A New Approach Based on Sinusoidal PWM Inverter with PI Controller for Vector control of Induction Motor

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ABSTRACT

This paper is mainly based on the vector control of Induction motor (IM). The analysis of mathematical model of IM, with the powerful simulation modeling capabilities of Matlab/Simulink is given here. The entire module of this IM is divided into several independent functional modules such as IM body module, Inverter module, coordinate transformation module and Sinusoidal pulse width modulation (SPWM) production module and so on. With the help of this module we can analyze a variety of different simulation waveforms, which provide an effective means for the analysis and design of the IM control system.

Keywords - Clarke Transformation, Park Transformation, Mathematical model of Induction motor, Sinusoidal pulse width modulation, Vector control.

I. INTRODUCTION

Among all types of ac machine, the Induction machine (IM), particularly the cage type, is most commonly used in industry. These machines are very economical, rugged and reliable and available in the ranges of fractional horse power (FHP) to multi-megawatt capacity. Low-power FHP machines are available in single-phase, but poly-phase (three-phase) machines are used most often in variable-speed drives. In vector control the IM can be controlled like a separately excited dc motor, brought a renaissance in the high-performance control of ac drives. In vector control the Magnetic field can be decoupled to get a good control performance, hence the torque and flux can be controlled independently [6]. The analysis of mathematical model of IM, with the powerful simulation modeling capabilities of Matlab/Simulink, the IM control system will be divided into several independent functional modules such as IM motor module, inverter module, coordinate transformation module and SPWM production module and so on. By combining these modules, the simulation model of IM control system can be built.

II. STATIONARY TO ROTATING REFERENCE FRAME TRANSFORMATION

Three phase ac machines conventionally use phase variable notation. For a balanced three phase, star connected machine we have:

\[ f_a + f_b + f_c = 0 \]

Fig.1. Current space vector in stationary and rotating reference frame
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Where \( f_a, f_b, \) and \( f_c \) denote any one of current, voltage and flux linkage [1].

### III. CLARK TRANSFORMATION

The transformation from three-phase to two-phase quantities can be written in matrix form as:

\[
\begin{bmatrix}
  f_a \\
  f_b \\
  f_c 
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2 \\
  -1/2 & \sqrt{3}/2 & -1/2
\end{bmatrix} \begin{bmatrix}
  f_\alpha \\
  f_\beta \\
  f_\gamma
\end{bmatrix}
\]

Where \( f_\alpha \) and \( f_\beta \) are orthogonal space phasor. The stator current space vector is defined as the complex quantity [1]:

\[
i_s = i_\alpha + f_i \beta
\]

It is possible to write (2) more compactly as:

\[
i_s = 2/3(i_a + a i_b + a^2 i_c)
\]

Where \( i_a, i_b, \) and \( i_c \) are instantaneous phase currents and \( a \) is a vector operator that produces a vector rotation of \( = (2\pi)/3 \). The choice of the constant in the transformation of equation (1) is somewhat arbitrary. Here, the value of \( 2/3 \) is chosen. Its main advantage is that magnitudes are preserved across the transformation. The inverse relationship is written as:

\[
\begin{bmatrix}
  f_\alpha \\
  f_\beta \\
  f_\gamma
\end{bmatrix} = \frac{1}{2/3} \begin{bmatrix}
  0 & 1 & -1/2 \\
  \sqrt{3}/2 & -1/2 & 0 \\
  -\sqrt{3}/2 & -1/2 & 0
\end{bmatrix} \begin{bmatrix}
  f_a \\
  f_b \\
  f_c
\end{bmatrix}
\]

Transformation equations (1) and (4) are known as the Forward Clarke Transformation and Reverse Clarke Transformation respectively.

### IV. PARK TRANSFORMATION

For the transformation of stationary stator variables to rotating reference frame we use park transformation and then we write in matrix form as:

\[
\begin{bmatrix}
  f_\alpha \\
  f_\beta
\end{bmatrix} = \begin{bmatrix}
  \cos \theta_r & \sin \theta_r \\
  -\sin \theta_r & \cos \theta_r
\end{bmatrix} \begin{bmatrix}
  f_a \\
  f_b
\end{bmatrix}
\]

Where \( \theta_r \) is the angle between stationary reference frames and rotating reference frame is shown in Fig.1. Transformation equation (5) is known as Park transformation.

### V. MATHEMATICAL MODEL OF INDUCTION MOTOR

The mathematical model of an IM [6] in terms of phase voltages for stator and rotor can be written as:

For stator:

\[
v_{abc} = R_i i_{abc} + p \lambda_{abc}
\]

For rotor:

\[
v_{abc} = R_r i_{abc} + p \lambda_{abc}
\]

The flux linkage can be written as in matrix form:

\[
\begin{bmatrix}
  \lambda_{abc} \\
  \lambda_{abc}
\end{bmatrix} = \begin{bmatrix}
  L_{ss} & L_{sr} \\
  L_{sr} & L_{rr}
\end{bmatrix} \begin{bmatrix}
  i_{abc} \\
  i_{abc}
\end{bmatrix}
\]

The flux linkage equations are:

\[
\lambda_{abc} = L_{ss} i_{abc} + L_{sr} i_{abc}
\]
\[ \lambda_{\alpha\beta\gamma\delta} = L_{rs} i_{\alpha\beta\gamma\delta} + L_{rr} i_{\alpha\beta\gamma\delta} \]  \hspace{1cm} (10)

Where,

\[ L_{ss} = \begin{bmatrix} l_{ls} + l_{ms} & -\frac{1}{2} l_{ms} & -\frac{1}{2} l_{ms} \\ -\frac{1}{2} l_{ms} & l_{ls} + l_{ms} & -\frac{1}{2} l_{ms} \\ -\frac{1}{2} l_{ms} & l_{ls} & l_{ls} + l_{ms} \end{bmatrix} \]  \hspace{1cm} (11)

\[ L_{sr} = L_{qr} \begin{bmatrix} \cos \theta_r & \cos(\theta_r + 120^\circ) & \cos(\theta_r - 120^\circ) \\ \cos(\theta_r - 120^\circ) & \cos \theta_r & \cos(\theta_r + 120^\circ) \\ \cos(\theta_r + 120^\circ) & \cos(\theta_r - 120^\circ) & \cos \theta_r \end{bmatrix} \]  \hspace{1cm} (12)

\[ L_{rr} = \begin{bmatrix} \cos \theta_r & \cos(\theta_r - 120^\circ) & \cos(\theta_r + 120^\circ) \\ \cos(\theta_r + 120^\circ) & \cos \theta_r & \cos(\theta_r - 120^\circ) \\ \cos(\theta_r - 120^\circ) & \cos(\theta_r + 120^\circ) & \cos \theta_r \end{bmatrix} \]  \hspace{1cm} (13)

\[ L_{rr} = \begin{bmatrix} l_{lr} + l_{mr} & -\frac{1}{2} l_{mr} & -\frac{1}{2} l_{mr} \\ -\frac{1}{2} l_{mr} & l_{lr} + l_{mr} & -\frac{1}{2} l_{mr} \\ -\frac{1}{2} l_{mr} & l_{lr} & l_{lr} + l_{mr} \end{bmatrix} \]  \hspace{1cm} (14)

Now applying the transformations (1) and (5) to voltages, flux linkages and currents in the equations (6), (7), (9) and (10) we get a set of simple transformed equations as:

The voltage equation in q-d reference frame:

\[ v_{qs} = r_s i_{qs} + p \lambda_{qs} + \omega \lambda_{ds} \]  \hspace{1cm} (15)

\[ v_{ds} = r_s i_{ds} + p \lambda_{ds} - \omega \lambda_{qs} \]  \hspace{1cm} (16)

\[ v_{qr} = \tau_r i_{qr} + p \lambda_{qr} + (\omega - \omega_r) \lambda_{dr} \]  \hspace{1cm} (17)

\[ v_{dr} = \tau_r i_{dr} + p \lambda_{dr} - (\omega - \omega_r) \lambda_{qr} \]  \hspace{1cm} (18)

The flux linkage equation in q-d reference frame:

\[ \lambda_{qs} = (l_{ls} + L_M) i_{qs} + L_M i_{qr} \]  \hspace{1cm} (19)

\[ \lambda_{ds} = (l_{ls} + L_M) i_{ds} + L_M i_{dr} \]  \hspace{1cm} (20)

\[ \lambda_{qr} = L_M i_{qs} + (l_{lr} + L_M) i_{qr} \]  \hspace{1cm} (21)

\[ \lambda_{dr} = L_M i_{ds} + (l_{lr} + L_M) i_{dr} \]  \hspace{1cm} (22)

Here, \( l_{ls} \) and \( L_M \) are leakage and mutual inductance respectively. \( \omega \) is the synchronous speed and \( \omega_r \) is the speed of the rotor.

The electromagnetic torque \( (T_e) \) for rotor current can be written as:

\[ T_e = (3/2)(P/2)(\lambda_{drm} i_{qr} - \lambda_{dsm} i_{dr}) \]  \hspace{1cm} (23)

The electromagnetic torque \( (T_e) \) for stator current can be written as:
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\[ T_e = \frac{3}{2}(P/2)(\lambda_{dm}^2 q^2 - \lambda_{dm}^2 d^2) \]  

(24)

The equation for motor dynamics is:

\[ T_e = J_p \omega_r + B \omega_r + T_i \]  

(25)

VI. VECTOR CONTROL

Vector control is also known as decoupling or field oriented control. Vector control decouples three-phase stator current into two phase d-q axis current, one producing flux and other producing torque [2]. This allows direct control of flux and torque [5]. In case of vector control of an IM drives operates similarly like a separately excited dc motor drive. In a dc machine, neglecting the armature reaction effect and field saturation, the developed torque is given by

\[ T_e = K_t I_a I_f \]  

(26)

Where \( I_a \) = armature current and \( I_f \) = field current. The construction of a dc machine is such that the field flux \( \varphi_f \) produced by the current \( I_f \) is perpendicular to the armature flux \( \varphi_a \), which is produced by the armature current \( I_a \) [3]. These space vectors, which are stationary in space, are orthogonal or decoupled in nature. So in case of an IM the phase currents are decoupled in q-d reference frame for vector controlling. The dc motor model is shown in the fig. 2.

![Fig.2. Dc motor model](image)

![Fig.3. Phasor diagram of a field orientated system](image)

When three-phase voltages are applied to the machine, they produced three-phase fluxes both in the stator and the rotor. The three-phase fluxes can be represented in a two-phase stationary (\( \alpha - \beta \)) frame. If these two phase fluxes along (\( \alpha - \beta \)) axes are represented by a single-vector then all the machine flux will be aligned along that vector. This vector is commonly specified as d-axis which makes an angle with the stationary frame \( \alpha \)-axis, as shown in the fig. 3. The q-axis is set perpendicular to the d-axis. The flux along the q-axis in case will obviously zero. The phasor diagram fig.3 presents these axes. When the machine input current change sinusoidally in time, the angle keeps changing. Thus the problem is to know the angle accurately, so that the d-axis of d-q frame is locked with the flux vector.

The controller part of the vector control of IM is inverse transformed from q-d reference frame to \( \alpha - \beta \) reference frame and then to the stationary phase voltage frame which is then given through a inverter to the machine model part.
VII. SINUSOIDAL PWM

Fig. 4 shows circuit model of three phase PWM inverter [5] and Fig. 4 shows waveforms of carrier signal (V \text{tri}) and control signal (V \text{control}), inverter output line to neutral voltages are VAO, VBO, VCO, inverter output line to line voltages are VAB, VBC, VCA respectively.

![Three phase PWM inverter](image1)

The inverter output voltages are determined as follows:

When \( V_{\text{control}} > V_{\text{tri}} \), \( V_{AO} = V_{DC}/2 \)

When \( V_{\text{control}} < V_{\text{tri}} \), \( V_{AO} = -V_{DC}/2 \)

Where \( V_{AB} = V_{AO} - V_{BO} \), \( V_{BC} = V_{BO} - V_{CO} \), \( V_{CA} = V_{CO} - V_{AO} \).

VIII. FIGURES AND TABLES

Here we take the base speed as 200rpm and we observe from the waveform of the speed that the rotor speed almost gives the desired response. The total simulation time of the motor is \( t=0.4 \) sec. In current waveform we see that the three phases current almost perfectly merge with each other. We also observe the electromagnetic torque, inverter output voltage waveform and dq axis current waveform.

![Waveforms of three phase sinusoidal PWM inverter](image2)

We take Stator resistance \((r_s) = 6.03\), Rotor resistance \((r_r) = 6.085\), Stator inductance \((L_s) = 489.3e-3\), Rotor inductance \((L_r) = 489.3e-3\), Poles \((P) = 4\),
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Fig 6: Three Phase Current response curve

Fig 7: Speed Response curve

Fig 8: Electromagnetic Response Curve

Fig 9: Inverter Output Voltage Waveform

Fig 10: d-axis current Waveform
IX. CONCLUSION

In this paper vector control has been described in adequate detail and has been implemented on IM in real time. The performance of the IM is quite satisfactory by using PI controller and Sinusoidal pulse width modulated inverter. The selection of the value of the proportional gain and integral gain has a significant impact on the model performance. By varying the inverter voltage we can get steady response for high speeds also. The performance of vector control is quite satisfactory for achieving fast reversal of IM even at very high speed range.

REFERENCES