque at rotor mmf= 240At and 600At

### VIII. CONCLUSION

A method for estimating detent/cogging torque using measured flux-linkage data has been presented. The method used for measuring the flux data that constitute detent torque is shown. In the method, the flux is measured from the series connection of the phase windings. The measured flux-linkage data is differentiated numerically to produce magnetic coenergy, which when multiplied by one-half of the rotor mmf yields detent torque. As it has been clearly demonstrated, the measured and computed characteristics are in good agreement. This shows that the method is valid and can be used for the prediction of detent torque/rotor angular position characteristics for the HSM.

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**Author's Profile**



**Jonathan Uhaa Agber** received M.Sc. degree in Electromechanical Engineering from the Moscow Power Engineering Institute, Moscow, Russia in 1976, PhD degree in Electrical Engineering from the University of Newcastle-Upon-Tyne in the UK in 1985. He is a Senior Lecturer in the Department of Electrical/Electronics Engineering, The Federal University of Agriculture, Makurdi, Benue State, Nigeria. He is a registered Engineer and Member of the Nigerian Society of Engineers. His research interest includes CAD simulation and control of incremental motion devices and software development.